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INEQUALITIES ON EXPECTATIONS BASED ON THE KNOWLEDGE OF MULTIVARIATE MOMENTS[1]

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The paper deals with discrete moment problems where the possible values of a random vector form a known finite set. First, some earlier results concerning the one dimensional discrete moment problem are summarized. Then, restricting the discussion to the two-dimensional case, for the sake of simplicity, two different discrete moment problems are formulated: (a) the known moments are those where the exponents of the random variables are chosen between 0 and some upper bounds; (b) the sum of the exponents is less than or equal to a given number. The bounds that can be obtained by our technique include bounds for probabilities and expectation.

1. Introduction

The one-dimensional discrete moment problem can be formulated in the following manner. Given a random variable ξ, the possible values of which are known to be $z_0 < z_1 < \cdots < z_n$ and a function $f(z)$, $z \in \{z_0, z_1, \ldots, z_n\}$. We want to give lower and upper bounds for $E[f(\xi)]$, based on the knowledge of the moments $\mu_k = E[\xi^k]$, $k = 1, 2, \ldots, m$, while the probability distribution of ξ is unknown.

Introducing the notations $p_i = P\{\xi = z_i\}$, $f_i = f(z_i)$, $i = 0, 1, \ldots, n$, $\mu_0 = 1$, we obtain the above mentioned bounds by solving the linear programming problems

$$\min\,(\max) \sum_{i=0}^{n} f_i p_i \tag{1.1}$$

subject to

$$\sum_{i=0}^{n} z_i^k p_i = \mu_k, \qquad k = 0, 1, \ldots, m$$

$$p_i \geq 0, \qquad i = 1, 2, \ldots, n,$$

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