Research Article

Quality Improvement Policies in a Supply Chain with Stackelberg Games

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We first analyze quality and price decisions in a supply chain with two Stackelberg games: Manufacturer's Stackelberg (MS) and Supplier's Stackelberg (SS). Then, we investigate how equilibrium solutions are influenced by proposed quality improvement policies: coordination and manufacturer's involvement. Also, we derive the conditions under which the policies can be implemented in both MS and SS strategies. Numerical experiments illustrate the problems and several related issues are discussed. The results suggest that proposed quality improvement policies can realize Pareto improvement for the supply chain performance.

1. Introduction

Quality plays an important role in the competitive business environment of the twenty-first century, and firms are using improved quality as a strategic weapon to enhance their competitiveness against rival firms [1–3]. As the procedure from raw materials to products is not within a single firm but throughout a supply chain, quality of a manufacturer's products depends on not only its own process quality but also the quality of its supplier(s) [4–6]. Quality practices must advance from traditional firm centric and product-based mindsets to an interorganizational supply chain orientation involving customers, suppliers, and other partners [7, 8].

However, there are many quality incidents caused by low quality of raw materials or spare parts. For example, in 2009, millions of automobiles were recalled by Toyota because of unqualified spare parts from its suppliers [9]. In China, there were incidents of poisonous powdered milk caused by low quality milk produced by inferior cattle, milk adulterated with melamine, and nonstandard production processes used in the milk industry [10]. Therefore, quality improvement is an important issue in supply chain quality management. A natural question is what kind of policies can improve quality of products effectively. In the international trade, a typical trade scenario is that supplier-manufacturer supply chains in China produce goods according to orders placed by buyers from around the world [11]. In this paper, we investigate quality improvement policies of a make-to-order (MTO) supply chain where quality of products is mainly decided by that of raw materials or spare parts provided by the supplier.

In a decentralized supply chain, the objectives of players are to maximize their respective expected utilities or profits, which lead to noncooperative games between players and low quality of products. Therefore, coordination mechanisms have been designed for quality improvement in decentralized supply chains [12]. Forker [13] linked quality management with process optimization to address both effectiveness and efficiency concerns. The paper suggested that system performance was affected by transaction-specific investments in coordination. From a supply chain perspective, Singer et al. [14] derived the conditions under which the supplier and the retailer might devise a mutually beneficial contract that simultaneously increases profit and improves quality. Chao et al. [15] proposed a contract with selective root cause analysis which differentiated early failures from late failures to coordinate quality improvement efforts of supply chain members. Xu [16] studied a joint pricing and product quality decision problem in a distribution channel, in which a manufacturer sells a product through a retailer. The manufacturer determines both wholesale price and quality of the product, while the retailer determines the retail price. The investigation showed that marginal revenue function is closely related to the distribution channel structure. However, coordination does not necessarily lead to quality improvement [16].

Another approach to quality improvement is partner's involvement. Zhu et al. [17] explored the roles of different parties in a supply chain in quality improvement and showed that the buyer's involvement could have a significant impact on profits of both parties and of the supply chain as a whole. Different from quality management in a single supply chain, Xie et al. [18] investigated quality improvement in competing supply chains so as to discover which supply chain structure and quality improvement strategy will be selected by the competing supply chains.

The paper closest to ours is Xie et al.'s [11], which investigated quality investment and price decision of a maketo-order (MTO) supply chain with uncertain demand in international trade. The paper shows that both supply chain strategy and risk-averse behavior have significant impacts on quality investment and pricing. Compared with a riskneutral supply chain, a risk-averse supply chain has lower, same, and higher quality of products in VI, MS, and SS, respectively. Also, we derive the conditions under which the supply chain strategy is implemented in a decentralized setting. However, Xie et al. [11] did not consider the problem of quality improvement in a supply chain.

Although higher quality can be a reason for higher price, it can also result in higher costs. At the same time, quality and price influence demand and profits [19]. Just like the definition of quality in [20], we use the term "quality" to refer to both design and performance quality characteristics of interest to the consumer when evaluating the product offered by the supply chain. "Quality" may refer to characteristics such as the performance of a computer, the energy efficiency of a vehicle, or the nutritional ingredients of a particular food [21–23].

