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Research Article

Best Proximity Point for Generalized Proximal Weak Contractions in Complete Metric Space

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We introduce a new class of nonself-mappings, generalized proximal weak contraction mappings, and prove the existence and uniqueness of best proximity point for such mappings in the context of complete metric spaces. Moreover, we state an algorithm to determine such an optimal approximate solution designed as a best proximity point. We establish also an example to illustrate our main results. Our result provides an extension of the related results in the literature.

1. Introduction and Preliminaries

A self-mapping T, defined on a metric space (X, d), is said to be a contraction if there exists a constant $k \in [0, 1)$ such that the inequality $d(Tx, Ty) \le kd(x, y)$ holds for all $x, y \in X$. Moreover, a self-mapping T is called a contractive mapping if $d(Tx, Ty) < d(x, y)$ holds for all $x, y \in X$ with $x \neq y$.

The celebrated Banach contraction principle says that if X is complete, then every contraction has a unique fixed point. In fact, the fixed point of a contraction mapping T is obtained as a limit of repeated iteration of the mapping for any (initial) point of X. Let Φ be the class of continuous, nondecreasing mapping $\phi : [0, \infty) \to [0, \infty)$ such that ϕ is positive on $(0, \infty)$ and $\phi(0) = 0$. A function $\phi \in \Phi$ is called an altering distance function.

A mapping $T : X \to X$ is called a weak-ϕ contraction if there exists a $\phi \in \Phi$ such that $d(Tx, Ty) \le d(x, y) - \phi(d(x, y))$ for each $x, y \in X$. The notion of weak-ϕ contraction was defined by Alber and Guerre-Delabriere [1] to generalize the well-known Banach contraction principle in the setting of Hilbert spaces. Later, Rhoades [2] noticed that most of the results of Alber and Guerre-Delabriere [1] are valid for any Banach space. Rhoades also proved the following generalization of the Banach contraction principle (see also [3–7]).

Theorem 1. *Let (X, d) be a nonempty complete metric space and let $T : X \to X$ be a weak-ϕ contraction on X; then T has a unique fixed point.*

Recently, Dutta and Choudhury [8] proved the following generalization of Theorem 1 by using (ψ, ϕ)-weak contraction map.

Theorem 2. *Let (X, d) be a nonempty complete metric space and let $T : X \to X$ be a self-mapping satisfying the inequality*

$$\psi(d(Tx, Ty)) \le \psi(d(x, y)) - \phi(d(x, y)), \tag{1}$$

for all $x, y \in X$, where of all function. Then T has a unique fixed point.

Let Γ be the class of all function; $\varphi : [0, \infty) \to [0, \infty)$ is a lower semicontinuous with $\varphi(t) = 0$ if and only if $t = 0$. In [9] Dorić proved the following generalization of Theorem 2 by using generalized (ψ, ϕ)-weak contractions which contains the (ψ, ϕ)-weak contractions as a subclass.

Theorem 3. *Let (X, d) be a nonempty complete metric space and let $T : X \to X$ be a generalized (ψ, ϕ)-weak contraction map; that is, T satisfies the following inequality:*

$$\psi(d(Tx, Ty)) \le \psi(m(x, y)) - \varphi(m(x, y)), \tag{2}$$

where $\psi \in \Phi$, $\varphi \in \Gamma$, *and* $m(x, y) = \max\{d(x, y), d(Tx, x), d(Ty, y), (1/2)[d(y, Tx) + d(x, Ty)]\}$ *for all* $x, y \in X$. *Then* T *has a unique fixed point.*

One of the aims of this paper is to extend Theorem 3 via best proximity point. For this purpose, we recollect the basic definitions and fundamentals results as follows.

1.1. Best Proximity Point Theorems. We first recall the notion of best proximity point for nonself-mappings.

Definition 4. Let (A, B) be a pair of two nonempty subsets of a metric space X. An element $x \in A$ is said to be a best proximity point of the nonself-mappings $T : A \to B$ if it satisfies the condition that $d(x, Tx) = d(A, B)$ where A and B are nonempty subsets of a metric space.

