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The Zeroes of Characteristic Function χ_f for the Exponents of a Hypersurface Isolated Singular Point

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§ 0. Introduction

In this note, exponents mean a certain set of numbers $\alpha_1, \cdots, \alpha_\mu$ associated with an isolated critical point of a holomorphic function f defined in an open neighbourhood of 0 in C^{n+1}. For instance, if f is a simple germ of a function of the type of Dynkin diagram A_l, D_l, E_6, E_7 or E_8, then the exponents of f are given by $m_j/h+n/2, j=1, \cdots, l=$(rank of the Dynkin diagram) where m_j, $j=1, \cdots, \mu$ are the Coxeter exponents and h is the Coxeter number of the diagram.

Such exponents are introduced in [6] (or see [7] for the summary) for the study of the period mapping, associated with f. In fact, roughly speaking, they are given as exponents of Fuchs type differential equation ($=$the Gauß-Manin connection of f) so that they become the exponents of the Fourier expansion of the period mapping associated to f. The existence and well definedness of exponents introduced above depend on the existence of another object, the primitive form $\zeta^{(0)}$, introduced there. Unfortunately the existence of such primitive forms is shown only for simple singularities and simple elliptic singularities for the moment, so that, rigorously speaking, the above definition of the exponents is valid only for that type of singularities.

Thus in this note, in § 1, we give a tentative way of defining exponents and discuss their relation to the characteristic pairs of the mixed Hodge structure by J. Steenbrink [13] (see also M. Saito [10]). A duality property of exponents will be explained in § 1 using local dualities (see (1.3)).

Then the main purpose of this note is to give a provisional report on some computer experiments concerning such exponents. More precisely, our interest in this note concentrates mainly on two problems, namely, one of the "distribution" of exponents in § 2 and the other—the zeroes of characteristic function χ_f introduced in § 3.

The exponents $\alpha_1, \cdots, \alpha_\mu$, which lie in the interval $(0, n+1)$ with

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