Introduction

The theory of manifolds of dimension three is very different from that of other dimensions. On the one hand we do not even have a conjectural list of all 3-manifolds. On the other hand, if Thurston's Geometrization Conjecture is true, then we have a very good structure theory.

The topology of compact surfaces is well understood. There is a well known topological classification theorem, based on a short list of easily computable topological invariants: orientability, number of boundary components and Euler characteristic. For closed surfaces (compact with no boundary) the fundamental group is a complete invariant. The geometry of surfaces is also well understood. Every closed surface admits a metric of constant curvature. Those with curvature $+1$ are called spherical, or elliptic, and comprise the sphere and projective plane. Those with curvature 0 are Euclidean and comprise the torus and Klein bottle. The remainder all admit a metric of curvature -1 and are called hyperbolic. The Gauss-Bonnet theorem relates the topology and geometry

$$\int_F K \, dA = 2\pi\chi(F)$$

where K is the curvature of a metric on the closed surface F of Euler characteristic $\chi(F)$. In particular this implies that the sign of a constant curvature metric is determined by the sign of the Euler characteristic. However in the Euclidean and hyperbolic cases, there are many constant curvature metrics on a given surface. These metrics are parametrized by a point in a Teichmüller space.

The topology of 3-dimensional manifolds is far more complex. At the time of writing there is no complete list of closed 3-manifolds and no *proven* complete set of topological invariants. However if Thurston's Geometrization Conjecture were true, then we would know a complete set of topological invariants. In particular for irreducible atoroidal 3-manifolds, with