Best proximity point theorems have been studied to find necessary condition such that the minimization problem $\min_{x\in A} d(x, Tx)$ has at least one solution.

Existence and convergence of best proximity point is an interesting topic of optimization theory which recently attracted the attention of many authors [10–15]. A best proximity point theorem for nonself-proximal contractions has been investigated in [16–18].

In this paper, let us consider the mappings $T : A \to B$, where A and B are nonempty subsets of a metric space X with generalized proximal weak contraction on T which ensure the existence of a unique point $x \in A$ which satisfies $d(x, Tx) = d(A, B)$. When the map T is considered to be self-map, then our result reduces to Theorem 3.

Given nonempty subsets A and B of a metric space X, the following notions are used subsequently:

$$\begin{gathered} d(A, B) := \inf\{d(x, y) : x \in A, y \in B\}, \\ A_0 = \{x \in A : d(x, y) = d(A, B) \text{ for some } y \in B\}, \\ B_0 = \{y \in B : d(x, y) = d(A, B) \text{ for some } x \in A\}. \end{gathered} \tag{3}$$

In [13], the authors discussed sufficient conditions which guarantee the nonemptiness of A_0 and B_0. Also, in [14], the authors proved that A_0 is contained in the boundary of A.

Definition 5. The set B is said to be approximatively compact with respect to A if every sequence $\{y_n\}$ in B satisfying the condition that $d(x, y_n) \to d(x, B)$ for some x in A has a convergent subsequence.

Note that every set is approximatively compact with respect to itself and that every compact set is approximatively compact. Further, A_0 and B_0 are nonempty if A is compact and B is approximatively compact with respect to A.

Let us define the notion of generalized proximal weak contraction maps as follows. For this goal, we first introduce the following class of the mapping. Let Ω be the set of all function; $\phi : [0, \infty) \to [0, \infty)$ is nondecreasing with the following property: $\phi(t) = 0$ if and only if $t = 0$.

Definition 6. A mapping $T : A \to B$ is said to be a generalized proximal weak contraction on A if there exists functions $\psi, \phi : [0, \infty) \to [0, \infty)$ satisfying the following condition:

$$\left.\begin{array}{l} d(u, Tx) = d(A, B) \\ d(v, Ty) = d(A, B) \end{array}\right\} \Longrightarrow \psi(d(u, v)) \le \psi(m(x, y)) - \phi(m(x, y)), \tag{4}$$

where $m(x, y) = \max\{d(x, y), d(x, u), d(y, v), (d(x, v) + d(y, u))/2\}$ for all u, v, x, y in A, where $\psi \in \Phi$ and $\phi \in \Omega$.

Remark 7. Definition 6 guarantees that if a mapping T has a best proximity point then it should be unique. Indeed, we can prove our claim easily. Let $x \in A$ be a best proximity point of T. Suppose, on the contrary, that there is another element y such that $d(y, Ty) = d(A, B)$. Since T is a generalized proximal weak contraction on A, we have

$$\psi(d(x, y)) \le \psi(m(x, y)) - \phi(m(x, y)), \tag{5}$$

where $m(x, y) = \max\{d(x, y), d(x, x), d(y, y), (d(x, y) + d(y, x))/2\} = d(x, y)$.

From (5), we obtain $\psi(d(x, y)) \le \psi(d(x, y)) - \phi(d(x, y))$, which implies $\phi(d(x, y)) = 0$, and by our assumption about ϕ, we get $d(x, y) = 0$, or equivalently, $x = y$.