In this paper, we consider quality and price decisions in a decentralized supply chain with two Stackelberg games: Manufacturer's Stackelberg (MS) and Supplier's Stackelberg (SS). Then, we investigate the impacts of coordination and manufacturer's involvement on equilibrium solutions. Also, we derive the conditions under which quality improvement policies can be implemented. Moreover, a numerical example is used to illustrate the related issues, and managerial insights are indicated for quality improvement. Finally, conclusions are drawn and some topics for future work are suggested.

The remainder of the paper is organized as follows. In Section 2, we describe the business flow and related decisions in a decentralized supply chain. Then, Manufacturer's Stackelberg (MS) and Supplier's Stackelberg (SS) in a decentralized supply chain are investigated in Section 3. In Section 4, we consider two quality improvement policies: coordination and manufacturer's involvement. A numerical example is used to illustrate the problem and some related issues are discussed in Section 5. Section 6 draws the conclusions.

2. Description of the Problem

We consider quality investment and price decision of a supply chain which produces goods against specific orders placed by buyers from around the world. Before orders are placed, quality of raw materials and prices of products are set by the supplier and the manufacturer, respectively. After orders are placed, the supply chain organizes production with respect to the orders. Moreover, we investigate the impacts of coordination and manufacturer's involvement on supply chain performance. Below we present the assumptions on the problem.

Assumption 1. Buyers can recognize the quality of products provided by the supply chain. Different from common consumers, buyers usually have more professional expertise for procurement. Hence, they can know about the quality of products in detail.

Assumption 2. There is no moral hazard between the supplier and the manufacturer in a make-to-order (MTO) supply chain. We use the term "quality" to refer to both design and conformance quality characteristics of interest to the consumer.

The following notations are used in the model.

- *i*: Supply chain strategy *i*, *i* = MS, SS, *C*, MI
- x_i : Quality of product in the *i*th supply chain strategy

p_i: Price per unit in the *i*th supply chain strategy

w: Wholesale price per unit of product to the manufacturer

 v_S : Variable production cost per unit of the supplier

 v_M : Variable production cost per unit of the manufacturer

- $C_{\rm S}$: Fixed cost related to quality of the supplier
- C_M : Fixed cost related to quality of the manufacturer
- α : The demand responsiveness to quality

 β : The demand responsiveness to price of product.

In this paper, x_i and p_i are decision variables and other variables are exogenous variables, known to both players in the supply chain. In addition, we assume that $p_i > w + v_M$ and $w > v_S$. These inequalities ensure that each firm makes a positive profit.

Extending the demand function in Banker et al. [20], we assume that the primary demand function D_i for the products is decided by price p_i and quality x_i as follows:

$$D_i = a + \alpha x_i - \beta p_i, \tag{1}$$

where *a* is potential intrinsic demand. Obviously, the demand D_i has a positive correlation with x_i and a negative correlation with p_i .

Business flow of an MTO supply chain is shown in Figure 1, where both the supplier and the manufacturer know the distribution of demand and they organize the production required for meeting the demand.



FIGURE 1: Business flow of an MTO supply chain.

In the following sections, we investigate quality investment and price decision in two supply chain strategies: SS and MS.

3. Stackelberg Games in the Supply Chain

We consider two supply chain structures: Manufacturer's Stackelberg (MS) and Supplier's Stackelberg (SS), where the optimal quality investment and price decision are analyzed.

In a decentralized supply chain, profit Π^M_D of the manufacturer is

$$\Pi_D^M = \left(p_i - w - v_M\right) \left(a + \alpha x_i - \beta p_i\right), \quad (i = \text{MS, SS}), \quad (2)$$

and profit Π_D^S of the supplier is

$$\Pi_{D}^{S} = (w - v_{S}) (a + \alpha x_{i} - \beta p_{i}) - f - C_{S} x_{i}^{2}.$$
 (3)

Here, *f* is fixed cost not related to quality x_i , and $C_S x_i^2$ is fixed cost related to x_i .

The objectives of the manufacturer and the supplier are to maximize their respective profits, that is, Π_D^M in (2) and Π_D^S in (3). Assuming there are two supply chain strategies, Manufacturer's Stackelberg (MS) and Supplier's Stackelberg (SS), we analyze the quality investment and price decision as follows.

3.1. Manufacturer's Stackelberg. In case of Manufacturer's Stackelberg, i = MS, the manufacturer is the leader of the supply chain and the decision-making process of players takes place in the following sequence.

- (i) The manufacturer tenders a price $p_{\rm MS}$ for the product.
- (ii) The supplier observes the price and selects the optimal quality x^{*}_{MS} for raw materials on the basis of its maximum profit.
- (iii) The manufacturer then selects the optimal price p_{MS}^* for the product again, based on the supplier's selected quality.
- (iv) Orders are then placed and the demand is realized, based on prices and quality levels set by the supply chain.

