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## Dynamical Systems on Foliations and Existence Problem of Transverse Foliations

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## § 0. Introduction and statements of results

Let M be an *n*-dimensional  $C^{\infty}$  manifold and let  $\mathscr{F}$  be a codimension one  $C^{r+1}$  foliation of M ( $r \ge 1$ ). It is well known that there exists always a one dimensional foliation of M transverse to  $\mathscr{F}$ . Thus the following question on the integrability of 2-plane fields comes to the front:

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