Definition 8. A mapping $T : A \to B$ is said to be a generalized proximal weak contraction on B if there exist functions $\psi, \phi : [0, \infty) \to [0, \infty)$ satisfying the following condition:

$$\left.\begin{array}{l} d(u, Tx) = d(A, B) \\ d(v, Ty) = d(A, B) \end{array}\right\} \Longrightarrow \psi(d(Tu, Tv)) \le \psi(m(Tx, Ty)) - \phi(m(Tx, Ty)), \tag{6}$$

where $m(Tx, Ty) = \max\{d(Tx, Ty), d(Tx, Tu), d(Ty, Tv), (d(Tx, Tv) + d(Ty, Tu))/2\}$ for all u, v, x, y in A, where $\psi \in \Phi$ and $\phi \in \Omega$.

For self-mappings, it is clear that every generalized proximal weak contraction on A is a generalized proximal weak contraction on B. An operator T is said to be a *generalized proximal weak contraction* if it is both generalized proximal weak contraction on A and generalized proximal weak contraction on B.

2. Main Results

We start this section with our main result.

Theorem 9. *Let* (A, B) *be a pair of two nonempty closed subsets of a complete metric space X such that* A_0 *is nonempty. Let* $T : A \to B$ *be a map satisfying the following conditions:*

(i) T *is a generalized proximal weak contraction,*

(ii) $T(A_0) \subseteq B_0$.

Then, there exists a unique $x \in A$ *such that* $d(x, Tx) = d(A, B)$. *Further, for any fixed element* $x_0 \in A_0$, *the sequence* $\{x_n\}$, *defined by* $d(x_{n+1}, Tx_n) = d(A, B)$, *converges to the element* x.

Proof. We prove the theorem in several steps.

Step 1. Let $x_0 \in A_0$. Since $Tx_0 \in T(A_0) \subseteq B_0$, there exists $x_1 \in A_0$ such that $d(x_1, Tx_0) = d(A, B)$. Due to the fact that $Tx_1 \in T(A_0) \subseteq B_0$, there exists $x_2 \in A_0$ such that $d(x_2, Tx_1) = d(A, B)$. Recursively, we find a sequence $\{x_n\}$ in A_0 such that

$$d(x_{n+1}, Tx_n) = d(A, B), \quad \forall n \in N. \tag{7}$$

If there exists n_0 such that $x_{n_0} = x_{n_0+1}$, then $d(x_{n_0+1}, Tx_{n_0}) = d(x_{n_0}, Tx_{n_0}) = d(A, B)$; that is, x_{n_0} is a best proximity point of T. Thus, the proof is finished. Hence, we suppose that $x_n \neq x_{n+1}$ for all n. Since T is a generalized proximal weak contraction on A, it follows that

$$\begin{aligned} \psi(d(x_n, x_{n+1})) &\leq \psi(m(x_{n-1}, x_n)) - \phi(m(x_{n-1}, x_n)) \\ &\leq \psi(m(x_{n-1}, x_n)). \end{aligned} \tag{8}$$

Using the monotone property of the ψ-function, we get $d(x_n, x_{n+1}) \leq m(x_{n-1}, x_n)$.

Now from the triangle inequality for d, we have

$$\begin{aligned} m(x_{n-1}, x_n) = &\max\left\{d(x_{n-1}, x_n), d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_{n+1}) + d(x_n, x_n)}{2}\right\} \\ = &\max\left\{d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_{n+1})}{2}\right\} \\ \leq &\max\left\{d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_n) + d(x_n, x_{n+1})}{2}\right\} \\ = &\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}. \end{aligned} \tag{9}$$

If $d(x_n, x_{n+1}) \geq d(x_{n-1}, x_n)$, then $m(x_{n-1}, x_n) = d(x_n, x_{n+1}) > 0$. From (8), we obtain $\psi(d(x_n, x_{n+1})) \leq \psi(d(x_n, x_{n+1})) - \phi(d(x_n, x_{n+1}))$, which is a contradiction. So, we have

$$d(x_n, x_{n+1}) \leq m(x_{n-1}, x_n) \leq d(x_{n-1}, x_n). \tag{10}$$

Hence, the sequence $\{d(x_n, x_{n+1})\}$ is monotone nonincreasing and bounded. Thus, there exists $r \geq 0$ such that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = \lim_{n\to\infty} m(x_{n-1}, x_n) = r \geq 0. \tag{11}$$

