APPENDIX C

# $p$-adic symmetric domains and Totaro's theorem

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This appendix is a short exposition of M. Rapoport and T. Zink's construction of $p$-adic symmetric domains [**RZ96**] and of B. Totaro's theorem [**Tot96**]. Let $G$ be a connected reductive algebraic group over $\mathbb{Q}_p$. The set $\mathcal{F}$ of filtrations on an $F$-isocrystal with $G$-structure has a structure of a homogeneous space. Rapoport and Zink introduced a $p$-adic rigid analytic structure on the set $\mathcal{F}^{wa}$ of weakly admissible points in $\mathcal{F}$. They conjectured that the point in $\mathcal{F}^{wa}$ is characterized by the semistability in the sense of the geometric invariant theory [**MFK94**] and Totaro proved this conjecture.

## 1. Weakly admissible filtered isocrystals.

We recall J.-M. Fontaine's definition of weakly admissible filtered $F$-isocrystals [**Fon79**].

**1.1.** Let $p$ be a prime number, $k$ a perfect field of characteristic $p$, $K_0$ an absolutely unramified discrete valuation field of mixed characteristics $(0, p)$ with residue field $k$, $\overline{K}_0$ an algebraic closure of $K_0$, and $\sigma$ the Frobenius automorphism on $K_0$.

**Definition 1.2.** (1) An $F$-isocrystal over $k$, (we simply say "isocrystal"), is a finite dimensional $K_0$-vector space $V$ with a bijective $\sigma$-linear endomorphism $\Phi : V \to V$. We denote the category of isocrystals over $k$ by $\mathrm{Isoc}(K_0)$.

(2) For a totally ramified finite extension $K$ of $K_0$ in $\overline{K}_0$, a filtered isocrystal $(V, \Phi, F^{\cdot})$ over $K$ is an isocrystal $(V, \Phi)$ with a decreasing filtration $F^{\cdot}$ on the $K$-vector space $V \otimes_{K_0} K$ such that $F^r = V \otimes_{K_0} K$ for $r \ll 0$ and $F^s = 0$ for $s \gg 0$. We denote the category of filtered isocrystals over $K$ by $MF(K)$.

Fontaine also introduced a filtered isocrystal with nilpotent operator $N$ [**Fon94**]. In this appendix we restrict our attension to filtered isocrystals with $N = 0$.

The category $MF(K)$ is a $\mathbb{Q}_p$-linear additive category with $\otimes$ and internal $Hom$'s, but not abelian. A subobject $(V', \Phi', F'^{\cdot})$ of a filtered isocrystal

$(V, \Phi, F^\bullet)$ is a $\Phi$-stable $K_0$-subspace $V'$ such that $\Phi' = \Phi|_{V'}$ and $F'^i = (V' \otimes_{K_0} K) \cap F^i$ for all $i$.

**Definition 1.3.** Let $K$ be a totally ramified finite extension of $K_0$ in $\overline{K}_0$. A filtered isocrystal $(V, \Phi, F^\bullet)$ over $K$ is weakly admissible if, for any subobject $(V', \Phi', F'^\bullet) \neq 0$, we have

$$\sum_i i \dim_{F'} \mathrm{gr}^i_{F'}(V' \otimes_{K_0} K) \leq \mathrm{ord}_p(\det(\Phi'))$$

and the equality holds for $(V', \Phi', F'^\bullet) = (V, \Phi, F^\bullet)$. Here $\mathrm{ord}_p$ is an additive valuation of $K_0$ normalized by $\mathrm{ord}_p(p) = 1$.

