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Research Article

Gutman Index and Detour Gutman Index of Pseudo-Regular Graphs

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The Gutman index of a connected graph G is defined as $\mathrm{Gut}(G) = \sum_{u \neq v} d(u)d(v)d(u,v)$, where $d(u)$ and $d(v)$ are the degree of the vertices u and v and $d(u,v)$ is the distance between vertices u and v. The Detour Gutman index of a connected graph G is defined as $\mathrm{Gut}(G) = \sum_{u \neq v} d(u)d(v)D(u,v)$, where $D(u,v)$ is the longest distance between vertices u and v. In this paper, the Gutman index and the Detour Gutman index of pseudo-regular graphs are determined.

1. Introduction

All graphs considered in this paper are simple, connected, and finite. A graph is a collection of points and lines connecting a subset of them. The points and lines of a graph G are also called vertices and edges of the graph and are denoted by $V(G)$ and $E(G)$, respectively. For $u, v \in V(G)$, the distance between u and v in G, denoted by $d(u,v)$, is the length of the shortest (u,v)-path in G. The number of vertices of G adjacent to a given vertex v is the degree of this vertex and it is denoted by $d(v)$.

A topological index is a real number related to a graph. It does not depend on the labeling or pictorial representation of a graph. The Wiener index of G, $W(G)$ is defined as $W(G) = \sum_{u \neq v} d(u,v)$, where the sum is taken through all unordered pairs of vertices of G. Wiener index was introduced by Wiener, as an aid to determine the boiling point of Paraffin [1]. It is related to boiling point, heat of evaporation, heat of formation, chromatographic relation times, surface tension, vapour pressure, partition coefficients, total electron energy of polymers, ultrasonic sound velocity, internal energy, and so on [2]. For this reason, Wiener index is widely studied by chemists.

In [3], Gutman defined the modified Schultz index, which is known as the Gutman index as a kind of a vertex-valency-weighted sum of the distance between all pair of vertices in a graph. Gutman revealed that, in the case of acyclic structures, the index is closely related to the Wiener index and reflects precisely the same structured features of a molecular as the Wiener index does.

The Gutman index of G, denoted by $\mathrm{Gut}(G)$, is defined as $\mathrm{Gut}(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u)d_G(v)d_G(u,v) = (1/2)\sum_{u,v \in V(G)} d_G(u)d_G(v)d_G(u,v)$ with the summation runs over all pair of vertices of G. Dankelmann et al. [4] presented an upper bound for the Gutman index and also established the relation between the Edge-Wiener index and Gutman index of graphs. Chen and Liu [5] studied the maximal and minimal Gutman index of unicyclic graphs, and they also determined the minimal Gutman index of bicyclic graphs [6].

If G is a tree on n vertices, then Wiener index and Gutman index are closely related by $\mathrm{Gut}(G) = 4W(G) - (2n-1)(n-1)$. For path graph P_n, $W(P_n) = n(n^2-1)/6$, and for star graph S_n, $W(S_n) = (n-1)^2$. Thus, $\mathrm{Gut}(P_n) = (n-1)(2n^2-4n+3)/3$ and $\mathrm{Gut}(S_n) = (2n-3)(n-1)$. Also, for every tree T_n of order n, $\mathrm{Gut}(S_n) \leq \mathrm{Gut}(T_n) \leq \mathrm{Gut}(P_n)$.

For any acyclic connected graph, the shortest distance $d(u,v)$ is the same as the longest distance $D(u,v)$. But for cyclic structured graphs, the shortest distance $d(u,v)$ naturally is not equal to the longest distance $D(u,v)$. In this paper, the new index which is Gutman index of graphs with