Theorem 3. In a supply chain with Stackelberg games, there is a pure-strategy Nash equilibrium between players in terms of quality investment and price decisions.

Proof. It is straightforward and therefore is omitted. \Box

From the sequence, Proposition 4 quantifies the optimal quality of the supplier and price of the manufacturer as follows.

Proposition 4. *In supply chain with Manufacturer's Stackelberg, we obtain the optimal quality of the supplier as*

$$x_{MS}^* = \frac{\alpha \left(w - v_S\right)}{2C_S},\tag{4}$$

and the optimal price of the manufacturer is

$$p_{MS}^{*} = w + v_{M} + \frac{x_{MS}^{*} + a - \beta \left(w + v_{M}\right)}{2\beta}.$$
 (5)

Proof. See the appendix.

3.2. Supplier's Stackelberg. In Supplier's Stackelberg, i = SS, the supplier is the leader of the supply chain and the decision-making process of players takes place in the following sequence.

- (i) The supplier gives a quality x_{SS} for products.
- (ii) The manufacturer observes quality x_{SS} of raw materials and then selects an optimal price p_{SS}^* on the basis of its maximum profit.
- (iii) The supplier observes price p_{SS}^* and then reselects optimal quality x_{SS}^* of products.
- (iv) Orders are then placed and the demand is realized, based on price p_{SS}^* and quality level x_{SS}^* set by the supply chain.

From the sequence, Proposition 5 quantifies the optimal quality of the supplier and price of the manufacturer as follows.

Proposition 5. *In supply chain strategy Supplier's Stackelberg, one obtains the optimal quality of the supplier as*

$$x_{SS}^* = \frac{\alpha \left(w - v_S \right)}{4C_S},\tag{6}$$

and the optimal price of the manufacturer is

$$p_{SS}^{*} = w + v_{M} + \frac{\alpha x_{SS}^{*} + a - \beta (w + v_{M})}{2\beta}.$$
 (7)

Proof. See the appendix.

After the decisions with two supply chain strategies are investigated, we continue to analyze quality improvement policies in the next section.

4. Quality Improvement Policies

In this section, we investigate the impacts of quality improvement policies on equilibrium solutions. Also, we derive the conditions under which the supply chain strategy is implemented in both MS and SS strategies. 4.1. Coordination. For a decentralized supply chain, the same performance as an integrated one can be realized by using a coordination contract. Different models of supply chain contracts have been developed in the literature, including quantity flexibility contracts [24], the backup agreements [25], the buy back or return policies [26], the incentive mechanisms [27] and the revenue sharing contracts [28, 29]. Usually, these models should satisfy the *win-win* condition, by supporting appropriate choice of contract parameters [30].

In a supply chain with coordination, i = C, decisions are made centrally. The decision-making process of players takes place in the following sequence.

- (i) The supplier and the manufacturer observe the distribution of demand.
- (ii) The optimal quality x_C^* and price p_C^* are decided for maximum profit of the supply chain.
- (iii) Orders are then placed and the demand is realized, based on prices and quality levels set by the supply chain.

Profit Π_C of the supply chain is

$$\Pi_{C} = \left(p_{C} - \nu\right) \left(a + \alpha x_{C} - \beta p_{C}\right) - f - c x_{C}^{2}, \qquad (8)$$

where $c = \min\{C_S, C_M\}$ and $v = v_S + v_M$.

Theorem 6. When $a > \beta v$ and $4\beta c > \alpha^2$, there are unique optimal solutions for quality and price.

Proof. See the appendix.
$$\Box$$

Proposition 7 quantifies the equilibrium quality and price.

Proposition 7. *In a supply chain with coordination, equilibrium solutions for quality and price of products are as follows:*

$$x_{C}^{*} = \frac{\alpha \left(a - \beta v\right)}{4\beta c - \alpha^{2}},$$

$$p_{C}^{*} = v + \frac{2c \left(a - \beta v\right)}{4\beta c - \alpha^{2}}.$$
(9)

Proof. See the appendix.