The category of weakly admissible filtered isocrystals is an abelian category which is closed under duals in the category of filtered isocrystals. Fontaine proved that an admissible filtered isocrystal over $K$, that means a filtered isocrystal arising from a crystalline representation of the absolute Galois group of $K$ via Fontaine's functor, is weakly admissible and conjectured that a weakly admissible filtered isocrystal is admissible in [**Fon79**]. The category of admissible filtered isocrystals is a $\mathbb{Q}_p$-linear abelian category with $\otimes$ and duals. Hence, he also conjectured that the category of weakly admissible filtered isocrystals is closed under $\otimes$, and this was proved by G. Faltings in [**Fal95**]. (See also [**Tot96**].)

In [**CF00**] P. Colmez and Fontaine proved a weakly admissible filtered isocrystal is admissible.

## 2. Filtered isocrystals with $G$-structure.

**2.1.** Let $G$ be a linear algebraic group over $\mathbb{Q}_p$ and denote by $\mathrm{Rep}_{\mathbb{Q}_p}(G)$ the category of finite dimensional $\mathbb{Q}_p$-rational representations of $G$. An exact faithful $\otimes$-functor $\mathrm{Rep}_{\mathbb{Q}_p}(G) \to \mathrm{Isoc}(K_0)$ is called an isocrystal with $G$-structure over $K_0$.

Let $b \in G(K_0)$. Then, the functor

$$\mathrm{Rep}_{\mathbb{Q}_p}(G) \to \mathrm{Isoc}(K_0)$$

associated to $b$, defined by $V \mapsto (V \otimes K_0, b(\mathrm{id} \otimes \sigma))$, is an isocrystal with $G$-structure over $K_0$. [**Kot85**] Two elements $b$ and $b'$ in $G(K_0)$ are conjugate if and only if there is an element $g \in G(K_0)$ such that $gb\sigma(g)^{-1} = b'$. In this case, $g$ defines an isomorphism between the isocrystals with $G$-structure associated to $b$ and $b'$.

If $G$ is connected and $k$ is algebraically closed, then any isocrystal with $G$-structure over $K_0$ is associated to an element $b \in G(K_0)$ as above. [**RR96**]

**2.2.** Let $\mathbb{D} = \varprojlim \mathbb{G}_m$ be the pro-algebraic group over $\mathbb{Q}$ whose character group is $\mathbb{Q}_p$. For an element $b \in G(K_0)$, R.E. Kottwitz defined a morphism

$$\nu : \mathbb{D} \to G_{K_0}$$

of algebraic groups over $K_0$ which is characterized by the property that, for any object $V$ in $\mathrm{Rep}_{\mathbb{Q}_p}(G)$, the $\mathbb{Q}$-grading of $V \otimes K_0$ associated to $-\nu$ is the slope grading of the isocrystal $(V \otimes K_0, b(\mathrm{id} \otimes \sigma))$. (The sign of our $\nu$ is different from the one in [**Kot85**].) For a suitable positive integer $s$, $s\nu$ is regarded as a one-parameter subgroup of $G$ over $K_0$.

**Definition 2.3.** A $\sigma$-conjugacy class $\bar{b}$ of $G(K_0)$ is decent if there is an element $b \in \bar{b}$ such that

$$(b\sigma)^s = s\nu(p)\sigma^s$$

for some positive integer $s$.

One knows that, for a decent $\sigma$-conjugacy class $\bar{b}$, $b$ and $\nu$ as above are defined over $\mathbb{Q}_{p^s}$. If $G$ is connected and $k$ is algebraically closed, then any $\sigma$-conjugacy class is decent. [**Kot85**]

**2.4.** Let $K$ be a totally ramified finite extension of $K_0$ in $\overline{K}_0$. For a one-parameter subgroup $\lambda : \mathbb{G}_m \to G$ over $K$ and an element $b \in G(K_0)$, we have an exact $\otimes$-functor

$$\mathcal{I} : \mathrm{Rep}_{\mathbb{Q}_p}(G) \to MF(K)$$

which is defined by $V \mapsto (V \otimes K_0, b(\mathrm{id} \otimes \sigma), F_\lambda^\bullet)$. Here $V_{K,\lambda,j}$ is the subspace of $V \otimes K$ of weight $j$ with respect to $\lambda$ and

$$F_\lambda^i = \bigoplus_{j \geq i} V_{K,\lambda,j}$$

is the weight filtration associated to $\lambda$.