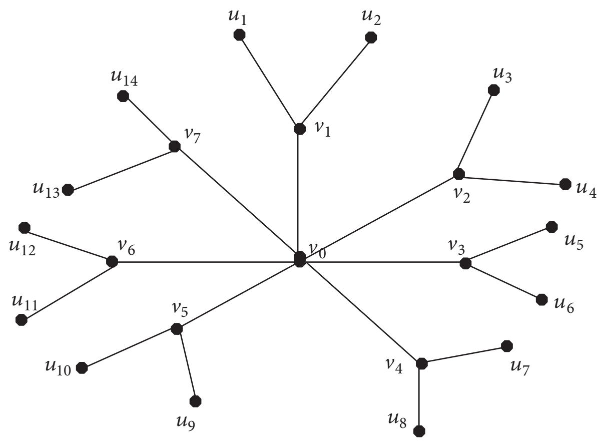

Figure 1

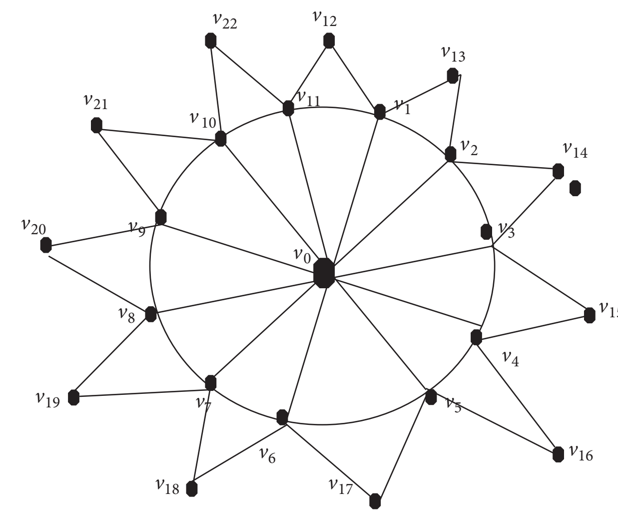

Figure 2

respect to detour distance [7, 8] is considered; this is named as Detour Gutman index which is defined as $D\,\mathrm{Gut}(G) = \sum_{u\neq v} d(u)d(v)D(u,v)$.

A graph G is called pseudo-regular graph [9, 10] if every vertex of G has equal average degree. The main goal of this paper is to find the exact formula for Gutman index and Detour Gutman index of pseudo-regular graphs.

2. Pseudo-Regular Graphs

Let $G = (V, E)$ be a simple, connected, undirected graph with n vertices and m edges. For any vertex $v_i \in V$, the degree of v_i is the number of edges incident on v_i. It is denoted by d_i or $d(v_i)$. A graph G is called regular if every vertex of G has equal degree. A bipartite graph is called semiregular if each vertex in the same part of a bipartition has the same degree. The 2-degree v_i [9] is the sum of the degree of the vertices adjacent to v_i and denoted by t_i [11]. The average degree of v_i is defined as t_i/d_i. For any vertex $v_i \in V$, the average degree of v_i is also denoted by $m(v_i) = t_i/d_i$.

A graph G is called pseudo-regular graph [9] if every vertex of G has equal average degree and $m(G) = (1/n)\sum_{v\in V(G)} m(u)$ is the average neighbor degree number of the graph G.

A graph is said to be r-regular if all its vertices are of equal degree r. Every regular graph is a pseudo-regular graph [10]. But the pseudo-regular graph need not be a regular graph. Pseudo-regular graph is shown in Figures 1 and 2.

In Figure 1, there are 14 vertices of degree 1, 7 vertices of degree 3, and 1 vertex of degree 7. So totally there are 22 vertices in the graph. Average degree of vertices of degree 1 is equal to 3/1 = 3. Average degree of vertices of degree 2 is equal to 9/3 = 3. Average degree of vertices of degree 7 is equal to 21/7 = 3. Therefore, average degree of each vertex is 3. Hence, it is a pseudo-regular graph. In Figure 2, average degree of each vertex is 5. Hence, the graph in Figure 2 is also a pseudo-regular graph.

The relevance of pseudo-regular graph for the theory of nanomolecules and nanostructure should become evident from the following. There exist polyhedral (planar, 3-connected) graphs and infinite periodic planar graphs belonging to the family of the pseudo-regular graphs. Among polyhedral, the deltoidal hexecontahedron possesses pseudo-regular property. The deltoidal hexecontahedron is a Catalan polyhedron with 60 deltoid faces, 120 edges, and 62 vertices with degrees 3, 4, and 5, and average degree of its vertices is 4.

In this paper, the Gutman index and Detour Gutman index for three types of pseudo-regular graphs are derived. For construction of pseudo-regular graphs, see [12].

3. Gutman Index and Detour Gutman Index of Type I Pseudo-Regular Graph

Theorem 1. *For $p \geq 2$, the Gutman index of type I pseudo-regular graph G_{I} is*

$$\mathrm{Gut}(G_{\mathrm{I}}) = \frac{2p^7 + 42p^6 - 275p^5 + 3006p^4 - 18889p^3 + 54808p^2 - 74014p + 38040}{2p}. \tag{1}$$

Proof. Let $G = G_{\mathrm{I}}$ be the type I pseudo-regular graph.

