Complete Lattices of Probability Measures with Applications to Martingale Theory

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Abstract

The set of probability measures on $\mathbb{R}$ with the stochastic order and the set of right-tail integrable probability measures on $\mathbb{R}$ with the convex order form complete lattices. Connections of these lattice structures to martingale theory and to the Hardy-Littlewood maximal function are exhibited.

AMS 1991 Classifications: Primary 60E15, 60G44; Secondary 60G40, 28A33

Key words and phrases: Martingales, stochastic order, convex order, complete lattices of probability measures, suprema of sets of probability measures, Hardy and Littlewood maximal function.

*Supported in part by NSF grants DMS-88-01818 and DMS-92-09586.
**This research was initiated while this author was at Institut für Mathematische Stochastik, Georg-August Universität, Göttingen.

1 Introduction

The comparison of random variables (r.v.'s) or distributions of random variables leads in a natural way to orderings of probability measures (p.m.'s). For probability measures μ_1 and μ_2 on $\mathbb{R}$, μ_1 is said to be smaller than μ_2