**Definition 2.5.** A pair $(\lambda, b)$ as above is weakly admissible if and only if the filtered isocrystal $\mathcal{I}(V)$ over $K$ is so for any object $V$ in $\mathrm{Rep}_{\mathbb{Q}_p}(G)$.

To see the weak admissibility for $(\lambda, b)$, it is enough to check the weak admissibility of $\mathcal{I}(V)$ for a faithful representation $V$ of $G$. Indeed, any representation of $G$ appears as a direct summand of $V^{\otimes m} \otimes (V^\vee)^{\otimes n}$ and $\mathcal{I}(V)^{\otimes m} \otimes (\mathcal{I}(V)^\vee)^{\otimes n}$ is weakly admissible by Faltings (see 1.3). Here $V^\vee$ (resp. $\mathcal{I}(V)^\vee$) is the dual of $V$ (resp. $\mathcal{I}(V)$).

## 3. Totaro's theorem.

In this section we assume that $k$ is algebraically closed.

**3.1.** Let $G$ be a reductive algebraic group over $\mathbb{Q}_p$. We fix a conjugacy class of a one-parameter subgroup $\lambda : \mathbb{G}_m \to G$ over $\overline{K}_0$. Here two one-parameter subgroups $\lambda, \lambda'$ are conjugate if and only if $g\lambda g^{-1} = \lambda'$ for some element $g \in G(\overline{K}_0)$. Then, there is a finite extension $E$ of $\mathbb{Q}_p$ in $\overline{K}_0$ such that the conjugacy class of $\lambda$ is defined over $E$. Let us suppose that $\lambda$ is defined over $E$ and denote by $\breve{E}$ the composite field $EK_0$ in $\overline{K}_0$

**3.2.** Two one-parameter subgroups of $G$ over $\overline{K}_0$ are equivalent if and only if they define the same weight filtration for any object in $\mathrm{Rep}_{\mathbb{Q}_p}(G)$. Note that, if two one-parameter subgroups are equivalent, then they belong to the same conjugacy class.

Consider the functor

$$R \mapsto \{\text{the equivalence classes in the conjugacy class of } \lambda \text{ defined over } R\}$$

on the category of $E$-algebras. If one defines an algebraic subgroup of $G$ over $E$ by

$$P(\lambda)(\overline{K}_0) = \{g \in G(\overline{K}_0) \mid g\lambda g^{-1} \text{ is equivalent to } \lambda\},$$

then $P(\lambda)$ is parabolic and the functor above is represented by the projective variety $G_E/P(\lambda)$. We denote this homogeneous space over $E$ by $\mathcal{F}_\lambda$. If $V$ is a faithful representation in $\mathrm{Rep}_{\mathbb{Q}_p}(G)$ and if we denote by $\mathrm{Flag}_\lambda(V)$ the flag variety over $\mathbb{Q}_p$ which represents the functor

$$R \mapsto \left\{ \begin{array}{l} \text{the filtrations } F^\bullet \text{ of } V \otimes R \text{ as } R\text{-modules such that} \\ F^i \text{ is a direct summand and } \mathrm{rank}_R F^i = \dim_{\overline{K}_0} F^i_\lambda(V \otimes \overline{K}_0) \end{array} \right\}$$

on the category of $\mathbb{Q}_p$-algebras, then there is a natural $E$-closed immersion

$$\mathcal{F}_\lambda \to \mathrm{Flag}_\lambda(V) \otimes_{\mathbb{Q}_p} E.$$

**3.3.** Let $b \in G(K_0)$. For a finite extension $K$ of $\breve{E}$, a point $\xi$ in $\mathcal{F}_\lambda(K)$ is called weakly admissible if and only if the pair $(\xi, b)$ is weakly admissible. This condition is independent of the choice of the representative in the equivalence class $\xi$. We denote by $\mathcal{F}^{wa}_{\lambda,b}(K)$ the subset of weakly admissible points. Totaro gave a characterization of $\mathcal{F}^{wa}_{\lambda,b}$ in the sense of geometric invariant theory. [**Tot96**] We explain his theory in the rest of this section.

