Advanced Studies in Pure Mathematics 17, 1989
Algebraic Number Theory — in honor of K. Iwasawa
pp. 97–137

Iwasawa Theory for p-adic Representations

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To Professor Kenkichi Iwasawa on his seventieth birthday

Several years ago Mazur and Wiles proved a fundamental conjecture of Iwasawa which gives a precise link between the critical values of the Riemann zeta function (and, more generally, Dirichlet L-functions) and the ideal class groups of certain towers of cyclotomic fields. Probably the first hint of such a link is Kummer's well-known criterion for irregularity of primes. In Iwasawa's theory one defines for each prime p certain modules over the Iwasawa algebra Λ (which we will describe in Section 1). Iwasawa's conjecture then relates the structure of these Λ-modules to the p-adic L-functions constructed by Kubota and Leopoldt which interpolate critical values of Dirichlet L-functions. Mazur realized that, following Iwasawa's model, one could formulate a similar conjecture for an elliptic curve E (defined over $\boldsymbol{Q}$) and for any prime p where E has good, ordinary reduction. Mazur and Swinnerton-Dyer constructed p-adic L-functions attached to E for such p (assuming E is a Weil curve). The Iwasawa modules which Mazur's conjecture relates to these p-adic L-functions are defined in terms of Selmer groups for E, again in towers of cyclotomic fields. This time the hint of such a relationship is the Birch and Swinnerton-Dyer conjecture.

There are now several other cases where p-adic analogues of complex L-functions have been constructed—for example, Manin's p-adic L-functions attached to classical modular forms. It seems worthwhile then to search for appropriate Iwasawa modules in a much more general context and that is our purpose in this paper. We will consider a compatible system of l-adic representations $V=\{V_l\}$ over $\boldsymbol{Q}$. Thus V_l is a finite dimensional vector space over $\boldsymbol{Q}_l$ (the l-adic numbers) of dimension $d=d_V$ on which $G_{\boldsymbol{Q}}=\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$ acts. (For any field k, G_k denotes $\mathrm{Gal}(\bar{k}/k)$, where $\bar{k}$ is an algebraic closure of k.) If q is any prime, then one can also consider V_l as a representation space of $G_{\boldsymbol{Q}_q}$ by choosing a place $\bar{q}$ of $\bar{\boldsymbol{Q}}$ over q and identifying $G_{\boldsymbol{Q}_q}$ with the decomposition group for that place. We will usually assume that p is an "ordinary" prime for V in the follow-

Received February 5, 1988.