Let $V(G) = \{v_0, v_1, v_2, \ldots, v_m, u_1, u_2, \ldots, u_{m(p-1)}\}$ be the vertex set of G and let v_0 be the central vertex of star graph $K_{1,m}$, $\{v_1, v_2, \ldots, v_m\}$ are pendant vertices of $K_{1,m}$, where $m = p^2 - p + 1$, and the $(p-1)$ pendant vertices $\{u_1, u_2, \ldots, u_{m(p-1)}\}$ are attached with m pendant vertices $v_1, v_2, \ldots, v_m$.

Let $E(G) = (v_0v_i; 1 \leq i \leq m\} \cup \{v_1u_j; 1 \leq j \leq p-1\} \cup \{v_2u_j; p \leq j \leq 2p-2\} \cup \{v_3u_j; 2p-1 \leq j \leq 3p-3\} \cup$

$\cdots\{v_m u_j; (m-1)p-m+2 \le j \le mp-m\}$. The pseudo-regular graph of type I for $p=3$ is shown in Figure 3.

The Gutman index of any graph G is given by $\mathrm{Gut}(G) = \sum_{u\neq v} d(u)d(v)d(u,v)$.

Now

$$\begin{aligned}
\mathrm{Gut}(G_{\mathrm{I}}) &= \sum_{i=1}^{m} d(v_0)\,d(v_i)\,d(v_0,v_i) + \sum_{i=1}^{m}\sum_{j=i+1}^{m} d(v_i)\,d(v_j)\,d(v_i,v_j) + \sum_{i=1}^{m(p-1)} d(v_0)\,d(u_i)\,d(v_0,u_i) \\
&\quad + \sum_{i=1}^{m}\sum_{j=1}^{m(p-1)} d(v_i)\,d(u_j)\,d(v_i,u_j) + \sum_{1\le i<j\le n} d(u_i)\,d(u_j)\,d(u_i,u_j) \\
&= \mathrm{Gut}(K_{1,m}) + \sum_{i=1}^{m(p-1)} d(v_0)\,d(u_i)\,d(v_0,u_i) + \sum_{i=1}^{m}\sum_{j=1}^{m(p-1)} d(v_i)\,d(u_j)\,d(v_i,u_j) + \sum_{1\le i<j\le n} d(u_i)\,d(u_j)\,d(u_i,u_j) \\
&= \left[p^6+p^5-13p^4+25p^3-15p^2+3p+8\right] + \left[2p^5-6p^4+10p^3-10p^2+6p-2\right] \\
&\quad + \frac{\left[1141p^4-896p^3+27083p^2-36931p+19020\right]}{p} + \frac{\left[36p^5-237p^4+654p^3-913p^2+624p-164\right]}{2}, \\
\mathrm{Gut}(G_{\mathrm{I}}) &= \frac{2p^7+42p^6-275p^5+3006p^4-18889p^3+54808p^2-74014p+38040}{2p}.
\end{aligned} \tag{2}$$

□

4. Gutman Index and Detour Gutman Index of Type II Pseudo-Regular Graph

Theorem 2. *For $p \ge 3$, the Gutman index of type II Pseudo-regular graph G_{II} is*

$$\mathrm{Gut}(G_{II}) = \frac{3p^9-93p^8+1181p^7-8026p^6+32047p^5-75192p^4+87593p^3+8025p^2-145324p+121314}{2(p-1)}. \tag{3}$$

Proof. Let $G=G_{\mathrm{II}}$ be a type II pseudo-regular graph.

Let $V(G) = \{v_0, v_1, v_2, \ldots, v_m, u_1, u_2, \ldots, u_{m(p-3)}\}$ be the vertex set of G and v_0 as the central vertex of wheel graph W_m and $\{v_1, v_2, \ldots, v_m\}$ are vertices of cycle c_m in the clockwise direction and $\{u_1, u_2, \ldots, u_{m(p-3)}\}$ are the pendant vertices joined to every vertex in the cycle except the central vertex, where $m = p^2-3p+3$.