**3.4.** For a maximal torus $T$ in $G_{\overline{K}_0}$, let $X^*(T)$ be the free abelian group of characters, $X_*(T)$ the free abelian group of one-parameter subgroups, and $< \, , \, >: X^*(T) \times X_*(T) \to \mathbb{Z}$ the perfect pairing with $\chi(\xi(t)) = t^{<\chi,\xi>}$. If $N(T)$ is the normalizer of $T$ in $G$, the Weyl group $W(T) = N(T)/T$ acts $X_*(T)$ via inner automorphisms.

Now we fix an invariant norm $|| \ ||$ on $G$, a non-negative real valued function on the set of one-parameter subgroups of $G_{\overline{K}_0}$, such that

a) $||g\xi g^{-1}|| = ||\xi||$ for any $g \in G(\overline{K}_0)$,

b) for any maximal torus $T$, there is a positive definite rational valued bilinear form $( \, , \, )$ on $X_*(T) \otimes \mathbb{Q}$ with $(\xi, \xi) = ||\xi||^2$,

c) $||\gamma(\xi)|| = ||\xi||$ for $\gamma \in \mathrm{Gal}(\overline{K}_0/K_0)$, where $\gamma(\xi)(t) = \gamma(\xi(t))$.

The bilinear form on $X_*(T) \otimes \mathbb{Q}$ as above is invariant under the action of the Weyl group by (a). For any maximal torus $T$, invariant norms are in one-to-one correspondence with $(\mathrm{Gal}(\overline{K}_0/K_0), W(T))$-invariant positive definite rational valued bilinear forms on $X_*(T) \otimes \mathbb{Q}$ since all maximal tori are conjugate and, if $g\xi g^{-1} \in X_*(T)$ for $\xi \in X_*(T)$ and $g \in G(\overline{K}_0)$, then there is $h \in W(T)$ with $g\xi g^{-1} = h\xi h^{-1}$ by [**MFK94**]. (See also [**Kem78**] and [**Tot96**].) Hence, such an invariant norm exists.

**3.5.** Now we assume that $G$ is connected. Let $U(\lambda)$ be the unipotent radical of $P(\lambda)$, whose elements act on the graded space gr $F^\bullet_\lambda$ trivially. Then there is a bijection between the set of maximal tori of $G$ in $P(\lambda)$ and the set of maximal tori of $P(\lambda)/U(\lambda)$ by the natural projection $T \mapsto \overline{T}$. Hence, the invariant norm on $G$ induces the one on $P(\lambda)/U(\lambda)$. Fix a maximal torus $T$ of $G$. Since the image of $\lambda$ is contained in the center of $P(\lambda)/U(\lambda)$, the perfect pairing associated to the invariant norm determines the dual of $\lambda$ in $X^*(\overline{T})$. This dual can extend to an element in $X^*(P(\lambda)/U(\lambda)) \otimes \mathbb{Q}$, which we call a character $\otimes\mathbb{Q}$ of $P(\lambda)/U(\lambda)$. Now we define a $G$-line bundle $\otimes\mathbb{Q}$ on $\mathcal{F}_\lambda$, $L_\lambda \in \mathrm{Pic}^G(\mathcal{F}_\lambda) \otimes \mathbb{Q}$, by the associated one to the negative of the dual character $\otimes\mathbb{Q}$ of $\lambda$. By construction, the line bundle $\otimes\mathbb{Q}$, $L_\lambda$, depends only on the conjugacy class of $\lambda$ and is ample.

