Background on the Orbifold Theorem

Many people have obtained partial results and developed related ideas. The following is a selective list. Some of this work is used in our approach, and other parts are used in the approach of Boileau and Porti. We have included some hearsay concerning the events surrounding the Orbifold theorem.

1978 The Smith Conjecture is proved [64].
This was a culmination of the work of many people and used a major part of the theory of 3-manifolds, in particular the work of Bass, Culler & Shalen, Gordon & Litherland, Meeks & Yau, Haken, Waldhausen, and Thurston. It is now a (very special) consequence of the Orbifold theorem.

1981 Thurston announces the Orbifold Theorem [81], [83].

Theorem A. [83] *Let M^3 be a prime, $\mathbb{P}^2$-irreducible, compact 3-manifold which admits a diffeomorphism ϕ ($\neq 1$) of finite order whose fixed point set is more than a finite set of points. Then M has a geometric decomposition.*

Theorem B. [83] *Suppose F is a finite group of diffeomorphisms of a compact 3-manifold M, of which some element $\phi \in F$ ($\phi \neq 1$) has more than a finite set of fixed points. Let $O = M/F$ and $\Sigma \subset O$ be the image of the union of fixed point sets of all elements $\phi \neq 1$ of F. Suppose that the 3-manifold $O - \Sigma$ is prime, and that any 2-sided projective plane in $O - \Sigma$ is homotopic to the boundary of a regular neighbourhood of an isolated point of Σ. Then M has a geometric decomposition which is invariant by F. More precisely, there is a collection of disjoint embedded spheres, projective planes, incompressible tori and incompressible Klein bottles, whose union is invariant by F and geometric structures on the pieces obtained by decomposition along the surfaces on which F acts isometrically.*

Interestingly, Thurston's original theorem pre-dated Hamilton's announce-