

Geometry of Laplace-Beltrami Operator on a Complete Riemannian Manifold

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§0. Introduction

This is a survey paper on recent developments of analytic and geometric aspects of the Laplace-Beltrami operator on a complete Riemannian manifold. Systematic treatments from a Riemannian geometric viewpoint have been already appeared in Berger, Gauduchon & Mazet [’71], Kotake, Maeda, Ozawa & Urakawa [’81], Bérard & Berger [’83], Bérard [’86], Chavel [’84], Gilkey [’84] and Sunada [’88]. But they are mainly concerned with compact case, except Chavel [’84]. In this paper, we shall focus on recent developments of spectral geometry of a *noncompact* complete Riemannian manifold. It seems that the materials may be divided into three parts:

- (1) the distribution of the (essential) spectrum of the Laplacian,
- (2) the heat kernel of a complete Riemannian manifold, and
- (3) harmonic functions, and Green functions on such a manifold.

More precisely,

(1) in §3, we treat mainly results on estimates of the bottom of the (essential) spectrum of the Laplacian of a noncompact complete Riemannian manifold.

(2) In §4, following Ito [’88], Dodziuk [’83], we construct the (minimal) heat kernel of a noncompact complete Riemannian manifold, and show results on uniqueness and estimates of such heat kernel, under certain curvature conditions.

(3) In §5, we will treat positive harmonic functions, the Martin boundary, and Liouville type theorems for harmonic functions on complete manifolds.

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