Advanced Studies in Pure Mathematics 9, 1986
Homotopy Theory and Related Topics
pp. 259–271

On p-Sylow Subgroups of Groups of Self Homotopy Equivalences of Sphere Bundles over Spheres

Mamoru Mimura and Norichika Sawashita

Dedicated to Professor Nobuo Shimada on his 60th birthday

Introduction

The set $\mathscr{E}(X)$ of homotopy classes of homotopy equivalences of a space X to itself forms a group under composition of maps. This group $\mathscr{E}(X)$ has been investigated by several authors (e.g. [2], [7] and [12]).

In the case where X is an S^m-bundle over S^n, the group $\mathscr{E}(X)$ has been investigated for $X=V_{n,2}$ and $W_{n,2}$ by Y. Nomura [10] and for X with $3<m+1<n<2m-2$ by S. Sasao [13], where $V_{n,2}=O(n)/O(n-2)$ and $W_{n,2}=U(n)/U(n-2)$ are the real and complex Stiefel manifolds respectively.

In this note, we study the p-Sylow subgroup of $\mathscr{E}(X)$ for an S^m-bundle X over S^n with a mod p H-structure such that $i_{(p)}\colon S^m_{(p)}\to X_{(p)}$ is an H-map, where m and n are odd integers, $S^m_{(p)}$ and $X_{(p)}$ are localizations of S^m and X at $\{p\}$ respectively and $i_{(p)}$ is the localization of the inclusion $i\colon S^m\subset X$ at $\{p\}$. Our main result is as follows:

Theorem 4.5. *Let m and n be odd integers such that $3\leqq m<n-1$, and let $S^m\xrightarrow{i}X\xrightarrow{q}S^n$ be an S^m-bundle over S^n. Let p be an odd prime. If $S^m_{(p)}$ and $X_{(p)}$ are H-spaces such that $i_{(p)}\colon S^m_{(p)}\to X_{(p)}$ is an H-map, then the group $\mathscr{E}(X)$ is a finite group with a unique p-Sylow subgroup $\tilde{S}_p$ given by the semi direct product*

$$\tilde{S}_p\cong\pi_{m+n}(X;p)\underset{T}{\times}\pi_n(S^m;p),$$

where $\alpha T\beta=\alpha+i\circ\beta\circ q\circ\alpha$ for $\alpha\in\pi_{m+n}(X;p)$ and $\beta\in\pi_n(S^m;p)$.

In Section 1, we determine the p-Sylow subgroup of $\mathscr{E}(S^m\cup e^n)$ (Proposition 1.3). In Section 2, we define a homomorphism $j^1\colon\mathscr{E}(X)\to\mathscr{E}(K)$ and study the p-Sylow subgroup of Im j^1 (Lemma 2.7). In Section

Received January 30, 1985.