Change-point Problems
IMS Lecture Notes – Monograph Series (Volume 23, 1994)

ON STOPPING RULES FOR THE SETS SCHEME: AN APPLICATION OF THE SUCCESS RUNS THEORY

BY GIOVANNA CAPIZZI
University of Padua

A stopping rule is proposed for the SETS scheme (Chen, 1978). The methods of generating functions and partial fractions are applied to the theory of the *success runs*. Under the hypothesis that a change in intensity of a Poisson process occurs very far from the origin of the observations, two different expressions are derived for the average delay in detecting increases in birth defect rates.

Comparisons are made with the CUSUM scheme, which appears to be more efficient than the SETS scheme, in detecting increases in malformation rates.

1. Introduction. Suppose we are monitoring the rate of occurrence of a rare health event in a specified community, e.g. a specific congenital malformation in a single hospital.

Under the null hypothesis H_0 of a homogeneous rate of birth defects, the number of normal births per unit of time is very large and the number of malformed births is small. Therefore the malformed births occur according to a realization of a Poisson process with parameter: λ_0. The constant λ_0 is the *baseline rate* of *failures* under the null hypothesis, i.e. λ_0 represents the expected number of births with the same malformation, per unit of time.

Suppose an epidemic situation occurs at an unknown instant of time and the normal rate is subject to an increase of $\gamma > 1$ times the probability of a birth defect, i.e. $\lambda_1 = \gamma\lambda_0$. Let ν be this change-point. The situations $\nu = 0$ and $\nu = \infty$ correspond to the situations of a change at the initial time of observation and of no-change or stationarity, respectively.

Sequential surveillance systems can make easier the detection of increases in the rare diseases intensity. Since the interarrival times for a homogeneous Poisson process are i.i.d. exponential random variables, inspections schemes with exponentially distributed observations arise naturally in the context of monitoring the occurrence rate of rare events. Therefore let $X_i, (i = 0, 1, 2, ..)$,

---

AMS 1991 Subject Classification: Primary 62L10; Secondary 62N10.

Key words and phrases: Change-point problem, CUSUM Schemes, sequential detection, SETS schemes.

be the waiting times between consecutive events of the Poisson process. They are independent random variables with a common exponential distribution F_0. Under the hypothesis that exactly one change occurs at ν, we have:

$$H_1 : X_1, \ldots, X_\nu \sim F_0 \text{ and } X_{\nu+1}, X_{\nu+2}, \ldots \sim F_1$$

where F_0 and F_1 are exponential distributions with parameters λ_0^{-1} and λ_1^{-1} respectively.

Monitoring procedures to detect changes in the parameters of distributions are designed to minimize the delay between the increase and its detection and to optimize the probabilities of real and false alarms. These conditions are satisfied by two different techniques, currently used for the surveillance of rare health events. The first, called SETS scheme (Chen, 1978), has been conceived specially for health monitoring. In this kind of surveillance system, analyses are carried out sequentially as soon as a new case is diagnosed and the decision is based on a fixed number of previous observations. The second procedure constitutes an adaptation of CUSUM scheme (Page, 1954), widely used in quality control systems. The decision that a change has occurred is based on all previous observations.

The problem of detecting the change at ν as soon as possible can be formalized for both these procedures by devising a *stopping rule*, i.e. a random variable N taking non negative integer values. For any observation a certain condition must be satisfied when the process is "in control" state while its violation implies that the process is "out of control" and an alarm should be declared as soon as possible. It is shown that a stopping rule N that minimizes the functional:

$$f(N) = \sup_{0 \leq \nu < \infty} E_\nu[N - \nu \mid N \geq \nu] \tag{1}$$

subject to a restriction on the frequency of false reactions:

$$E_\infty[N] \geq \Delta, \qquad \Delta > 0 \tag{2}$$

is optimal for detecting changes in ν (Lorden, 1971; Moustakides, 1986). Moreover, it has been shown that under certain conditions CUSUM and SETS schemes are "equalizer rules" (Pollak and Siegmund, 1975 and Radaelli, 1988), that is the upper extreme of the right part of (1) is equal to the *average run length* (ARL) out of control, $E_0[N]$. Therefore, for a fixed large value of the ARL in control, $E_\infty[N]$ (i.e. the expected number of observations until a first alarm, when $\nu = \infty$), the efficiency of different rules can be established in terms of minimum $E_0[N]$. This approach is based on the assumption that the epoch of the change ν is near zero.

