

RANDOMNESS AND FOUNDATIONS OF PROBABILITY: VON MISES' AXIOMATISATION OF RANDOM SEQUENCES

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Abstract

We discuss von Mises' notion of a random sequence in the context of his approach to probability theory. We claim that the acceptance of Kolmogorov's rival axiomatisation was due to a different intuition about probability getting the upper hand, as illustrated by the notion of a martingale. We also discuss the connection between randomness and the axiom of choice.

to David Blackwell

1. Introduction. In 1937, the Université de Genève organized a conference on the theory of probability, part of which was devoted to foundational problems (the proceedings of this part have been published as [9]). The focal point of the discussion was von Mises' axiomatisation of probability theory [21], and especially its relation to the newly published axiomatisation by Kolmogorov. In 1919 Richard von Mises (1883-1957) had published an (in fact the first) axiomatisation of probability theory, which was based on a particular type of disorderly sequences, so called *Kollektivs*. The two features characterizing Kollektivs are, on the one hand, existence of limiting relative frequencies within the sequence (global regularity) and, on the other hand, invariance of these limiting relative frequencies under the operation of "admissible place selection" (local irregularity). An admissible place selection is a procedure for selecting a subsequence of a given sequence x in such a way that the decision to select a term x_n does not depend on the value of x_n .

After several years of vigorous debate, which concerned not only von Mises' attempted characterisation of a class of random phenomena, but also his views on the interpretation of probability, it became clear that most probabilists were critical of von Mises' axiomatisation and preferred the simple

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