PREFACE

These are notes of lectures which I gave at the University of Notre Dame during the fall of 1966; they have been written and prepared for publication by Dr. M. Borelli, Assistant Professor at the University of Notre Dame, whom I heartily thank for the care with which he has accomplished his task and the many hours he has devoted to it.

The lectures were intended as an introduction to modern algebraic geometry, in order to familiarize with some of its most important concepts mathematicians who have had no previous contact with that theory. The scope of the book prevented me from giving anything like a complete exposition, and I have accordingly suppressed a large number of proofs, all of which can be found either in Bourbaki's "Algèbre commutative" or in Grothendieck's "Eléments de Géométrie algébrique". On the other hand, the proofs which are given have been made as explicit as possible, and Dr. Borelli has taken great pains to spell out many details which would be taken for granted by anybody having some familiarity with the material.

As the title indicates, the concepts which are studied are those which have to do with the properties of an algebraic variety (or scheme) at a point, or equivalently with the local ring of the scheme at that point. The most important of these concepts are dimension, depth, regularity, normality and completeness, and they are most of the time studied for noetherian local rings.

In the study of the dimension of a module M over a noetherian semi-local ring A (§1 and 2) we prove the Krull-Chevalley-Samuel theorem, which gives three different interpretations of dimension, namely as the Krull dimension, as the leading coefficient of the

Hilbert-Samuel polynomial, and as the smallest number of elements $x_1, \ldots, x_r$ of A such that the Module $M/x_1 M + \ldots + x_r M$ has finite length. The general form of the Hauptidealsatz is proved in §2.

§3 is devoted to the notion of depth and the study of the properties of Cohen-Macauley rings.

Regular rings are defined in §4. Here, in addition to giving the usual definition and properties of regular local rings, we characterize the regular local rings of classical Algebraic Geometry as those rings whose corresponding points are simple, i.e. the corresponding Jacobian matrix has maximal rank. The cohomological dimension of a ring is defined, and the Hilbert-Serre theorem concerning it is stated (but not proved). In this same §4 we characterize reduced and normal noetherian rings, the latter characterization due to Serre.

§5 concerns itself with the behavior of the above mentioned notions under local, flat morphisms, and in §6 we apply the results of §5 to the study of the completion and normalization of a noetherian local ring. The main results of §6 are Cohen's Structure Theorem for noetherian complete local rings, and Nagata's theorem that every noetherian, complete, local integral domain is Japanese. The notes end with the definition of Grothendieck's excellent rings and the statement of the theorem that localizations of finitely generated algebras over excellent rings are again excellent.

I have tried to give to the notes a geometrical flavor, in as much as possible, by examining, with examples and figures, most of the above notions in the context of classical Algebraic Geometry over the complexes.