Suppose that $\lim_{n\to\infty} d(x_n, x_{n+1}) = \lim_{n\to\infty} m(x_{n-1}, x_n) = r > 0$. Then the inequality

$$\begin{aligned} \psi(d(x_n, x_{n+1})) &\leq \psi(m(x_{n-1}, x_n)) - \phi(m(x_{n-1}, x_n)) \\ &\leq \psi(m(x_{n-1}, x_n)) \end{aligned} \tag{12}$$

implies that

$$\lim_{n\to\infty} \phi(m(x_{n-1}, x_n)) = 0. \tag{13}$$

But as $0 < r \leq d(x_n, x_{n+1}) \leq m(x_{n-1}, x_n)$ and ϕ is nondecreasing function,

$$0 < \phi(r) \leq \phi(m(x_{n-1}, x_n)), \tag{14}$$

and this gives us $\lim_{n\to\infty}\phi(m(x_{n-1}, x_n)) \geq \phi(r) > 0$ which contradicts to (13). Hence,

$$\lim_{n\to\infty} m(x_{n-1}, x_n) = 0 = \lim_{n\to\infty} d(x_n, x_{n+1}). \tag{15}$$

Step 2. We will show that $\{x_n\}$ is a Cauchy sequence. Suppose, on the contrary, that $\{x_n\}$ is not a Cauchy sequence. Thus, there exists $\epsilon > 0$ for which we can find subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ such that $n(k)$ is the smallest index for which $n(k) > m(k) > k$, $d(x_{m(k)}, x_{n(k)}) \geq \epsilon$.

This means that

$$\begin{aligned} &d(x_{m(k)}, x_{n(k)-1}) < \epsilon, \\ \epsilon \leq\, &d(x_{m(k)}, x_{n(k)}) \\ &\leq d(x_{m(k)}, x_{n(k)-1}) + d(x_{n(k)-1}, x_{n(k)}) \\ &< \epsilon + d(x_{n(k)-1}, x_{n(k)}). \end{aligned} \tag{16}$$

Letting $k \to \infty$ and by using (15), we conclude that

$$\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = \epsilon. \tag{17}$$

Again,

$$\begin{aligned} d(x_{m(k)}, x_{n(k)-1}) &\leq d(x_{m(k)}, x_{n(k)}) + d(x_{n(k)}, x_{n(k)-1}), \\ d(x_{m(k)}, x_{n(k)}) &\leq d(x_{m(k)}, x_{n(k)-1}) + d(x_{n(k)}, x_{n(k)-1}). \end{aligned} \tag{18}$$

Therefore,

$$\left|d(x_{m(k)}, x_{n(k)-1}) - d(x_{m(k)}, x_{n(k)})\right| \leq d(x_{n(k)}, x_{n(k)-1}). \tag{19}$$

Letting $k \to \infty$ and by using (17) together with (15), it follows that

$$\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)-1}) = \epsilon. \tag{20}$$

Similarly, we derive that

$$\begin{aligned} \lim_{k\to\infty} d(x_{m(k)-1}, x_{n(k)}) &= \lim_{k\to\infty} d(x_{m(k)-1}, x_{n(k)-1}) \\ &= \lim_{k\to\infty} d(x_{m(k)+1}, x_{n(k)}) \\ &= \lim_{k\to\infty} d(x_{m(k)}, x_{n(k)+1}) = \epsilon. \end{aligned} \tag{21}$$