However, it can be reasonably argued that a monitoring system starts in a control situation and therefore ν is usually large. Under the last assumption, this paper uses the theory of *success runs* (Feller, 1968) to define the optimality of a stopping rule for the SETS scheme.

Section 2 contains a brief description of the SETS procedure for a small scale system. In Section 3 the methods of the generating functions and the partial fractions are used to derive for Chen's procedure two different approximations for the out of control ARL, under the hypothesis that ν is very far from the origin. The CUSUM scheme is discussed in Section 4. Finally, optimal values for both procedures are compared. The results show that, even if a more reasonable hypothesis about the time of change is introduced, the CUSUM scheme is more efficient than the SETS scheme for any value of gamma and for the hypotheses of ν near the origin and far from it.

2. Set Technique. A small scale SETS scheme is based on the analysis of the number of consecutive normal births occurring between the birth of an infant with a specific malformation and the birth of another infant with the same malformation. These sequences of consecutive observations are called *Sets*. An alarm is declared whenever a *full* sequence of n sets, or time intervals, occurs such that each set has a size smaller than some specified *reference value* R, generally defined in terms of a multiple k of the expected interval $E[X_i] = 1/\lambda_0$. This monitoring system is reset to zero after each alarm. The number of included intervals n and the k value represent the SETS parameters. They are related to two basic characteristics of the surveillance schemes: the risk of false alarms and the expected delay for an alarm.

Under the null and the alternative hypotheses respectively, the probabilities $P_j, (j = \infty, 0)$, that a sequence of n consecutive time intervals signals an alarm are:

$$P_\infty(n) = p_0^n = [1 - \exp(-k)]^n \tag{3}$$

$$P_0(n) = p_1^n = [1 - \exp(-k\gamma)]^n. \tag{4}$$

Chen (1978, 1987) proposed combinations of the parameters n and k, imposing that under stable conditions the realization of the appearance of n consecutive short sets is rare, and relatively common in epidemic situations.

Kenett and Pollak (1983) introduced an equation to find the expected number of diagnoses between alarms for the Chen's procedure. This expected value, under the hypotheses H_0 and H_1 respectively, is equal to:

$$E_{s,p_j}[N] = \frac{1 - p_j^n}{p_j^n q_j} \tag{5}$$

with $s = \infty, 0$ and $q_j = 1 - p_j$, $j = 0, 1$.

A suitable stopping rule N for the SETS scheme might be the random variable counting the number of sets before the first alarm. The optimality of this stopping rule, considering the *waiting times* for the appearance of adjacent sets of consecutive events, with a preassigned integer size, can be evaluated solving problem (1) and (2). Gallus and Radaelli (1988) find in this class the following stopping rule. Let $\tau^{(m)}$ be the waiting times for the occurrence of a specified number $m \geq 1$ of consecutive events. The stopping rule is $N = \inf[s \geq n : \max_{i=s+1-n,\ldots,s} \tau_i^{(m)} < T]$. For this procedure these authors evaluate an analytical expression for the ARL, which is equivalent to (5) with $p_j = F_j(T) = P_j[\tau^{(m)} < T]$, $j = 0, 1$. They also use an iterative procedure to find optimal combinations of the parameters n and k. This combination allows to obtain the minimum value of the delay $E_0[N]$.

On the basis of all these previous works, it may be concluded that of the SETS and the CUSUM methods, the first procedure is to be preferred for detection of relatively large increases in the incidence of rare diseases. However observe that the CUSUM parameters, used in these comparisons, were taken from Ewan and Kemp's tables (1960). These approximate values are those associated with an expected interval of about 500 observations between false alarms and an expected delay until detection of about 3 or 7 units of time, but they are not the optimal values of parameters for a fixed level of the in control ARL.

In the next paragraph a stopping rule N, for a different form of the problem (1) and (2), is proposed using two different methods of the recurrent events theory. An analytical expression of the ARL, equivalent to (5), is derived. Values of the parameters n and k which minimize $E_0[N]$, for a fixed large value of $E_\infty[N]$, are determined. Then the results are not compared with the CUSUM values listed in Ewan and Kemp's tables, but with the "optimal" values of the stopping rule associated to this scheme, which were obtained by the Markov-chain approach (Brook and Evans, 1972).

