

# Contents

<b>Preface</b>	<b>VII</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Background . . . . .	7
1.2 Homogeneous symbols . . . . .	10
<b>2 Main estimates</b>	<b>15</b>
2.1 Away from the real axis: exponential decay . . . . .	17
2.2 Roots with non-degeneracies . . . . .	19
2.3 Roots meeting the real axis . . . . .	25
2.4 Application to the Cauchy problem . . . . .	28
2.5 Schematic of method . . . . .	31
2.6 Strichartz estimates and nonlinear problems . . . . .	32
<b>3 Properties of hyperbolic polynomials</b>	<b>35</b>
3.1 General properties . . . . .	35
3.2 Symbolic properties . . . . .	40
<b>4 Oscillatory integrals with convexity</b>	<b>53</b>
4.1 Estimates for oscillatory integrals . . . . .	53
4.2 Functions of convex type . . . . .	57
4.3 Convexity condition for real-valued phases . . . . .	60
<b>5 Oscillatory integrals without convexity</b>	<b>73</b>
5.1 Real-valued phase function . . . . .	73
<b>6 Decay of solutions to the Cauchy problem</b>	<b>79</b>
6.1 Representation of the solution . . . . .	79
6.2 Division of the integral . . . . .	81
6.3 Estimates for large frequencies . . . . .	84
6.4 Phase separated from the real axis: Theorem 2.1.1 . . . . .	85
6.5 Non-degenerate phase: Theorems 2.2.1 and 2.2.2 . . . . .	87

6.6	Phase satisfies the convexity condition: Theorem 2.2.6 . . . . .	91
6.7	Results without convexity: Theorem 2.2.10 . . . . .	97
6.8	Asymptotic properties of complex phase functions . . . . .	98
6.9	Estimates for bounded frequencies away from multiplicities . .	100
6.10	Phase separated from the real axis: Theorem 2.1.1 again . . .	101
6.11	Roots meeting the real axis: Theorem 2.3.2 . . . . .	102
6.12	Phase function lies on the real axis . . . . .	106
<b>7</b>	<b>Frequencies around multiplicities</b>	<b>107</b>
7.1	Resolution of multiple roots . . . . .	108
7.2	Phase separated from the real axis: Theorem 2.1.2 . . . . .	113
7.3	Phase meeting the real axis: Theorem 2.3.1 . . . . .	114
7.4	Phase on the real axis for bounded frequencies . . . . .	116
<b>8</b>	<b>Examples and extensions</b>	<b>131</b>
8.1	Wave equation with mass and dissipation . . . . .	131
8.2	Higher order equations . . . . .	133
8.3	Hyperbolic triples . . . . .	135
8.4	Strictly hyperbolic systems . . . . .	136
8.5	Application to Fokker–Planck equation . . . . .	137
	<b>Bibliography</b>	<b>143</b>