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INEQUALITIES FOR THE PARAMETERS $\lambda(F)$,$\mu(F)$ WITH APPLICATIONS IN NONPARAMETRIC STATISTICS[1]

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The parameter $\lambda(F) = P(X_1 < X_2+X_3-X_4; X_1 < X_5+X_6-X_7)$, where $X_1,\ldots,X_7$ are independent and identically distributed (iid) according to a continous distribution F, was first considered by Lehmann (1964) in the context of certain nonparametric methods for the two-way layout. The parameter $\mu(F) = P(X_1 < X_2; X_1 < X_3 + X_4 - X_5)$ was first studied by Hollander (1966), also in the context of nonparametric techniques for the two-way layout. The best known bounds on these probabilities are

$$.28254 \approx 89/315 \leq \lambda(F) \leq 7/24 \approx .29167,$$

and

$$3/10 \leq \mu(F) \leq (\sqrt{2}+6)/24 \approx .30893.$$

The upper bound on $\lambda(F)$ is due to Lehmann (1964), the lower bound on $\lambda(F)$ to Spurrier (1991), the upper bound on $\mu(F)$ to Hollander (1967), and the lower bound on $\mu(F)$ to Spurrier (1991). We briefly review the development of these bounds and then present some new applications motivated by the recent bounds due to Spurrier. The applications include studying the extent to which the new bounds can improve large sample approximations to certain nonparametric test statistics and providing tighter upper and lower bounds on certain correlation coefficients involving these parameters.

1. Introduction

Consider the two-way layout with one observation per cell. Let

$$X_{ij} = \mu + b_i + \theta_j + e_{ij}, i = 1, ..., n, j = 1, ..., k \ (\Sigma b_i = \Sigma\theta_j = 0) \tag{1.1}$$

where the θ's are the parameters of interest, the b's are the nuisance parameters, and the e's are iid according to a common continuous distribution F. Let $Y_{uv}^{(i)} = |X_{iu} - X_{iv}|$ and $R_{uv}^{(i)}$ = rank of $Y_{uv}^{(i)}$ in the ranking from

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