Let $E(G) = (v_0v_i; 1 \le i \le m\} \cup \{v_iv_{i+1}; 1 \le i \le m-1\} \cup \{v_mv_1\} \cup \{v_1u_j; 1 \le j \le p-3\} \cup \{v_2u_j; p-2 \le j \le 2(p-3)\} \cup \cdots \{v_mu_j; m \le j \le m(p-3)\}$. The pseudo-regular graph of type II for $p=5$ is shown in Figure 4.

The Gutman index of G is given by $\mathrm{Gut}(G) = \sum_{u\neq v} d(u)d(v)d(u,v)$.

Now

$$\begin{aligned}
\mathrm{Gut}(G_{\mathrm{II}}) &= \sum_{i=1}^{m} d(v_0,v_i)\,d(v_0)\,d(v_i) + \sum_{i=1}^{m}\sum_{j=i+1}^{m} d(v_i)\,d(v_j)\,d(v_i,v_j) + \sum_{i=1}^{m(p-3)} d(v_0)\,d(u_i)\,d(v_0,u_i) \\
&\quad + \sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} d(v_i)\,d(u_j)\,d(v_i,u_j) + \sum_{1\le i<j\le m(p-3)} d(u_i)\,d(u_j)\,d(u_i,u_j) \\
&= \mathrm{Gut}(K_1+C_m) + \sum_{i=1}^{m(p-3)} d(v_0)\,d(u_i)\,d(v_0,u_i) + \sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} d(v_i)\,d(u_j)\,d(v_i,u_j)
\end{aligned}$$

$$+\sum_{1\le i<j\le m(p-3)} d(u_i)\,d(u_j)\,d(u_i,u_j)$$

$$= \left[\frac{-10p^6+393p^5-3650p^4+14717p^3-27872p^2+20382p}{2(p-1)}\right]$$

$$+\left[12p^5-148p^4+671p^3-1292p^2+759p+306\right]$$

$$+\left[\frac{3p^9-93p^8+1173p^7-7920p^6+31232p^5-70772p^4+72458p^3+36097p^2-167010p+122400}{2(p-1)}\right]$$

$$+\left[4p^6-56p^5+315p^4-88p^3+1283p^2-868p+237\right],$$

$$\operatorname{Gut}(G_{\mathrm{II}}) = \frac{3p^9-93p^8+1181p^7-8026p^6+32047p^5-75192p^4+87593p^3+8025p^2-145324p+121314}{2(p-1)}. \tag{4}$$

□

Theorem 3. *For $p \ge 3$, the Detour Gutman index of type II pseudo-regular graph G_{II} is*

$$D\operatorname{Gut}(G_{II}) = \frac{3p^{13}-150p^{12}+3267p^{11}-40822p^{10}+324929p^9-1730282p^8+6302777p^7-15784266p^6+26967288p^5-31025576p^4+24249128p^3-14093944p^2+6618144p-1661472}{16(p-1)(p-2)}. \tag{5}$$

Proof. Let $G = G_{\mathrm{II}}$ be a type II pseudo-regular graph.

Let $V(G) = \{v_0, v_1, v_2, \ldots, v_m, u_1, u_2, \ldots, u_{m(p-3)}\}$ be the vertex set of G and v_0 as the central vertex of wheel graph W_m and $\{v_1, v_2, \ldots, v_m\}$ are vertices of cycle c_m in the clockwise direction and $\{u_1, u_2, \ldots, u_{m(p-3)}\}$ are the pendant vertices joined to every vertex in the cycle except the central vertex, where $m = p^2 - 3p + 3$.

Let $E(G) = (v_0v_i; 1 \le i \le m\} \cup \{v_iv_{i+1}; 1 \le i \le m-1\} \cup \{v_mv_1\} \cup \{v_1u_j; 1 \le j \le p-3\} \cup \{v_2u_j; p-2 \le j \le 2(p-3)\} \cup \cdots \{v_mu_j; m \le j \le m(p-3)\}$.

The Detour Gutman index of G is given by $D\operatorname{Gut}(G) = \sum_{u\neq v} d(u)d(v)D(u,v)$.