3. A Stopping Rule for the SETS Scheme. Define as "success" the event $\mathbf{S} = \{$the time interval, between two consecutive events of a Poisson process, is shorter than the reference value $R = k/\lambda_0\}$. The alarm situation can be considered equivalent to the *recurrent event:* $\zeta = \{$*a success run* of length n occurs$\}$. It is possible to associate with ζ two sequences of numbers defined for $s = 1, 2 \ldots, u_s = P(A) = \mathbf{P}\{e$ occurs at the sth trial$\}$ and $f_s = P(B) = \mathbf{P}\{$the *first success run of length* n occurs at the sth trial$\}$. For convenience $u_0 = 1$ and $f_0 = 0$.

The generating functions are $F(a) = \sum_{i=0}^{\infty} f_i a^i$ and $U(a) = \sum_{i=0}^{\infty} u_i a^i$ respectively. Events like A are mutually exclusive and so we have $f = F(1) = \sum_1^{\infty} f_i \leq 1$.

It is possible to introduce a random variable T representing the number of trials up to the first occurrence of ζ and having distribution $f_s = P\{T = s\}$. The random variable T is an improper random variable because, under the hypothesis that ζ never occurs, it does not assume a numerical value with probability $(1-f)$. The r.v. T is by definition equivalent to the stopping rule:

$$T = \inf\{s : X_s < kE_0[X_i], X_{s-1} < kE_0[X_i], \ldots\ldots, X_{s-n+1} < kE_0[X_i]\}. \quad (6)$$

It is proved that the relationship existing between the generating functions of $\{u_s\}$ and $\{f_s\}$, can be used to find the generating function of the *recurrence times* T_r, standing for the waiting times between the successive occurrence of ζ (Feller, 1968):

$$F(a) = \frac{p_j^n a^n (1-p_j)}{1 - a + q_j p_j^n a^{n+1}} \quad (7)$$

where p_j and q_j are given by (3) and (4) for $j = 0, 1$.

Differentiation of (7) leads to the *mean* of the recurrence times of runs of length n, respectively under H_0 and H_1, that are equal to the in and out ARL, derived by Kenett and Pollak (1983) and Gallus and Radaelli (1988).

It can be useful to show the same results for the general Markov chain associated to the success runs. The SETS procedure forms a discrete Markov chain with $(n+1)$ possible states $i, i = 0, 1, .., \text{n}$. Beginning from the state $i = 0$ (counting as a failure) the system, after each observation, goes to the state $(i+1)$ with probability p, if the observed time interval is shorter than R, or comes back to $i = 0$ with probability q. Finally, when i is equal to n, an alarm is declared and the system is reset to zero, i.e. $i = 0$. The corresponding transition matrix is

$$\|P_{ij}\| = \begin{bmatrix} q & p & 0 & \cdots & 0 \\ q & 0 & p & \cdots & 0 \\ . & . & . & . & . \\ q & 0 & 0 & \cdots & p \\ q & p & 0 & \cdots & 0 \end{bmatrix}. \quad (8)$$

The long-run stationary distribution corresponding to (8), π, determines the relative frequency of the possible states. The $(n+1)$ elements of the vector π are

$$\pi_i = \begin{cases} 1-p & i = 0 \\ \frac{p^i(1-p)}{1-p^n} & i = 1, 2, \ldots, n. \end{cases} \quad (9)$$

The relative frequency of alarms is represented by the element π_n, while the expected number of diagnoses until an alarm is declared is equal to $D = \pi_n^{-1}$, which is equivalent to (5).

Under stationary conditions and starting from the state zero (or n), D represents the in control average run length. On the other hand, supposing there is an abrupt change in the baseline rate, when the system is in the state zero, the number of diagnoses until an alarm is declared is equal to the out of control average run length, that is D with p given in (3) and (4) respectively. Several papers (Kenett and Pollak, 1983, Gallus and Radaelli, 1988) are based on this assumption.

However the change point ν can occur in any unknown instant of time so it may be convenient to consider the probability that the system is in a certain state i before an increase occurs and the expected number of further observations until an alarm is declared, given the number of consecutive successes. Let ξ_ν be the state of the observed process at time ν, that is the number of consecutive successes ending at the trial corresponding to the time ν. For a fixed ν, the delay between the rise in the incidence and the first alarm may be expressed by $E_\nu[T-\nu \mid T>\nu]$. We define the optimality of the stopping rule T solving a problem similar to the (1) and (2) and defining two different forms for the functional (1). In order to reduce the delay, we propose for the original SETS procedure the minimization of the following functional

$$\lim_{\nu\to\infty} E_\nu[T-\nu \mid T>\nu] = \sum_{i=0}^{n-1} \lim_{\nu\to\infty} P_\nu\{\xi_\nu = i \mid T>\nu\} E_0[T \mid \xi_0 = i] \qquad (10)$$

subject to

$$E_{\lambda_0}[T] = \frac{1-p_0^n}{q_0 p_0^n} = B \qquad (11)$$

with B a positive large constant and T given by (6).

