Sequential Selection of an Increasing Subsequence From a Random Sample with Geometrically Distributed Sample-Size

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Following a long-standing suggestion by Samuels and Steele we study the problem of sequential selection of an increasing subsequence from a random sample of size N, where N is geometrically distributed with parameter p. The maximum expected length of a subsequence which can be selected by a nonanticipating policy is shown to be asymptotic to $p^{-1/2}$, as $p \to 0$.

Running Title: Selecting Increasing Sequence

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1. Introduction. There has been a great deal of interest in long increasing subsequences of a random sequence (see surveys [10], [1]). A central result in this vein says that the expected length of the longest increasing subsequence in a sequence of n random items is asymptotic to $2n^{1/2}$ (see [8],[11]).

Samuels and Steele [9] studied selection of a long increasing subsequence as a dynamical decision problem. In their model, n i.i.d. items with known continuous distribution are inspected in strict succession. Each item can be selected or rejected only at the time it is inspected: once rejected, the item cannot be recalled and if accepted cannot be discarded. The selected sequence must increase. The objective it to maximize the expected length of the selected sequence. Samuels and Steele showed that for n large the maximum expected length is $v \sim (2n)^{1/2}$ and demonstrated a policy which attains this value asymptotically. Comparing v with the length of the longest increasing subsequence, Samuels and Steele interpreted the ratio $2 : 2^{1/2}$ as the long-run advantage of a prophet with complete foresight of the sequence over an intelligent but non-clairvoyant gambler, who observes the items in