Now

$$D\operatorname{Gut}(G_{\mathrm{II}}) = \sum_{i=1}^{m} D(v_0,v_i)\,d(v_0)\,d(v_i) + \sum_{i=1}^{m}\sum_{j=i+1}^{m} d(v_i)\,d(v_j)\,D(v_i,v_j) + \sum_{i=1}^{m(p-3)} d(v_0)\,d(u_i)\,D(v_0,u_i)$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} d(v_i)\,d(u_j)\,D(v_i,u_j) + \sum_{1\le i<j\le m(p-3)} d(u_i)\,d(u_j)\,D(u_i,u_j) = D\operatorname{Gut}(K_1+C_m) + \sum_{i=1}^{m(p-3)} d(v_0)\,d(u_i)\,D(v_0,u_i)$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} d(v_i)\,d(u_j)\,D(v_i,u_j) + \sum_{1\le i<j\le m(p-3)} d(u_i)\,d(u_j)\,D(u_i,u_j)$$

$$= \left[\frac{3p^{12}-147p^{11}+3120p^{10}-37710p^9+287235p^8-1441911p^7+4846138p^6-10839136p^5+15691368p^4-14003680p^3+}{16(p-2)}\right.$$

$$\left.+\frac{7515504p^2-3133728p+1306368}{16(p-2)}\right] + \left[42p^5-539p^4+2912p^3-8407p^2+13020p-8568\right]$$

$$+\left[\frac{2p^{10}-46p^9+490p^8-3020p^7+11314p^6-24572p^5+22173p^4+22916p^3-75151p^2+53958p}{(p-1)(p-2)}\right]$$

$$+\left[\frac{-6p^8+162p^7-1746p^6+10032p^5-33756p^4+67700p^3-77828p^2+45678p-10116}{4}\right],$$

$$D\,\mathrm{Gut}\left(G_{\mathrm{II}}\right)=\left(3p^{13}-150p^{12}+3267p^{11}-40822p^{10}+324929p^{9}-1730282p^{8}+6302777p^{7}-15784266p^{6}+26967288p^{5}\right.$$
$$\left.-31025576p^{4}+24249128p^{3}-14093944p^{2}+6618144p-1661472\right)\left(16\left(p-1\right)\left(p-2\right)\right)^{-1}. \tag{6}$$

□

5. Gutman Index of Pseudo-Regular Graph of Type III

Theorem 4. *For $p \geq 5$, the Gutman index of type III pseudo-regular graph G_{III} is*

$$\begin{aligned}\mathrm{Gut}\left(G_{III}\right) &= 9p^{8}-237p^{7}+2670p^{6}-16723p^{5}\\ &\quad +63422p^{4}-148307p^{3}+207979p^{2}\\ &\quad -161586p+55987.\end{aligned} \tag{7}$$

Proof. Let $G = G_{\mathrm{III}}$ be a type III pseudo-regular graph.

Let $V(G) = \{v_0, v_1, v_2, \ldots, v_m, u_1, u_2, \ldots, u_{m(p-5)}, w_1, w_2, \ldots, w_m\}$ be the vertex set of G and v_0 as the central vertex of wheel graph w_m, where $m = p^2 - 3p + 1$.

Let $E(G) = (v_0v_i; 1 \leq i \leq m\} \cup \{v_iv_{i+1}; 1 \leq i \leq m-1\} \cup \{v_mv_1; 1 \leq j \leq m(p-5)\} \cup \{u_1v_i; 1 \leq i \leq 2\} \cup \{u_2v_i; 2 \leq i \leq 3\}, \ldots, \{u_mv_i; m-1 \leq i \leq m\} \cup \{v_iw_j; 1 \leq j \leq p-5\} \cup \{v_2w_j; p-4 \leq j \leq 2(p-5)\} \cup \{v_mw_j; m \leq j \leq m(p-5)\}$. The pseudo-regular graph of type III for $p = 6$ is shown in Figure 5.

The Gutman index of G is given by $\mathrm{Gut}(G) = \sum_{u \neq v} d(u)d(v)d(u,v)$.