Suppose $\{\xi_\nu = i\}$. An alarm is declared if, after time ν, a success run of length $(n-i)$ occurs. We can describe any possible results in the following scheme:

Observed frequency after $\xi_\nu = i$	Number of trials until ζ occurs $(T-\nu)$	Probability
$(n-i)$ successes	$(n-i)$	p_1^{n-i}
$(n-i-1)$ successes 1 failure	$(n-i)+E_{\lambda_1}[T \mid \xi_0 = 0]$	$p_1^{n-i-1}q_1$
$(n-i-2)$ successes 1 failure	$(n-i-1)+E_{\lambda_1}[T \mid \xi_0 = 0]$	$p_1^{n-i-2}q_1$
......	......	......
1 success 1 failure	$2+E_{\lambda_1}[T \mid \xi_0 = 0]$	$p_1 q_1$
1 failure	$1+E_{\lambda_1}[T \mid \xi_0 = 0]$	q_1

where $E_{\lambda_1}[T \mid \xi_0 = 0] = [1 - p_1^n]/[q_1 p_1 n]$ stands for the "start over". In fact, when an interval larger than the reference value occurs after the i-th success, the monitoring system is re-set to zero.

The distribution $P\{\xi_\nu = i \mid T > \nu\}$ can be approximated by following two different approaches. The first approach uses, as a reasonable approximation justified by Pollak and Siegmund (1986), the stationary distribution π. Therefore the functional (10) can be expressed as follows:

$$\lim_{\nu\to\infty} E_\nu[T - \nu \mid T > \nu] \cong \sum_{i=0}^{n-1} \pi_i E_0[T \mid \xi_0 = i] =$$

$$\sum_{i=0}^{n-1} \left\{ p_0^i \frac{1-p_0}{1-p_0^n} \left[q_1 \sum_{j=1}^{n-i} \left(j + \frac{1-p_1^n}{q_1 p_1^n} \right) p_1^{j-1} + p_1^{n-i}(n-i) \right] \right\}. \tag{12}$$

Another expression for the functional (10) can be derived using the method of partial fractions, proposed by Feller (1968), to find an approximation for the probability of no success run of length n in s trials. Let W be a random variable assuming values 0 and 1 with probabilities q_0 and p_0 when a failure and a success occurs respectively.. The event $\{\xi_\nu = i\}$ is equivalent to the situation in which we have observed i consecutive successes at times $\nu - i + 1, \nu - i + 2, \ldots \nu$ and a failure at time $\nu - i$, that is it is equivalent to the event:

$$A_{\nu,i} = \{W_\nu = \ldots = W_{\nu-i+1} = 1, W_{\nu-i} = 0 \mid T > \nu\}$$

The probability $P\{\xi_\nu = i \mid T > \nu\}$ is equal to:

$$\begin{aligned} P_\nu\{A_{\nu,i} \mid T > \nu\} &= \frac{P_\infty\{A_{\nu,i} \cap T > \nu\}}{P_\infty\{T > \nu\}} = \frac{P_\infty\{A_{\nu,i} \cap T \geq \nu - i\}}{P_\infty\{T > \nu\}} \\ &= \frac{P_\infty\{A_{\nu,i} \cap T > \nu - i - 1\}}{P_\infty\{T > \nu\}} = \frac{P_\infty\{A_{\nu,i}\} P_\infty\{T > \nu - i - 1\}}{P_\infty\{T > \nu\}}. \end{aligned} \tag{13}$$

The probability $P_\infty\{A_{\nu,i}\}$ is equal to $q_0 p_0^i$, while the other two components, $q_{\nu-i-1} = P_\infty\{T > \nu - i - 1\}$ and $q_\nu = P_\infty\{T > \nu\}$, may be easily computed by the method of partial fractions. This method provides good approximations for the probabilities of no success runs of length n in $(\nu - i - 1)$ and in ν trials (Feller, 1968). In particular we find:

$$q_{\nu-i-1} \approx \frac{1 - p_0 x}{(n + 1 - nx) q_0 x^{\nu-i}}, \quad q_\nu \approx \frac{1 - p_0 x}{(n + 1 - nx) q_0 x^{\nu+1}}$$

where x is the smallest root greater than one of the equation: $1 - x + q_0 p_0^n x^{n+1} = 0$.

