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Note on Stable Homotopy Types of Stunted Quaternionic Spherical Space Forms

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§ 1. Introduction

Let $H_m=\{x, y \mid x^{2^{m-1}}=y^2, xyx=y\}$ be the generalized quaternion group of order 2^{m+1} $(m \geqq 2)$. An element of H_m is uniquely expressed as $x^u y^v$ for $0 \leqq u < 2^m$, $v=0, 1$. Let $d_1: H_m \to S^3 = Sp(1) = SU(2)$ be the natural inclusion map defined by $d_1(x)=\exp(2\pi i/2^m)$, $d_1(y)=j$. Then H_m acts freely on the unit sphere S^{4n+3} in the quaternion $(n+1)$-space H^{n+1} by the diagonal action $(n+1)d_1: H_m \to Sp(n+1)$. The quotient manifold S^{4n+3}/H_m is called the quaternionic spherical space form and is denoted by $N^n(m)$. If $n \geqq 0$, we have the natural inclusion map $N^{k-1}(m) \subset N^{n+k}(m)$, and denote by $N_k^{n+k}(m)$ the quotient space $N^{n+k}(m)/N^{k-1}(m)$.

The purpose of this note is to study the stable homotopy types of the stunted quaternionic spherical space forms $N_k^{n+k}(m)$. We have

Theorem 1.1. *If $N_j^{n+j}(m)$ and $N_k^{n+k}(m)$ are of the same stable homotopy type, then* $j \equiv k \mod 2^{2n-2}$.

This is proved in the way of H. Ōshima [10, Theorem 8.4] (cf. [8, Theorem 1.1]), and is a generalization of the Ōshima's result. As for the converse, we obtain

Theorem 1.2. *If* $j \equiv k \mod 2^{2n+m-2+\varepsilon}$, *then $N_j^{n+j}(m)$ and $N_k^{n+k}(m)$ are of the same stable homotopy type, where $\varepsilon=1$ if n is odd, and $\varepsilon=0$ if n is even* >0.

This is a consequence of the results of M. F. Atiyah [2, Proposition 2.6], H. Ōshima [10, Theorem 2.1 and Proposition 8.2] and [7, Corollary 1.7], and is also a generalization of Theorem 8.3 (iii) in [10].

We recall in Section 2 the representation rings $R_F(H_m)$ of H_m, where F denotes the field R of the real numbers or the field C of the complex numbers, according to [4], [5], [6] and [11]. In Section 3 we study Adams operations [1] in $K_F(N^n(m))$. The proofs of Theorems 1.1 and 1.2 depend

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