A DECISION APPROACH TO ORDERING STOCHASTIC DEPENDENCE

BY T. BROMEK AND T. KOWALCZYK

Polish Academy of Sciences

Editors' Note: This paper is being published posthumously and in dedicated remembrance of Tadeusz Bromek, who died in an automobile accident on August 23, 1988 in Warsaw. He was very active in the Polish statistical community as well as the international statistical community.

An ordering of global dependence is defined on the basis of a natural ordering of pairs of distributions describing two classes of objects. Its properties are investigated; the links with orderings of multinormal and 2×2 distributions are shown.

1. Introduction. Traditionally, two types of stochastic dependence of components of a vector $\mathbf{X}$ have been distinguished in statistical literature, namely monotone and global dependence. Orderings for monotone dependence were considered by many authors; an overview was given by Yanagimoto (1990). Kimeldorf and Sampson (1987) introduced an axiomatic approach to the matter of orders of monotone dependence. The abundance of formalizations for orderings of monotone dependence contrasts with the silence concerning orderings of global dependence (see Dabrowska (1985)). It seems that a good starting point could be two-class discriminant analysis, with one class reserved for the distribution of the vector $\mathbf{X}$ and the other class reserved for the respective product of marginal distributions of $\mathbf{X}$. Thus, a natural ordering of pairs of distributions describing two classes of objects (Niewiadomska-Bugaj (1987)), called prognostic ordering and denoted $\leq_p$, may be a base to define an ordering of global dependence, called global ordering and denoted $\leq_g$.

In Section 2 we recall the definition of $\leq_p$ and prove some of its new properties. Section 3 contains the definition and properties of $\leq_g$.

2. Prognostic Order $\leq_p$. Consider a two-class discriminant problem corresponding to a pair $(\mathbf{Z}_1, \mathbf{Z}_2)$ where $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are random vectors supported on $\mathcal{Z} \subset R^k$. Distribution P_i of $\mathbf{Z}_i$ describes the ith class of the considered population $(i = 1, 2)$. Let a classification rule for $(\mathbf{Z}_1, \mathbf{Z}_2)$ be a Borel measurable function

AMS 1980 subject classifications. 62H20.

Key words and phrases. Global dependence, monotone dependence, likelihood ratio, two-class discriminant analysis.