Given these approximations the probability (13) is equal to:

$$P_\nu\{A_{\nu,i} \mid T > \nu\} \approx q_0 p_0^i x^{i+1}$$

and another approximation for the functional (10) may be obtained:

$$E_\nu[T - \nu \mid T > \nu] = \sum_{i=0}^{n-1} P_\nu\{A_{\nu,i} \mid T > \nu\} E_\nu[\xi_\nu = i \mid T > \nu] \approx$$

$$\sum_{i=0}^{n-1} \left\{ q_0 p_0^i x^{i+1} \left[q_1 \sum_{j=1}^{n-i} \left(j + \frac{1 - p_1^n}{q_1 p_1^n} \right) p_1^{j-1} + p_1^{n-i}(n-i) \right] \right\}. \tag{14}$$

4. Computation of CUSUM Parameters. Consider a CUSUM scheme with exponentially distributed random variables. The stopping rule for this scheme is: $T = \inf_i\{i : S_i \geq h\}$, where $S_i = \max\{0, S_{i-1} + k - x_i\}$, $S_0 = 0$ and x_i are the observed data. A Markov chain approach (Brook and Evans, 1972) may be used to obtain the average run length. The Markov chain is based on $m+2$ states; each state is defined by a reference value u_i ($u_i = 0$ if $i = 1, m+2$ and $(i-1)\Delta$ otherwise with $\Delta = 2h/[2m+1]$) and an interval I_i ($I_1 = (-\infty, \Delta/2]$, $I_2 = (\Delta/2, 3\Delta/2], \ldots, I_{m+1} = (h - \Delta, h], I_{m+2} = (h, +\infty)$). This approach leads to the exact average run length of the approximating process: $S_i^* = u(S_{i-1}^*) + k - x_i$, where $u(y)$ is equal to u_i, if $y \in I_i$. Since the scheme is reset to zero whenever an alarm is declared, the reference value of the last state (out of control situation) is equal to zero. By the standard Markov theory, it is possible to derive the stationary distribution, π, of the Markov chain formed by the sequence of intervals reached by S_i^*, and the average number of steps to reach the $(m+2)$-th state, starting from the state i, $\mu_i(\gamma)$. Therefore the ARL^*, that is the analogous of (12), is given by:

$$(\pi_1 + \pi_{m+2})\mu_1(\gamma) + \pi_2\mu_2(\gamma) + \cdots + \pi_{m+1}\mu_{m+1}(\gamma).$$

Observe that π_{m+2} multiplies $\mu_1(\gamma)$ since we are assuming that the CUSUM is reset to zero when an alarm is declared.

5. Results and Discussion. Let B_1 and B_2 denote two larges values of the constant B, 500 and 750. First, we find values (n, k) such that (11) is equal to $B_i, i = 1, 2$. Then for fixed values of the increase rate γ, i.e. $1.5, 2.00, \ldots, 7.00$, minimum values D^* of (12) and (14) are derived (Tables I and II). It is possible to observe that as γ increases, n and k decrease until $\gamma = 5.00$, while after that threshold their values begin to be constant while the corresponding values of (12) and (14) are still decreasing. Observe that (12) and (14) lead to close values of n and k. Indeed, (12) and (14) give almost similar results for all values of n and k with (14) typically smaller. This is probably due to the dependence of (14) only on ν, while the approximation (12) assumes that both ν and B diverge. Tables I and II also show the corresponding optimal values of the CUSUM scheme (h,k,ARL^*) obtained setting $m = 120$. It should be noted that the performance of this scheme is better than (12) and (14) for all values of γ.

Moreover, recall that a minimax criterion was used in the stopping rule solving problems (1) and (2). This formulation has the drawback that an optimal value of the functional is attained when the true change-point is equal to zero. Table III gives the corresponding values of n and k and the functional denoted by ARL_0. It is evident, from tables I, II and III that the different criteria do not lead to essentially different results when γ is greater than 3. In other words, in order to detect a large increase, the solutions of problems (1), (2) and (10), (11) are nearly optimal for all change-points, both close to zero and close to infinity.