Due to the complete and symmetric information assumption, lump sum transfer contracts can be used for supply chain coordination. To ensure that the manufacturer and the retailer in a decentralized supply chain both have incentives to accept the coordination contract, the profits of the manufacturer and the retailer should be no less than those before coordination, that is, $\Pi_C^S \ge \Pi_D^{S^*}$ and $\Pi_C^M \ge \Pi_D^{M^*}$. This problem can be easily solved by offering a lump sum fee $F(\underline{F} \le F \le \overline{F})$. Here, $\underline{F} = \max\{\Pi_D^{S^*} - \Pi_C^{S^*}, \Pi_D^{M^*} - \Pi_C^{M^*}, 0\}$ and $\overline{F} = \max\{\Pi_C^{S^*} - \Pi_D^{S^*}, \Pi_C^{M^*} - \Pi_D^{M^*}, 0\}$, where $\Pi_C^{S^*}$ and $\Pi_C^{M^*}$ are optimal profits of the supplier and the manufacturer after coordination. When $\Pi_D^{S^*} \ge \Pi_C^{S^*}$, that is, the supplier earns less with the coordination contract, the manufacturer should pay the lump sum fee F to the supplier. Then the profits of

the supplier and the manufacturer are $\Pi_C^S = \Pi_C^{S^*} + F$ and $\Pi_C^M = \Pi_C^{M^*} - F$. Otherwise, the supplier should pay the lump sum fee *F* to the manufacturer. Then the profits of the supplier and the manufacturer are $\Pi_C^S = \Pi_C^{S^*} - F$ and $\Pi_C^M = \Pi_C^{M^*} + F$. Then $\Pi_C^S \ge \Pi_D^{S^*}$ and $\Pi_C^M \ge \Pi_D^{M^*}$ are satisfied. Therefore, we propose Corollary 8 as follows.

Corollary 8. In a lump sum transfer contract for supply chain coordination, when $\Pi_D^{M^*} \ge \Pi_C^{M^*}$, the supplier should pay the lump sum fee F to the manufacturer; otherwise, when $\Pi_D^{R^*} \ge \Pi_C^{R^*}$, the manufacturer should pay the lump sum fee F to the supplier, where F meets $\underline{F} \le F \le \overline{F}$, $\underline{F} = \max\{\Pi_D^{S^*} - \Pi_C^{S^*}, \Pi_D^{M^*} - \Pi_C^{M^*}, 0\}$, and $\overline{F} = \max\{\Pi_C^{S^*} - \Pi_D^{S^*}, \Pi_C^{M^*} - \Pi_D^{M^*}, 0\}$.

Proof. It is straightforward and therefore is omitted. \Box

4.2. Manufacturer's Involvement. When the manufacturer involves in quality improvement of the supplier, i = MI, the decision-making process takes place in the following sequence.

- (i) The manufacturer observes the demand.
- (ii) The manufacturer invests in quality improvement of raw materials on the basis of its maximum profit.
- (iii) The optimal quality x_{MI}^* and price p_{MI}^* are decided.
- (iv) Orders are then placed and the demand is realized, based on quality x_{MI}^* and price p_{MI}^* set by the supply chain.

Profit Π_{MI}^{M} of the manufacturer is

$$\Pi_{\rm MI}^{M} = (p_{\rm MI} - w - v_{M}) (a + \alpha x_{\rm MI} - \beta p_{\rm MI}) - C_{M} [x_{\rm MI}^{2} - (x_{i}^{*})^{2}], \quad (i = \rm MS, SS)$$
(10)

and profit Π_{MI}^{S} of the supplier is

$$\Pi_{\rm MI}^{\rm S} = (w - v_{\rm S}) \left(a + \alpha x_{\rm MI} - \beta p_{\rm MI} \right) - f - C_{\rm S} (x_i^*)^2, (i = {\rm MS, SS}).$$
(11)

Theorem 9. When $a > \beta(w + v_M)$ and $4\beta C_M > \alpha^2$, there are unique optimal solutions for quality and price.

Proof. See the appendix.
$$\Box$$

From the sequence, Proposition 10 quantifies the optimal quality of the supplier and price of the manufacturer as follows.