Let $J$ be a smooth affine group scheme over $\mathbb{Q}_p$ such that

$$J(\mathbb{Q}_p) = \{g \in G(K_0) \mid g(b\sigma) = (b\sigma)g\}$$

(which is introduced in [**RZ96**]). Since $J_{K_0} \subset G_{K_0}$, the pull back $L_{\lambda\breve{E}}$ of $L_\lambda$ on $\mathcal{F}_{\lambda\breve{E}}$ is an ample $J_{\breve{E}}$-line bundle.

By the same construction as above, $\nu$ in 2.2 gives a character $\otimes\mathbb{Q}$ of $P(\nu)$. The opposite of this character $\otimes\mathbb{Q}$ determines a $J_{\breve{E}}$-action $\otimes\mathbb{Q}$ on the trivial line bundle on $\mathcal{F}_{\lambda\breve{E}}$ since $J_{K_0} \subset P(\nu)$. We denote it by $L^0_\nu$.

We put a $J_{\breve{E}}$-line bundle $\otimes\mathbb{Q}$, $L = L_{\lambda\breve{E}} \otimes L^0_\nu$, on $\mathcal{F}_{\lambda\breve{E}}$. Then it is ample and depends only on $b$ and the conjugacy class of $\lambda$. We denote by $\mathcal{F}^{ss}_\lambda(L)$ the set of semistable points in $\mathcal{F}_\lambda$ with respect to $L$ in the sense of D. Mumford [**MFK94**].

**Theorem 3.6.** [**Tot96**] *Suppose that $G$ is connected and reductive. For any finite extension $K$ of $\breve{E}$, we have*

$$\mathcal{F}^{wa}_{\lambda,b}(K) = \mathcal{F}^{ss}_{\lambda\breve{E}}(L)(K).$$

We shall sketch Totaro's proof. First, let $G = GL(n)$ and let us consider the invariant norm induced by the pairing

$$(\alpha, \beta) = \sum_{i,j} ij \dim_K \mathrm{gr}^i_{F^\cdot_\alpha} \mathrm{gr}^j_{F^\cdot_\beta}(V)$$

for one-parameter subgroups $\otimes\mathbb{Q}$, $\alpha, \beta$ of $G = GL(V)$ over $K$. If one puts $\mu_\alpha(V) = \sum_i i \dim_K \mathrm{gr}^i_{F^\cdot_\alpha}(V)/\dim_K V$, then one has

$$(\alpha, \beta) = \int (\mu_\alpha(F^j_\beta) - \mu_\alpha(V)) \dim_K F^j_\beta dj + \mu_\alpha(V)\mu_\beta(V) \dim_K V.$$

By using the notation in 2.2, $(\xi, b)$ is weakly admissible if and only if $(\xi, \alpha) + (\nu, \alpha) \leq 0$ for any one-parameter subgroup $\alpha$ of $G$ over $K_0$ with the filtration $F^\cdot_\alpha$ as subisocrystals. In other words, $(\xi, b)$ is weakly admissible if and only if $(\xi, \alpha) + (\nu, \alpha) \leq 0$ for any one-parameter subgroup $\alpha$ of $J$ over $\mathbb{Q}_p$. Hence, the assertion follows from the calculation of Mumford's numerical invariant below.

*Lemma* 3.7. *If $\mu(\xi, \alpha, L)$ is Mumford's numerical invariant of $\xi \in \mathcal{F}_\lambda$ for a one-parameter subgroup $\alpha$ of $J$ over $\mathbb{Q}_p$, then*

