Estimating Functions, Partial Sufficiency and Q-Sufficiency in the Presence of Nuisance Parameters

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Abstract

When there exists a statistic which has its distribution free of nuisance parameters, the optimality of the marginal score function can be investigated in the context of generalized Fisher information for parameters of interest. In the case of a partially sufficient statistic, i.e. a statistic sufficient for parameters of interest, the marginal score function is the optimal estimating function. With the new concept of q-sufficiency for parameters of interest, the marginal score function is operationally equivalent to the optimal estimating function.

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1 INTRODUCTION

The optimality of the conditional score function as an estimating function for the parameter of interest, in the presence of unknown nuisance parameters, was established by Godambe (1976) in the situation where there exists a statistic which is ancillary for the parameter of interest and which is also complete for nuisance parameters. Such a statistic has been termed a complete p-ancillary statistic in an earlier paper (Bhapkar, 1989).

Refer to Liang and Zeger (1995) for a review of estimating functions theory, some discussion of optimality of the conditional score function under the conditions assumed by Godambe, and references to further work (for example, Lindsay 1982) to find approximately optimal estimating functions in more general situations.

The analogous question concerning optimality of the marginal score function in the *complete case*, has not yet been satisfactorily resolved. Lloyd (1987) considered this question; however, his assertion of optimality of the