Furthermore, observe that CUSUM and SETS schemes are usually compared for specific values of B and γ. However, since the true value of γ is unknown, it is important to evaluate a procedure which is optimal for one choice of γ, at other values of γ as well. As the two criteria (12) and (14) lead to identical schemes, for two values of γ, i.e. 2 and 5 and for B equal to 750, these values have been chosen to compare the performance of both procedures. In particular the optimal CUSUM and SETS schemes are compared for all values of γ, in terms of expected number of observations until an alarm (Tables IV and V). The result show that the optimal values of CUSUM always lead to a more efficient solution over the optimal SETS method, for any γ.

Table I. Minimum values of (12) and (14) and corresponding values of the parameters n and k, for $B = 500$

γ	n	k	$D^*_{(12)}$	n	k	$D^*_{(14)}$	h	k	ARL^*
1.5	25	1.8232	59.69	25	1.8232	59.59	8.3296	0.8529	33.81
2.0	15	1.2686	27.59	16	1.3347	27.56	4.8451	0.7309	17.42
2.5	12	1.0496	18.03	12	1.0496	18.02	3.4347	0.6459	12.23
3.0	9	0.7922	13.67	10	0.8830	13.67	2.6572	0.5815	9.72
3.5	8	0.6959	11.18	8	0.6959	11.17	2.1679	0.5312	8.23
4.0	7	0.5937	9.61	7	0.5937	9.60	1.8270	0.4901	7.25
4.5	7	0.5937	8.53	7	0.5937	8.58	1.5818	0.4565	6.56
5.0	6	0.4855	7.69	6	0.4855	7.69	1.3922	0.4275	6.03
5.5	6	0.4855	7.09	6	0.4855	7.09	1.2415	0.4025	5.62
6.0	5	0.3720	6.61	5	0.3720	6.60	1.1128	0.3795	5.29
6.5	5	0.3720	6.18	5	0.3720	6.17	1.0179	0.3617	5.02
7.0	5	0.3720	5.85	5	0.3720	5.84	0.9344	0.3447	4.79

Table II. Minimum values of (12) and (14) and corresponding values of the parameters n and k, for B = 750

γ	n	k	$D^*_{(12)}$	n	k	$D^*_{(14)}$	h	k	ARL^*
1.5	29	1.8925	72.97	27	1.8116	73.01	9.0120	0.8460	38.60
2.0	18	1.3742	32.06	18	1.3742	32.03	5.2122	0.7259	19.37
2.5	13	1.0541	20.49	13	1.0541	20.47	3.6815	0.6413	13.45
3.0	11	0.9025	15.35	11	0.9025	15.34	2.8461	0.5778	10.62
3.5	9	0.7334	12.46	9	0.7334	12.45	2.3189	0.5278	8.96
4.0	8	0.6414	10.64	8	0.6414	10.6	1.9567	0.4872	7.87
4.5	7	0.5441	9.39	7	0.5441	9.38	1.6880	0.4532	7.10
5.0	7	0.5441	8.48	7	0.5441	8.48	1.4861	0.4247	6.52
5.5	6	0.4415	7.77	6	0.4415	7.76	1.3268	0.4000	6.07
6.0	6	0.4415	7.21	6	0.4415	7.20	1.1982	0.3786	5.70
6.5	6	0.4415	6.80	6	0.4415	6.80	1.0895	0.3594	5.40
7.0	5	0.3345	6.39	5	0.3345	6.39	0.9939	0.3417	5.15

Table III. Minimum values of ARL_0 and corresponding values of the parameters n and k, for B = 500 and B = 750

	$B = 500$			$B = 750$		
γ	n	k	ARL_0	n	k	ARL_0
1.5	21	1.6262	66.45	25	1.7257	80.48
2.0	14	1.1992	30.75	16	1.2549	35.46
2.5	11	0.9686	20.05	12	0.9803	22.62
3.0	9	0.7922	15.12	10	0.8204	16.87
3.5	8	0.6959	12.34	8	0.6414	13.67
4.0	7	0.5937	10.54	8	0.6414	11.65
4.5	6	0.4855	9.29	7	0.5441	10.21
5.0	6	0.4855	8.39	6	0.4415	9.19
5.5	5	0.3720	7.72	6	0.4415	8.39
6.0	5	0.3720	7.12	6	0.4415	7.81
6.5	5	0.3720	6.67	5	0.3345	7.28
7.0	5	0.3720	6.33	5	0.3345	6.84

