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The Orbital Decomposition of Some Prehomogeneous Vector Spaces

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Introduction

Let G' be a semi-simple algebraic group, $\rho': G' \to GL(V)$ its finite-dimensional rational representation, all defined over $\mathbf{C}$. Then, we have $\rho' = \rho_1 \oplus \cdots \oplus \rho_k$, $V = V_1 \oplus \cdots \oplus V_k$ where $\rho_i: G' \to GL(V_i)$ is an irreducible representation for $i = 1, \cdots, k$. Put $G = GL(1)^k \times G'$ and let ρ be the composition of ρ' and the scalar multiplications $GL(1)^k$ on each irreducible components. In [7], the classification of such triplets (G, ρ, V) which admit only a finite number of orbits has been discussed. To complete this classification, one must give the orbital decomposition of some spaces, which will be done in this paper. We give the orbital decomposition of the following spaces. We use the same notations as in [7] (See Definition 1.10 in [7]).

(1.1) $\overset{G_2}{\circ}\overset{\Lambda_2}{\text{———}}\overset{2}{\circ}$, (1.2) $\overset{1}{\circ}\overset{\Lambda}{\text{———}}\overset{\mathrm{Spin}(7)}{\circ}\overset{\Lambda'}{\text{———}}\overset{2}{\circ}$, (1.3) $\overset{\mathrm{Spin}(7)}{\circ}\overset{\Lambda}{\text{———}}\overset{2}{\circ}$,

(1.4) $\overset{\mathrm{Spin}(7)}{\circ}\overset{\Lambda}{\text{———}}\overset{3}{\circ}$, (1.5) $\overset{1}{\circ}\overset{\Lambda}{\text{———}}\overset{\mathrm{Spin}(8)}{\circ}\overset{\Lambda'}{\text{———}}\overset{3}{\circ}$,

(1.6) $\overset{1}{\circ}\overset{\Lambda}{\text{———}}\overset{\mathrm{Spin}(8)}{\circ}\overset{\Lambda'}{\text{———}}\overset{2}{\circ}$, (1.7) $\overset{1}{\circ}\overset{\Lambda}{\text{———}}\overset{\mathrm{Spin}(10)}{\circ}\overset{\Lambda'}{\text{———}}\overset{3}{\circ}$

(1.8) $\overset{\mathrm{Sp}(n)}{\circ}\text{———}\overset{\mathrm{SO}(3)}{\circ}$, (1.9) $\overset{1}{\circ}\overset{\Lambda_1}{\text{———}}\overset{\mathrm{Sp}(2)}{\circ}\overset{\Lambda_2}{\text{———}}\overset{2}{\circ}$,

(2.1) $\overset{5}{\circ}\overset{\Lambda_2}{\text{———}}\overset{2}{\circ}$, (2.2) $\overset{6}{\circ}\overset{\Lambda_2}{\text{———}}\overset{2}{\circ}$, (2.3) $\overset{7}{\circ}\overset{\Lambda_2}{\text{———}}\overset{2}{\circ}$,

(2.4) $\overset{1}{\circ}\text{———}\overset{7}{\circ}\overset{\Lambda_2}{\text{———}}\overset{2}{\circ}$, (2.5) $\overset{1}{\circ}\text{———}\overset{5}{\circ}\overset{\Lambda_2}{\text{———}}\overset{2}{\circ}$, (2.6) $\overset{2}{\circ}\overset{\Lambda_2}{\text{———}}\overset{5}{\circ}\text{———}\overset{2}{\circ}$,

(2.7) $\overset{4}{\circ}\overset{\Lambda_2}{\text{———}}\overset{n}{\circ}$ $(n=3, 4)$, (2.8) $\overset{1}{\circ}\text{———}\overset{4}{\circ}\overset{\Lambda_2}{\text{———}}\overset{n}{\circ}$ $(n=3, 4)$,

(2.9) $\overset{1}{\circ}\overset{\Lambda_3}{\text{———}}\overset{7}{\circ}\text{———}\overset{2}{\circ}$,

(3.1) $\overset{1}{\circ}\text{———}\overset{n}{\circ}\overset{\Lambda_2}{\text{———}}\overset{1}{\circ}$, (3.2) $\overset{1}{\circ}\text{———}\overset{6}{\circ}\overset{\Lambda_3}{\text{———}}\overset{1}{\circ}$, (3.3) $\overset{1}{\circ}\text{———}\overset{7}{\circ}\overset{\Lambda_3}{\text{———}}\overset{1}{\circ}$,

(3.4) $\overset{1}{\circ}\text{———}\overset{\mathrm{Sp}(3)}{\circ}\overset{\Lambda_3}{\text{———}}\overset{1}{\circ}$, (3.5) $\overset{1}{\circ}\overset{\Lambda}{\text{———}}\overset{\mathrm{Spin}(10)}{\circ}\overset{\Lambda'}{\text{———}}\overset{1}{\circ}$,

(3.6) $\overset{1}{\circ}\overset{\Lambda}{\text{———}}\overset{\mathrm{Spin}(12)}{\circ}\overset{\Lambda'}{\text{———}}\overset{1}{\circ}$,

where Λ (resp. Λ') denotes the (half-)spin (resp. vector) representation of Spin (n),

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