$$\mu(\xi, \alpha, L) = -(\xi, \alpha) - (\nu, \alpha).$$

Next, let $G$ be arbitrary, $V$ a faithful representation of $G$, and consider the invariant norm on $G$ induced from the above norm by the natural immersion $G \to GL(V)$. Mumford's numerical invariant of weakly admissible points is non-negative for any one-parameter subgroup of $J(GL(V))$. Hence it is so for any one-parameter subgroup of $J$, and the weak admissibility implies the semistability. To see the converse, one needs to show that, if $\xi \in \mathcal{F}_\lambda^{ss}(L)(K)$, $(\xi, \alpha) + (\nu, \alpha) \leq 0$ for any one-parameter subgroup $\alpha$ of $J(GL(V))$ over $\mathbb{Q}_p$. If $\alpha$ is semistable for the $G_K$-line bundle $L_\alpha$ on $\mathcal{F}_\alpha$, the assertion follows from the first part. In the case where $\alpha$ is not semistable, one can use Kempf's filtration [**Kem78**] and Ramanan and Ramanathan's work [**RR84**], and obtains the required inequality.

Finally one needs to prove the independence of the choice of the norm. Suppose that the identity is valid for the particular norm. Since $G$ is a quotient of a product of a torus and some simple algebraic groups by a finite central subgroup [**BT65**], one can reduce the assertion in the case of tori and simple groups. In the case of tori it was proved in [**RZ96**], and in the case of simple groups it is true since the norm of the simple group comes from the Killing form up to a positive rational multiple. □

## 4. $p$-adic symmetric domains.

Let $k$ be the algebraic closure of the prime field $\mathbb{F}_p$, $\mathbb{C}_p$ the $p$-adic completion of a fixed algebraic closure $\overline{\mathbb{Q}}_p$ of $\mathbb{Q}_p$, and $K_0 = \widehat{\mathbb{Q}}_p^{ur}$ the $p$-adic completion of the maximum unramified extension of $\mathbb{Q}_p$ in $\mathbb{C}_p$.

**4.1.** Let $G$ be a reductive group over $\mathbb{Q}_p$, $b \in G(K_0)$, and fix a conjugacy class $\{\lambda\}$ of a one-parameter subgroup $\lambda$ of $G$ over $\overline{\mathbb{Q}}_p$.

Rapoport and Zink gave a rigid analytic structure on $\mathcal{F}_{\lambda,b}^{wa}$ as an admissible open subset in $\mathcal{F}_{\lambda\breve{E}}$ and call it the $p$-adic symmetric domain associated to the triple $(G, \{\lambda\}, b)$ in [**RZ96**]. This notion of $p$-adic symmetric domains is different from that of M. van der Put and H. Voskuil in [**vdPV92**]. Indeed, for any discrete co-compact subgroup $\Gamma$ of $G(\breve{E})$, the quotient $\mathcal{F}_{\lambda,b}^{wa}/\Gamma$ is not always a proper analytic space over $\breve{E}$.

**Theorem 4.2.** [**RZ96**] *The set $\mathcal{F}_{\lambda,b}^{wa}$ of weakly admissible points with respect to $b$ in $\mathcal{F}_\lambda(\mathbb{C}_p)$ is an admissible open subset of $\mathcal{F}_{\lambda\breve{E}}$ as a rigid analytic space.*

Now we sketch the proof of the theorem in [**RZ96**]. By [**Kot85**] one may assume that the $\sigma$-conjugacy class of $b$ is decent with the decent equation $(b\sigma)^s = s\nu(p)\sigma^s$ as in 2.3. Let $V$ be a faithful representation in $\mathrm{Rep}_{\mathbb{Q}_p}(G)$, $V_s = V \otimes \mathbb{Q}_{p^s}$, and $\Phi_s = b(\mathrm{id} \otimes \sigma)$. Then $(V \otimes K_0, b(\mathrm{id} \otimes \sigma)) = (V_s, \Phi_s) \otimes_{\mathbb{Q}_{p^s}} K_0$. Put $V_s = \bigoplus_\lambda V_{s,j}$ to be the isotypical decomposition for $\Phi_s$. The functor

