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On p-adic L-functions Attached to Motives over $\boldsymbol{Q}$

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To K. Iwasawa

Introduction

It is a pleasure to dedicate this paper to K. Iwasawa in recognition of his great work on the mysterious connexion between special values of the Riemann zeta function and certain Galois modules, which we now call Iwasawa modules. His ideas have enormously influenced our own work, and the work of many others. If one seeks to generalize Iwasawa's work, one of the first problems which arises is that of constructing p-adic analogues of the complex L-functions of number theory. It is this question which we address in this highly conjectural paper. We have been tempted into making such broad and sweeping conjectures because of the success and usefulness of similar general conjectures about complex L-functions, which were made by Serre [10], and refined by Deligne [2], in 1970. The framework in which we work, following [10], is that of motives over $\boldsymbol{Q}$—roughly speaking, such a motive V consists of a compatible system of l-adic representations of the Galois group $G=G(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$, together with a suitable Hodge structure at the infinite prime (see § 2 for more details). Let $L(V, s)$ be the complex L-function attached to V by Serre. The definition of $L(V, s)$ as an Euler product of factors determined by purely local data is simply not applicable in the p-adic case, and so, like all previous authors, we are forced to use p-adic interpolation to define the p-adic analogue of $L(V, s)$. Our conjectures are subject to two important restrictions on V and the prime p. Firstly, we must assume that $L(V, s)$ admits at least one critical point in the sense of Deligne [3]. Secondly, we must assume that V is ordinary at p (see § 4). There is little doubt that p-adic analogues of $L(V, s)$ should exist without these restrictions, but we are unable to formulate precise conjectures at present. For simplicity, we have deliberately not treated the case of motives defined over a finite extension of $\boldsymbol{Q}$ in this paper. When $F\neq\boldsymbol{Q}$, new features arise in the theory because we must take our p-adic L-functions

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