Chapter 7

The Determinant of a Period Matrix

In this chapter we assume that $\mathcal{A}$ is an essential complexified real arrangement and follow [DT]. Using the β**nbc** bases and the hypergeometric pairing of Definition 2.3.3, we obtain a period matrix whose rows and columns are labeled by β**nbc**. The entries are hypergeometric integrals. In general these individual entries cannot be calculated in closed form. The main result is a formula for the determinant of this period matrix. The formula was conjectured by Varchenko in [V1] who proved it for arrangements of general position as well as arrangements in $\mathbb{R}^2$. We assume throughout this chapter that the weights are in

$$\mathbf{U}_{\mathbb{Z}} = \{\lambda \in \mathbb{C}^n \mid \lambda_X \notin \mathbb{Z},\, X \in \mathsf{D}(\mathcal{A}_\infty)\}$$

so the weights are nonresonant.

7.1 The Period Matrix

The next result is due to Kohno [Ko1].

Theorem 7.1.1. *If $\lambda \in \mathbf{U}_{\mathbb{Z}}$, then*

$$i_c : H^p_c(M, \mathcal{L}) \to H^p(M, \mathcal{L}) \quad i_h : H_p(M, \mathcal{L}^\vee) \to H^{lf}_p(M, \mathcal{L}^\vee)$$

are isomorphisms for all p.

Proof. Let $j : M \to \bar{X}$ denote the inclusion where $\bar{X}$ is the resolution constructed in Theorem 4.2.3. Since 1 is not an eigenvalue of the monodromy along any irreducible component of Y, $j_*\mathcal{L} = j_!\mathcal{L}$, where $j_!$ is extension by zero. This provides isomorphisms:

$$H^p(M, \mathcal{L}) \simeq H^p(\bar{X}, j_*\mathcal{L}) \simeq H^p(\bar{X}, j_!\mathcal{L}) \simeq H^p_c(M, \mathcal{L}).$$