Now

$$\begin{aligned}\mathrm{Gut}\left(G_{\mathrm{III}}\right) &= \sum_{i=1}^{m} d\left(v_0\right)d\left(v_i\right)d\left(v_0,v_i\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=i+1}^{m} d\left(v_i\right)d\left(v_j\right)d\left(v_i,v_j\right)\\
&\quad+\sum_{i=1}^{m(p-5)} d\left(v_0\right)d\left(w_i\right)d\left(v_0,w_i\right)\\
&\quad+\sum_{i=1}^{m} d\left(v_0\right)d\left(u_i\right)d\left(v_0,u_i\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d\left(v_i\right)d\left(w_j\right)d\left(v_i,w_j\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m} d\left(v_i\right)d\left(u_j\right)d\left(v_i,u_j\right)\\
&\quad+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)} d\left(w_i\right)d\left(w_j\right)d\left(w_i,w_j\right)\\
&\quad+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m} d\left(w_i\right)d\left(u_j\right)d\left(w_i,u_j\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d\left(u_i\right)d\left(w_j\right)d\left(u_i,w_j\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m} d\left(u_i\right)d\left(u_j\right)d\left(u_i,u_j\right)\\
\mathrm{Gut}\left(G_{\mathrm{III}}\right) &= \mathrm{Gut}\left(K_1+C_m\right)\\
&\quad+\sum_{i=1}^{m(p-5)} d\left(v_0\right)d\left(w_i\right)d\left(v_0,w_i\right)\\
&\quad+\sum_{i=1}^{m} d\left(v_0\right)d\left(u_i\right)d\left(v_0,u_i\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d\left(v_i\right)d\left(w_j\right)d\left(v_i,w_j\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m} d\left(v_i\right)d\left(u_j\right)d\left(v_i,u_j\right)\\
&\quad+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)} d\left(w_i\right)d\left(w_j\right)d\left(w_i,w_j\right)\\
&\quad+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m} d\left(w_i\right)d\left(u_j\right)d\left(w_i,u_j\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d\left(u_i\right)d\left(w_j\right)d\left(u_i,w_j\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m} d\left(u_i\right)d\left(u_j\right)d\left(u_i,u_j\right)\\
\mathrm{Gut}\left(G_{\mathrm{III}}\right) &= \left[9p^{5}-59p^{4}+91p^{3}+18p^{2}+p\right]+\left[2p^{6}\right.\\
&\quad\left.-32p^{5}+192p^{4}-532p^{3}+672p^{2}-320p+50\right]\\
&\quad+\left[4p^{4}-24p^{3}+44p^{2}-24p+4\right]+\left[6p^{5}-36p^{4}\right.\\
&\quad\left.+54p^{3}-6p\right]+\left[17p^{5}-288p^{4}+1823p^{3}-5197p^{2}\right.\\
&\quad\left.+6120p-1675\right]+\left[4p^{8}-112p^{7}+1352p^{6}\right.\\
&\quad-9171p^{5}+38131p^{4}-99177p^{3}+156993p^{2}\\
&\quad\left.-137780p+51200\right]+\left[372p^{2}-2332p+3108\right].\\
\mathrm{Gut}\left(G_{\mathrm{III}}\right) &= 9p^{8}-237p^{7}+2670p^{6}-16723p^{5}\\
&\quad+63422p^{4}-148307p^{3}+207979p^{2}-161586p\\
&\quad+55987.\end{aligned} \tag{8}$$

□

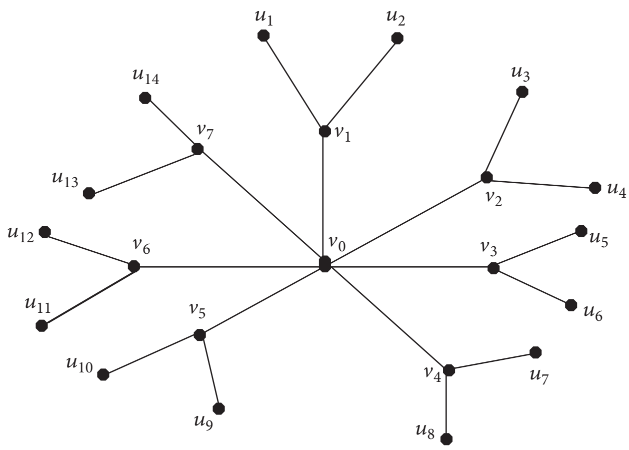

Figure 3: Pseudo-regular graph of type I at $p = 3$.

