Statistics, Probability and Game Theory
IMS Lecture Notes – Monograph Series (1996) Volume 30

EXPONENTIAL PROBABILITY INEQUALITIES WITH SOME APPLICATIONS

GEORGE G. ROUSSAS
University of California, Davis

Abstract.

A brief review of the Bennett and Hoeffding inequalities is presented, as they apply to independent random variables, for the purpose of identifying the point where independence is actually utilized. On the basis of an observation, it follows that the inequalities remain in force whenever the expectation of a certain product is bounded by the product of the expectations of the factors involved. This requirement is satisfied, for example, when the underlying random variables are negatively associated. By a counterexample, it is demonstrated that the inequalities need not hold for positively associated random variables. Next, a Hoeffding-type inequality is established for a strong mixing sequence of random variables. The paper is concluded with the utilization of the Hoeffding inequality in order to construct a minimum distance estimate of the probability measure governing a sequence of negatively associated random variables.

1. Introduction and Summary. Let $X_1, X_2, \ldots$ be (real-valued) random variables (r.v.) defined on the underlying probability space $(\Omega, \mathcal{A}, P)$, and set S_n for the sum of the first n r.v.'s, $\sum_{i=1}^{n} X_i$, and $\bar{S}_n$ for S_n/n. The problem of providing exponential bounds for the probabilities $P(|S_n| \geq \varepsilon)$ $(\varepsilon > 0)$ is of paramount importance, both in Probability and Statistics. From a statistical viewpoint, such inequalities can be used, among other things, for the purpose of providing rates of convergence (both in the probability sense and almost surely) for estimates of various quantities. Especially so in a nonparametric setting, where the advantages of a parametric structure are not available to the investigator.

In Section 2, a brief review is presented of the Bennett and the Hoeffding inequalities in the framework of independent r.v.'s, primarily for the purpose of isolating the point, where independence is utilized. This point is inequality (2.12) stated as a corollary to two propositions. In the following section, it is shown that inequality (2.12) is, indeed, satisfied for r.v.'s which are negatively associated. As a consequence of it, such r.v.'s satisfy the Bennett and Hoeffding inequalities. It is then shown, by means of a counterexample, that positively associated r.v.'s need not, in general, satisfy the Hoeffding inequality. This conclusion is an easy consequence of a result for positively