Table IV. Comparison of SET and CUSUM schemes optimal for $\gamma = 2$, $B = 750$

	SET			CUSUM	
γ	ARL_0	$D_{(12)}$	$D_{(14)}$	ARL_0	ARL^*
1.0	750.00	735.91	735.73	750.00	733.78
1.5	83.23	78.06	78.03	48.70	44.01
2.0	35.77	32.06	32.04	22.39	19.37
2.5	24.92	21.68	21.67	16.37	13.95
3.0	21.10	18.06	18.04	13.80	11.70
3.5	19.47	16.53	16.51	12.40	10.48
4.0	18.72	15.82	15.81	11.51	9.72
4.5	18.36	15.48	15.46	10.90	9.20
5.0	18.18	15.31	15.30	10.45	8.82
5.5	18.09	15.23	15.21	10.12	8.54
6.0	18.04	15.18	15.17	9.85	8.31
6.5	18.02	15.16	15.15	9.64	8.13
7.0	18.01	15.15	15.14	9.46	7.98

Table V. Comparison of SET and CUSUM schemes for $\gamma = 5$, $B = 750$

	SET			CUSUM	
γ	ARL_0	$D_{(12)}$	$D_{(14)}$	ARL_0	ARL^*
1.0	750.00	744.73	744.72	750.00	744.86
1.5	132.22	129.94	129.97	110.58	108.36
2.0	49.65	48.10	48.10	36.50	35.07
2.5	27.16	25.94	25.94	19.23	18.15
3.0	18.33	17.29	17.29	13.13	12.23
3.5	14.04	13.11	13.11	10.30	9.52
4.0	11.66	10.79	10.79	8.75	8.04
4.5	10.21	9.389	9.387	7.79	7.13
5.0	9.277	8.484	8.482	7.13	6.52
5.5	8.645	7.874	7.873	6.67	6.08
6.0	8.205	7.450	7.449	6.32	5.76
6.5	7.892	7.149	7.147	6.05	5.51
7.0	7.665	6.931	6.929	5.83	5.31

REFERENCES

Brooks, D. and Evans, D. A. (1972), An Approach to the Probability Distribution of Cusum Run Length, *Biometrika* **59**, 539–549.

Chen, R. (1978), A Surveillance System for Congenital Malformations *J. Amer. Statist. Assoc.* **73**, 323–327.

CHEN, R. (1987), The relative efficency of the sets and the Cusum techniques in monitoring the occurrence of a rare event, *Statist. in Med.* **6**, 517–525.

EWAN, W. D. and KEMP, K. W. (1960), Sampling inspection of continuous processes with no autocorrelation between successive results, *Biometrika* **47**, 363–380.

FELLER, W. (1968), *An Introduction to Probability Theory and Its Applications*, Vol. 1, III Edition, New York: John Wiley & Sons.

GALLUS G. and RADAELLI, G. (1988), Schemi di sorveglianza per individuare un cambiamento nella dinamica di eventi sanitari rari, *Atti della XXXIV Riunione Scientifica della Societá Italiana di Statistica*, Siena 27-30 Aprile 1988, 1, 313–320.

KENETT, R. and POLLAK, M. (1983), On Sequential Detection of a Shift in the Probability of a Rare Event, *J. Amer. Statist. Assoc.* **78**, 389–395.

LORDEN, G. (1971), Procedures for reacting to a change in distribution, *Ann. Statist.* **42**, 1897–1908.

MOUSTAKIDES,G. V. (1986), Optimal stopping times for detecting changes in distributions, *Ann. Statist.* **14**, 1379–1387.

POLLAK, M. and SIEGMUND, D. O. (1975), Approximations to the expected sample size of certain sequential tests, *Ann. Statist.* **3**, 1267–1282.

POLLAK, M.and SIEGMUND, D. O. (1986), Convergence of quasi-stationary to stationary for stochastically monotone Markov processes, *J. Appl. Probab.* **23**, 215–220.

RADAELLI, G. (1988), On the stopping rule defining sets schemes for detecting a change in distribution, *Statistica*, anno XLVIII, 229–233.

DIPARTIMENTO DI SCIENZE STATISTICHE
UNIVERSITÀ DI PADOVA
VIA SAN FRANCESCO 33
PADUA 35121, ITALY