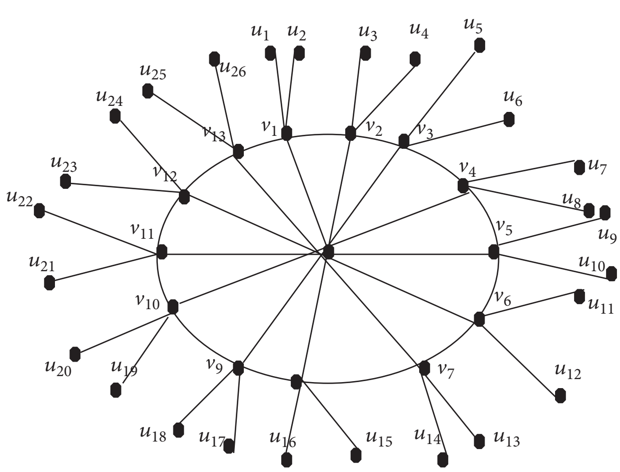

Figure 4: Pseudo-regular graph of type II at $p = 5$.

Theorem 5. *For $p \geq 5$, the Detour Gutman index of type III pseudo-regular graph G_{III} is*

$$\begin{aligned} D\,\mathrm{Gut}(G_{III}) = {} & 2p^{12} - 66p^{11} + 834p^{10} - 3163p^{9} \\ & - 42773p^{8} + 740044p^{7} - 5734272p^{6} \\ & + 28024578p^{5} - 92778639p^{4} \\ & + 209280657p^{3} - 309883869p^{2} \\ & + 272578823p - 108183686. \end{aligned} \tag{9}$$

Proof. Let $G = G_{\mathrm{III}}$ be a type III pseudo-regular graph.

Let $V(G) = \{v_0, v_1, v_2, \ldots, v_m, u_1, u_2, \ldots, u_{m(p-5)}, w_1, w_2, \ldots, w_m\}$ be the vertex set of G and v_0 as the central vertex of wheel graph w_m, where $m = p^2 - 3p + 1$ and $\{u_1, u_2, \ldots, u_{m(p-5)}\}$ are the pendant vertices and $\{w_1, w_2, \ldots, w_m\}$ are the vertices joined to the end vertices of each edge of a wheel graph except the central vertex.

Let $E(G) = (v_0v_i; 1 \leq i \leq m\} \cup \{v_iv_{i+1}; 1 \leq i \leq m-1\} \cup \{v_mv_1; 1 \leq j \leq m(p-5)\} \cup \{u_1v_i; 1 \leq i \leq 2\} \cup \{u_2v_i; 2 \leq i \leq 32\}, \ldots, \{u_mv_i; m-1 \leq i \leq m\} \cup \{v_iw_j; 1 \leq j \leq p-5\} \cup \{v_2w_j; p-4 \leq j \leq 2(p-5)\} \cup \{v_mw_j; m \leq j \leq m(p-5)\}$.

The Detour Gutman index of G is given by $D\,\mathrm{Gut}(G) = \sum_{u \neq v} d(u)d(v)D(u,v)$.

Now

$$\begin{aligned} D\,\mathrm{Gut}(G_{\mathrm{III}}) = {} & \sum_{i=1}^{m} d(v_0)\,d(v_i)\,D(v_0,v_i) \\ & + \sum_{i=1}^{m}\sum_{j=i+1}^{m} d(v_i)\,d(v_j)\,D(v_i,v_j) \\ & + \sum_{i=1}^{m(p-5)} d(v_0)\,d(w_i)\,D(v_0,w_i) \\ & + \sum_{i=1}^{m} d(v_0)\,d(u_i)\,D(v_0,u_i) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d(v_i)\,d(w_j)\,D(v_i,w_j) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{m} d(v_i)\,d(u_j)\,D(v_i,u_j) \\ & + \sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)} d(w_i)\,d(w_j)\,D(w_i,w_j) \\ & + \sum_{i=1}^{m(p-5)}\sum_{j=1}^{m} d(w_i)\,d(u_j)\,D(w_i,u_j) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d(u_i)\,d(w_j)\,D(u_i,w_j) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{m} d(u_i)\,d(u_j)\,D(u_i,u_j) \end{aligned}$$

$$\begin{aligned} D\,\mathrm{Gut}(G_{\mathrm{III}}) = {} & D\,\mathrm{Gut}(K_1 + C_m) \\ & + \sum_{i=1}^{m(p-5)} d(v_0)\,d(w_i)\,D(v_0,w_i) \\ & + \sum_{i=1}^{m} d(v_0)\,d(u_i)\,D(v_0,u_i) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d(v_i)\,d(w_j)\,D(v_i,w_j) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{m} d(v_i)\,d(u_j)\,D(v_i,u_j) \\ & + \sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)} d(w_i)\,d(w_j)\,D(w_i,w_j) \\ & + \sum_{i=1}^{m(p-5)}\sum_{j=1}^{m} d(w_i)\,d(u_j)\,D(w_i,u_j) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d(u_i)\,d(w_j)\,D(u_i,w_j) \end{aligned}$$

