

Mechanical models in nonparametric regression

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This note is devoted to an extrapolation of a solid-mechanic motif from nonparametric regression (or, as it is often referred to, smoothing) and numerical interpolation, a theme originating in the exposition of spline-based methods. The attention is turned to the fact that a mechanical analogy, akin to that from elasticity theory for quadratic spline penalties, can be elucidated in plasticity theory for certain penalties of different type. It is this—in the (slightly altered) words of a referee, “what would happen if the metaphor underlying splines were replaced by a metaphor of plastic energy”—what is the objective of what follows; everything else, in particular discussion of the origins of (certain types of) splines, or an exhaustive bibliographical account, is outside our scope.

To give an idea what mechanical analogies we have in mind, let us start by the quote from the Oxford English Dictionary, referencing (univariate) “draftsman spline” as “*a flexible strip of wood or hard rubber used by draftsmen in laying out broad sweeping curves*”. Bivariate case brings more sophistication: the designation “thin-plate spline” is regularly explained by a story about the deformation of an elastic flat thin plate—see page 139 of Green and Silverman [12] or page 108 of Small [31]: *if the plate is deformed to the shape of the function ϵf , and ϵ is small, then the bending energy is (up to the first order) proportional to the smoothing penalty*. While it is questionable how much of scientific utility such trivia may carry—for some discussion, see Bookstein [3], Dryden and Mardia [7], Wahba [35], Green and Silverman [12], and the references there—their mere existence may provoke inquiring minds. While a curious individual may pick up the “physics” of the elastic parable pretty much from the standard textbooks (either of engineering [34] or theoretical flavor [21]), comprehension of the pertinent plastic parallel is much more perplexing; even if the topic is perhaps not unknown to the specialized literature, the latter may be quite impenetrable for a person with standard statistical education. Fortunately, with a little help from his friend (the first author of this note—a theoretical physicist, albeit with principal interests in gravitation and cosmology), the second author was not only able to arrive to certain joy of cognition, but also to

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