Proposition 10. In supply chain strategy Manufacturer's Stackelberg, when the manufacturer involves in quality improvement, one obtains the optimal quality of product as

$$x_{MI}^{*} = \frac{\alpha \left[a - \beta \left(w + v_{M}\right)\right]}{4\beta C_{M} - \alpha^{2}},$$
(12)

and the optimal price of the manufacturer is

$$p_{MI}^{*} = w + v_{M} + \frac{2C_{M} \left[a - \beta \left(w + v_{M} \right) \right]}{4\beta C_{M} - \alpha^{2}}.$$
 (13)

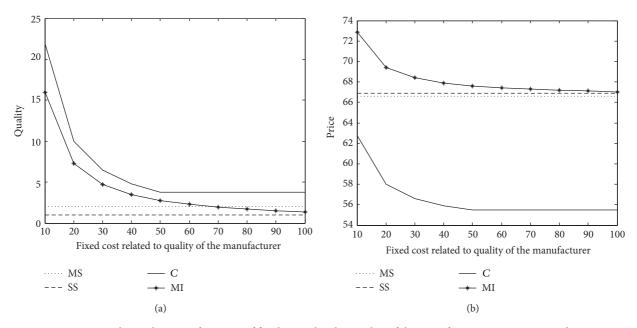


FIGURE 2: Quality and price as functions of fixed cost related to quality of the manufacturer in MS, SS, C, and MI.

Proof. See the appendix.

Also, we propose the conditions of manufacturer's involvement in Corollary 11 as follows.

Corollary 11. Only when $\Pi_{MI}^M \ge \Pi_D^{M^*}$ and $\Pi_{MI}^S \ge \Pi_D^{S^*}$ can the manufacturer invest in quality improvement and the supplier accept it.

Proof. It is straightforward and therefore is omitted. \Box

5. Analysis with Experiments

In this section, using numerical experiments, we illustrate some related issues in a supply chain with Stackelberg games. Let a = 1000, $\alpha = 8$, $\beta = 10$, $C_S = 50$, $v_S = 5$, $v_M = 3$, f = 800, and w = 30. Here $\alpha < \beta$ indicates that orders are more sensitive to price than to quality, which is particularly reasonable in the current trade environment characterized by financial crisis and recession [31–36]. As cost has significant impacts on quality decisions [37], we set fixed cost related to quality of the manufacturer $C_M \in [10, 100]$ to make a sensitivity analysis.

5.1. Quality and Price. Effects of C_M on quality and price are shown in Figure 2, where curves MS, SS, C, and MI denote supply chain strategies manufacturer's Stackelberg, supplier's Stackelberg, coordination, and manufacturer's involvement in quality improvement, respectively.

From Figure 2, we can observe that $x_C^* > x_{MI}^*$, $x_{MS}^* > x_{SS}^*$ when C_M is fixed. Also, there are

$$x_{\text{MI}}^* > x_{\text{MS}}^*, \quad 10 \le C_M \le 60,$$

 $x_{\text{MI}}^* < x_{\text{MS}}^*, \quad 70 \le C_M \le 100.$ (14)

As a result, the centralized decision-maker can invest in quality with lower fixed cost and achieve the highest quality. However, when C_M is fixed, there are $p_{MI}^* > p_{SS}^* > p_{MS}^* > p_C^*$. Particularly, in a supply chain with coordination, there is $c = C_S = 50$, $x_C^* = 3.8$, and $p_C^* = 55.52$ when $50 \le C_M \le 100$. Also, x_C^* , p_C^* ($10 \le C_M \le 50$), x_{MI}^* , and p_{MI}^* decrease in C_M , which indicates that a manufacturer with higher fixed cost related to quality tends to set a lower quality and price in supply chain strategies *C* and MI.

5.2. Demand. From (1), we obtain the demand $D_i^* = a + \alpha x_i^* - \beta p_i^*$ (i = MS, SS, C, MI) as shown in Figure 3, and the results suggest that $D_C^* > D_{MI}^*, D_{MS}^* > D_{SS}^*$ when C_M is fixed. Also, the increase of C_M will reduce D_C^* ($10 \le C_M \le 50$) and D_{MI}^* . When $40 \le C_M \le 100$, there are $D_{MS}^* > D_{MI}^*$. The reason is that supply chain strategy MS has a lower price than MI.

5.3. Profits. The profits of the manufacturer and the supplier in MS and MIMS are shown in Figure 4, where curves MMS, SMS, MMIMS, and SMIMS denote profits of the manufacturer in MS, the supplier in MS, the manufacturer in MIMS, and the supplier in MIMS, respectively. Here, MIMS means MI under MS. From Figure 4, we can observe that $\Pi_{\text{MIMS}}^{M^*} > \Pi_{\text{MS}}^{S^*} > \Pi_{\text{MIMS}}^{S^*}$, $\Pi_{\text{MS}}^{S^*}$ when C_M is fixed. Also, $\Pi_{\text{MIMS}}^{S^*} > \Pi_{\text{MS}}^{S^*} > \Pi_{\text{MIMS}}^{S^*}$. The reason may be that the demand decreases in C_M .