$$R \mapsto \{V' \subset V_s \otimes_{\mathbb{Q}_{p^s}} R \mid V' \text{ is a direct summand with } V' = \bigoplus_j V' \cap (V_{s,j} \otimes_{\mathbb{Q}_{p^s}} R)\}$$

on the category of $\mathbb{Q}_{p^s}$-algebras is represented by a disjoint sum $T'$ of closed subschemes of Grassmannians of $V_s$. $T'$ descends to a $\mathbb{Q}_p$-variety $T$ and one has

$$T(\mathbb{Q}_p) = \{\Phi_s\text{-stable subspaces of } V_s\}.$$

Indeed, $\Phi_s$ gives a descent datum $\alpha : T' \to T'^{\sigma}$, where $T'^{\sigma}(R)$ is a set of direct summands of $V_{s,\lambda} \otimes_{\mathbb{Q}_{p^s},\sigma} R$ with the isotypical decomposition, and $\alpha^{s-1} \circ \cdots \circ \alpha : T' \to T'$ is the identity by the decent equation.

Consider the closed subscheme over $\mathbb{Q}_{p^s}$

$$\mathcal{H} \subset (\mathrm{Flag}_\lambda(V) \times T) \otimes_{\mathbb{Q}_p} \mathbb{Q}_{p^s}$$

which consists of pairs $(F^\bullet, V')$ such that

$$\sum_i i \,\mathrm{rank}\ \mathrm{gr}^i_{F^\bullet \cap V'}(V') > \mathrm{ord}_p(\det(\Phi_s|_{V'_j})).$$

Then, by the definition of weak admissibility, one has

$$\mathcal{F}^{wa}_{\lambda,b}(\mathbb{C}_p) = \mathcal{F}_\lambda(\mathbb{C}_p) \cap \left(\mathrm{Flag}_\lambda(V)(\mathbb{C}_p) - \bigcup_{t \in T(\mathbb{Q}_p)} \mathcal{H}_t\right),$$

where $\mathcal{F}_\lambda(\mathbb{C}_p)$ is identified with the image of the immersion $\mathcal{F}_\lambda(\mathbb{C}_p) \subset \mathrm{Flag}_\lambda(\mathbb{C}_p)$.

Fix embeddings of $\mathrm{Flag}_\lambda(V)$ and $T$ in projective spaces over $\mathbb{Q}_p$ and a finite set $\{f_j\}$ of bi-homogeneous polynomials of definition of $\mathrm{Flag}_\lambda(V) \times T$ with integral coefficients. For $\epsilon > 0$, consider a tubular neighbourhood

$$\mathcal{H}_t(\epsilon) = \{x \in \mathrm{Flag}_\lambda(V)(\mathbb{C}_p) \mid |f_j(x,t)| < \epsilon \text{ for all } j\}$$

of $\mathcal{H}_t$. Here we choose unimodular representatives for $x$ and $t$'s and $|\ |$ is an absolute value on $\mathbb{C}_p$. Then there is a finite set $S \subset T(\mathbb{Q}_p)$ such that $\bigcup_{t \in T(\mathbb{Q}_p)} \mathcal{H}_t(\epsilon) = \bigcup_{t \in S} \mathcal{H}_t(\epsilon)$ for the local compactness. $\mathcal{F}_\epsilon = \mathcal{F}_\lambda(\mathbb{C}_p) \cap (\mathrm{Flag}_\lambda(V)(\mathbb{C}_p) - \bigcup_{t \in T(\mathbb{Q}_p)} \mathcal{H}_t(\epsilon))$ is an admissible open subset of $\mathcal{F}^{wa}_{\lambda,b\breve{E}}$, hence

$$\mathcal{F}_1 \subset \mathcal{F}_{\frac{1}{2}} \subset \mathcal{F}_{\frac{1}{3}} \subset \cdots$$

is an admissible covering of $\mathcal{F}^{wa}_{\lambda,b\breve{E}}$. □