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## Theory of Automorphic Forms of Weight 1

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## Dedicated to Professor Setsuya Seki on his 60th birthday

In this paper a brief exposition is given of some recent developments in the theory of automorphic forms of weight 1 of a complex variable and their applications to number theory. The main contents of this paper are based on my lectures given at Kobe University in 1982–86, at Nagoya University in 1983, and at Tokyo Metropolitan University in 1984.

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