The profits of the manufacturer and the supplier in SS and MISS are shown in Figure 5, where curves MSS, SSS, MMISS, and SMISS denote profits of the manufacturer in SS, the supplier in SS, the manufacturer in MISS, and the supplier in MISS, respectively. Here, MISS means MI under SS. From

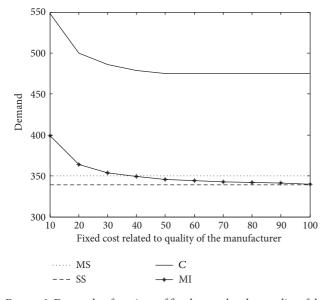


FIGURE 3: Demand as functions of fixed cost related to quality of the manufacturer in MS, SS, *C*, and MI.

Figure 5, we can observe that $\Pi_{\text{MISS}}^{M^*} > \Pi_{\text{SS}}^{M^*} > \Pi_{\text{MISS}}^{S^*} > \Pi_{\text{SS}}^{S^*}$ when C_M is fixed. Also, $\Pi_{\text{MISS}}^{M^*}$ and $\Pi_{\text{MISS}}^{S^*}$ decrease in C_M . However, there are $\Pi_{\text{MIMS}}^{M^*} > \Pi_{\text{MISS}}^{M^*}$ and $\Pi_{\text{MISS}}^{S^*} > \Pi_{\text{MIMS}}^{S^*}$. The profits of the supply chain in MS, SS, C, MIMS, and

The profits of the supply chain in MS, SS, C, MIMS, and MISS are shown in Figure 6. We can observe that $\Pi_{C}^{*} > \Pi_{\text{MIMS}}^{*}, \Pi_{\text{MISS}}^{*}, \Pi_{\text{MS}}^{*} > \Pi_{\text{SS}}^{*}$ when C_{M} is fixed, and Π_{MISS}^{*} decreases in C_{M} . Also, there are

$$\Pi_{\text{MISS}}^{*} > \Pi_{\text{MIMS}}^{*}, \quad 10 \le C_{M} \le 40,$$

$$\Pi_{\text{MIMS}}^{*} = \Pi_{\text{MISS}}^{*}, \quad C_{M} = 50,$$

$$\Pi_{\text{MIMS}}^{*} > \Pi_{\text{MISS}}^{*}, \quad 60 \le C_{M} \le 100,$$

$$\Pi_{\text{MS}}^{*} < \Pi_{\text{MIMS}}^{*}, \Pi_{\text{MISS}}^{*}, \quad 10 \le C_{M} \le 40,$$

$$\Pi_{\text{MS}}^{*} > \Pi_{\text{MIMS}}^{*}, \Pi_{\text{MISS}}^{*}, \quad 50 \le C_{M} \le 100.$$
(15)

The observations suggest that coordination brings the highest profit to the supply chain. When C_M is bigger than C_S , manufacturer's involvement (MI) in quality improvement may reduce the profit of the supplier. Then, the supplier will not accept the MI policy. Therefore, the manufacturer should pay more attention to lower C_M , when MI policy is preferred.

5.4. Consumer Surplus. As there are same quality, price, demand, and consumer surplus in both MIMS and MISS, we use MI to represent both. The consumer surplus of the supply chain in MS, SS, C, and MI are shown in Figure 7. We can observe that $CS_{\rm C}^* > CS_{\rm MI}^*, CS_{\rm MS}^* > CS_{\rm SS}^*$ when C_M is fixed, and $CS_{\rm MI}^*$ decreases in C_M . Also, there are

$$CS_{MS}^* < CS_{MI}^*$$
, $10 \le C_M \le 60$,
 $CS_{MS}^* > CS_{MI}^*$, $70 \le C_M \le 100$. (16)

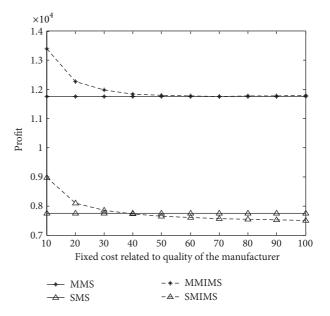


FIGURE 4: Players' profits as functions of fixed cost related to quality of the manufacturer in MS and MIMS.

