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On a Calculation of Vertex Operators for $E_n^{(1)}$ $(n=6, 7, 8)$

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In this note we give explicit formulas of vertex operators for $E_n^{(1)}$ $(n=6, 7, 8)$.

Realization of basic representations of Euclidean Lie algebras was initiated by Lepowsky-Wilson [5] for $A_1^{(1)}$. In their work, a differential operator of infinite order in infinitely many variables, called vertex operator, played an important role (vertex representation). Subsequently this construction was generalized to almost all Euclidean Lie algebras by Kac-Kazhdan-Lepowsky-Wilson [3].

Lepowsky-Wilson [6] used vertex representations to study Rogers-Ramanujan type identities from the viewpoint of the theory of Lie algebras.

Meanwhile through the work [2], it has been shown that representation theory of Euclidean Lie algebras are intimately related to the theory of solitons. In this connection explicit forms of vertex operators directly relate to the expressions of the so-called multi-soliton solutions of soliton equations.

Therefore explicit forms of vertex operators may be of some interest not only for the theory of Euclidean Lie algebras but also for the theory of solitons.

Procedure for calculating vertex operators are given in [3]. On the other hand, in [2], vertex operators for some of Euclidean Lie algebras (mainly of the classical type) are derived from those for $\mathfrak{gl}(\infty)$, $\mathfrak{go}(\infty)$ or $\mathfrak{go}(2\infty)$ by the process of "reduction". At present it is not clear whether vertex operators for Euclidean Lie algebras not appeared in [2] (mainly of the exceptional type) are also obtained from those for $\mathfrak{gl}(\infty)$, $\mathfrak{go}(\infty)$ or $\mathfrak{go}(2\infty)$, or not.

In this note we describe a procedure for calculating vertex operators, which supplements the procedure given in [3]. This procedure can be applied to affine Lie algebras and makes use of the relations between the notion of Coxeter transformations and the notion of apposition of Cartan subalgebras studied by Kostant [4].

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