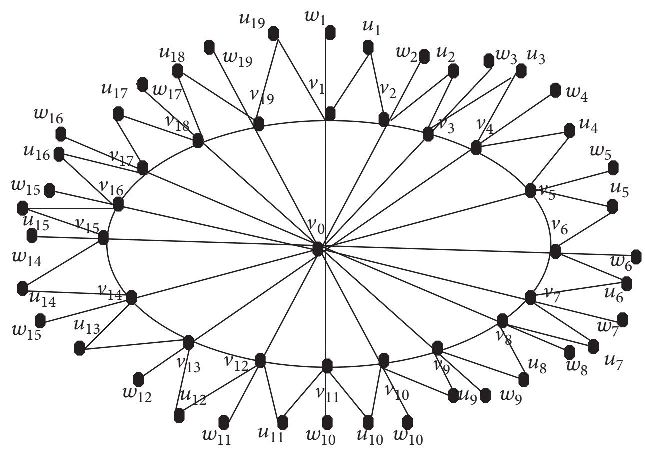

FIGURE 5: Pseudo-regular graph of type III at $p = 6$.

$$+\sum_{i=1}^{m}\sum_{j=1}^{m} d\left(u_i\right) d\left(u_j\right) D\left(u_i, u_j\right)$$

$$\begin{aligned}
D\,\mathrm{Gut}\left(G_{\mathrm{III}}\right) = {} & \left[8p^{10} - 200p^{9} + 1912p^{8} - 6812p^{7}\right.\\
& - 20084p^{6} + 305108p^{5} - 1363182p^{4} + 3178959p^{3}\\
& \left. - 3854871p^{2} + 1778689p + 261036\right] + \left[18p^{11}\right.\\
& - 826p^{10} + 17250p^{9} - 216444p^{8} + 1813319p^{7}\\
& - 10651871p^{6} + 44774333p^{5} - 134688490p^{4}\\
& + 284180079p^{3} - 400550125p^{2} + 339452825p\\
& \left. - 131036500\right] + \left[8p^{9} - 248p^{8} + 3384p^{7}\right.\\
& - 26584p^{6} + 131820p^{5} - 424452p^{4} + 875734p^{3}\\
& \left. - 1089150p^{2} + 701402p - 149768\right] + \left[2p^{12}\right.\\
& - 84p^{11} + 1652p^{10} - 20230p^{9} + 172232p^{8}\\
& - 1072340p^{7} + 4980735p^{6} - 17261562p^{5}\\
& + 43954031p^{4} - 79624939p^{3} + 96831776p^{2}\\
& \left. - 70644085p + 23295900\right] + \left[9p^{9} - 225p^{8}\right.\\
& + 2421p^{7} - 14604p^{6} + 53841p^{5} - 123786p^{4}\\
& \left. + 172950p^{3} - 135549p^{2} + 50376p - 6864\right]\\
& + \left[162p^{5} - 2736p^{4} + 17262p^{3} - 49050p^{2}\right.\\
& \left. + 57600p - 15750\right] + [72p^{7} - 1864p^{6} + 20812p^{5}\\
& - 129160p^{4} + 476036p^{3} - 1025500p^{2}\\
& + 1169800p - 528500 + \left[64p^{5} - 864p^{4} + 4576p^{3}\right.\\
& \left. - 11400p^{2} + 12216p - 3240\right],
\end{aligned}$$

$$\begin{aligned}
D\,\mathrm{Gut}\left(G_{\mathrm{III}}\right) = {} & 2p^{12} - 66p^{11} + 834p^{10} - 3163p^{9}\\
& - 42773p^{8} + 740044p^{7} - 5734272p^{6}\\
& + 28024578p^{5} - 92778639p^{4} + 209280657p^{3}\\
& - 309883869p^{2} + 272578823p - 108183686.
\end{aligned} \tag{10}$$

□

6. Conclusion

In this paper, we clearly determined the Gutman index of pseudo-regular graphs and also determined the Detour Gutman index of pseudo-regular graphs.

Conflicts of Interest

The authors S. Kavithaa and V. Kaladevi declare that there are no conflicts of interest regarding the publication of this article.

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