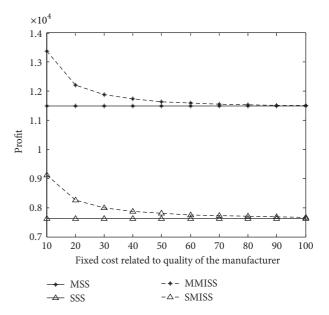


FIGURE 5: Players' profits as functions of fixed cost related to quality of the manufacturer in SS and MISS.

From these observations, we can conclude that coordination can achieve the highest consumer surplus. Only when C_M is less than a certain degree, MI can bring more consumer surplus.

6. Conclusions

In this paper, we first analyzed quality and price decisions in a supply chain with two Stackelberg games: Manufacturer's Stackelberg (MS) and Supplier's Stackelberg (SS). Then, we investigated how equilibrium solutions are influenced by

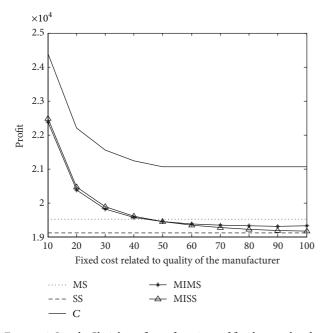


FIGURE 6: Supply Chain's profits as functions of fixed cost related to quality of the manufacturer.

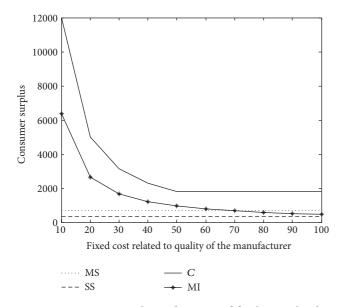


FIGURE 7: Consumer surplus as functions of fixed cost related to quality of the manufacturer in MS, SS, *C*, and MI.

proposed quality improvement policies: coordination and manufacturer's involvement. Also, we derived the conditions under which polices can be implemented in both MS and SS strategies. Numerical experiments illustrated the problems and some related issues were discussed. The results suggested that proposed quality improvement policies can realize Pareto improvement for the supply chain performance.

The investigation suggests that both cost and supply chain strategies have significant impacts on quality, price, demand, profits, and consumer surplus. The proposed quality improvement policies can realize Pareto improvement for the supply chain performance. Among all supply chain strategies, coordination can offer the highest quality and the lowest price. As a result, coordination achieves the biggest demand, consumer surplus, and brings the highest profit to the supply chain. However, manufacturer's involvement (MI) in quality improvement may reduce the profit of the supplier when fixed cost related to quality of the manufacturer is more than a certain degree, which leads to the supplier not accepting the MI policy. Therefore, the manufacturer should pay more attention to lower its own fixed cost related to quality, when MI policy is preferred.

Appendix

Proof of Proposition 4. According to the price given by the manufacturer, the supplier sets the optimal quality. We differentiate (3) with respect to x_{MS} and obtain

$$\frac{\partial \Pi_D^S}{\partial x_{\rm MS}} = (w - v_S) \alpha - 2C_S x_{\rm MS}. \tag{A.1}$$

Let $\partial \Pi_D^S / \partial x_{MS} = 0$; we obtain the optimal quality as

$$x_{\rm MS}^* = \frac{\alpha \left(w - v_{\rm S}\right)}{2C_{\rm S}}.$$
 (A.2)

In a supply chain strategy of Manufacturer's Stackelberg, we introduce x_{MS}^* into (2) and differentiate Π_D^M with respect to p_{MS} as

$$\frac{\partial \Pi_D^M}{\partial p_{\rm MS}} = a + \alpha x_{\rm MS}^* + (w + v_M) \beta - 2\beta p_{\rm MS}. \tag{A.3}$$

Let $\partial \Pi_D^M / \partial p_{\rm MS} = 0$; we obtain the optimal price of the manufacturer as

$$p_{\rm MS}^* = w + v_M + \frac{a + \alpha x_{\rm MS}^* - \beta (w + v_M)}{2\beta}.$$
 (A.4)

Since $\partial^2 \Pi_D^M / \partial p_{MS}^2 = -2\beta < 0$, the profit function is strictly concave in prices.

Proof of Proposition 5. According to the quality set by the supplier, the manufacturer sets the optimal price. By differentiating (2) with respect to p_{SS} and equating it to zero, we obtain the following function:

$$a + \alpha x_{\rm SS} + (w + v_M)\beta - 2\beta p_{\rm SS} = 0.$$
 (A.5)

Then, we obtain the optimal price p_{SS}^* as

$$p_{\rm SS}^* = p\left(x_{\rm SS}\right) = \frac{a + \alpha x_{\rm SS} + \beta\left(w + v_M\right)}{2\beta}.$$
 (A.6)

Since $\partial^2 \Pi_D^M / \partial p_{SS}^2 = -2\beta < 0$, profit function Π_D^M is strictly concave in pricing.

From (3), we can obtain profit Π_D^S of the supplier as follows:

$$\Pi_{D}^{S} = (w - v_{S}) \left[a + \alpha x_{SS} - \beta p \left(x_{SS} \right) \right] - f - C_{S} x_{SS}^{2}.$$
 (A.7)

Differentiating Π_D^S with respect to x_{SS} , we obtain

$$\frac{\partial \Pi_{D}^{S}}{\partial x_{\rm SS}} = (w - v_{\rm S}) \left[\alpha - \beta p'(x_{\rm SS}) \right] - 2C_{\rm S} x_{\rm SS}, \qquad (A.8)$$

where

$$p'(x_{\rm SS}) = \frac{\partial p_{\rm SS}^*}{\partial x_{\rm SS}} = \frac{\alpha}{2\beta}.$$
 (A.9)

Equating $\partial \Pi_D^S / \partial x_{SS}$ to zero, we obtain the following equation:

$$x_{\rm SS}^* = \frac{\alpha \left(w - v_{\rm S}\right)}{4C_{\rm S}},\tag{A.10}$$

Since $\partial^2 \Pi_D^S / \partial x_{SS}^2 = -2C_S < 0$, function Π_D^S is strictly concave in quality. Introducing x_{SS}^* into $p(x_{SS})$, we obtain the following equation:

$$p_{SS}^{*} = w + v_{M} + \frac{a + \alpha x_{SS}^{*} - \beta (w + v_{M})}{2\beta}.$$
 (A.11)

Proof of Theorem 6 and Proposition 7. From (8), we obtain partial derivatives of Π_C with respect to p_C and x_C as follows:

$$\frac{\partial \Pi_C}{\partial p_C} = a + \beta v + \alpha x_C - 2\beta p_C,$$

$$\frac{\partial \Pi_C}{\partial x_C} = \alpha \left(p_C - v \right) - 2c x_C,$$

$$\frac{\partial^2 \Pi_C}{\partial x_C^2} = -2c,$$

$$\frac{\partial^2 \Pi_C}{\partial p_C^2} = -2\beta,$$

$$\frac{\partial^2 \Pi_C}{\partial x_C \partial p_C} = \alpha.$$
(A.12)

Hessian matrix H_C of $\Pi_C(x_C, p_C)$ is $H_C = \begin{pmatrix} -2c, & \alpha \\ \alpha & -2\beta \end{pmatrix}$. To make sure that Π_C is maximum in (x_C^*, p_C^*) , Hessian matrix H_C should be negative definite. Therefore, when $(-2c)(-2\beta) > \alpha^2$, that is, $4\beta c > \alpha^2$, there are unique optimal solutions for quality and price.

Let $\partial \Pi_C / \partial x_C = 0$ and $\partial \Pi_C / \partial p_C = 0$; we have

$$x_{C}^{*} = \frac{\alpha \left(a - \beta v\right)}{4\beta c - \alpha^{2}},$$

$$p_{C}^{*} = v + \frac{2c \left(a - \beta v\right)}{4\beta c - \alpha^{2}}.$$
(A.13)

As $x_C^* > 0$, there is $a > \beta v$.

Proof of Theorem 9 and Proposition 10. Following the Proof of Proposition 7, from (11), we can prove that unique optimal solutions for quality and price exist when $4\beta C_M > \alpha^2$.

Let
$$\partial \Pi_{\mathrm{MI}}^{M} / \partial x_{\mathrm{MI}} = 0$$
 and $\partial \Pi_{\mathrm{MI}}^{M} / \partial p_{\mathrm{MI}} = 0$; we have

$$x_{\mathrm{MI}}^{*} = \frac{\alpha \left[a - \beta \left(w + v_{M} \right) \right]}{4\beta C_{M} - \alpha^{2}},$$

$$p_{\mathrm{MI}}^{*} = w + v_{M} + \frac{2C_{M} \left[a - \beta \left(w + v_{M} \right) \right]}{4\beta C_{M} - \alpha^{2}}.$$
(A.14)

As $x_{\text{MI}}^* > 0$, there is $a > \beta(w + v_M)$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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