Then, we have

$$\lim_{k\to\infty} m(x_{m(k)-1}, x_{n(k)-1}) = \epsilon. \tag{22}$$

Using the fact that T is generalized proximal weak contraction on A for $d(x_{m(k)}, Tx_{m(k)-1}) = d(A,B)$ and $d(x_{n(k)}, Tx_{n(k)-1}) = d(A,B)$, we obtain

$$\begin{aligned} 0 < \psi(\epsilon) &\leq \psi(d(x_{m(k)}, x_{n(k)})) \\ &\leq \psi(m(x_{m(k)-1}, x_{n(k)-1})) \\ &\quad - \phi(m(x_{m(k)-1}, x_{n(k)-1})) \\ &\leq \psi(m(x_{m(k)-1}, x_{n(k)-1})). \end{aligned} \tag{23}$$

By using (22) and the continuity of ψ in the above inequality, we find that

$$\lim_{k\to\infty} \phi(m(x_{m(k)-1}, x_{n(k)-1})) = 0. \tag{24}$$

But from $\lim_{k\to\infty} m(x_{m(k)-1}, x_{n(k)-1}) = \epsilon$, we can find $k_0 \in \mathbb{N}$ such that for any $k \geq k_0$

$$\frac{\epsilon}{2} \leq m(x_{m(k)-1}, x_{n(k)-1}), \tag{25}$$

and consequently,

$$0 < \phi\left(\frac{\epsilon}{2}\right) \leq \phi(m(x_{m(k)-1}, x_{n(k)-1})) \quad \text{for } k \geq k_0. \tag{26}$$

Therefore, $0 < \phi(\epsilon/2) \leq \phi(m(x_{m(k)-1}, x_{n(k)-1}))$ and this contradicts to (24). Thus, $\{x_n\}$ is a Cauchy sequence in A and hence converges to some element x in A. Analogously, by using the fact that T is generalized proximal weak contraction on B, we conclude that $\{Tx_n\}$ is a Cauchy sequence in B. Hence, $\{Tx_n\}$ converges to some element y in B.

Step 3. Let us now prove that x is best proximity point for T.

Recall that $x_n \to x$ in A and $Tx_n \to y$ in B. Therefore, from (7), we get $d(x,y) = d(A,B)$ and hence x is a member of A_0. Since $T(A_0) \subseteq B_0$, we get $Tx \in B_0$; hence there exists $z \in A$ such that

$$d(z, Tx) = d(A,B). \tag{27}$$

Since T is a generalized proximal weak contraction on A, we obtain

$$\begin{aligned} \psi(d(x_{n+1}, z)) &\leq \psi(m(x_n, x)) - \phi(m(x_n, x)) \\ &\leq \psi(m(x_n, x)), \end{aligned} \tag{28}$$

where $m(x_n, x) = \max\{d(x_n, x), d(x_n, x_{n+1}), d(x,z), (d(x_n, z) + d(x, x_{n+1}))/2\}$.

By using the fact that $x_n \to x$, we get

$$\lim_{n\to\infty} m(x_n, x) = d(x,z). \tag{29}$$

Regarding (29) and continuity of ψ in (28), we can obtain

$$\lim_{n\to\infty} \phi(m(x_n, x)) = 0. \tag{30}$$

But from $\lim_{n\to\infty} m(x_n, x) = d(x,z)$ we find $n_0 \in \mathbb{N}$ such that for any $n \geq n_0$

$$\frac{d(x,z)}{2} \leq m(x_n, x) \tag{31}$$

and consequently, since ϕ is nondecreasing, we get

$$\phi\left(\frac{d(x,z)}{2}\right) \leq \phi(m(x_n, x)) \quad \forall n \geq n_0. \tag{32}$$

By using (30) in the inequality above, we get $\phi(d(x,z)/2) = 0$ and from the property of ϕ, we obtain $d(x,z)/2 = 0$ or equivalently, $x = z$. Hence, from (27), we have $d(x, Tx) = d(A,B)$. □

Example 10. Consider the complete space $X = \mathbb{R}^2$ with usual metric.

Suppose that $A := \{(x,0) : x \in [-2,-1]\}$ and $B := \{(1,x) : x \in [3/2, 2]\}$. Then A and B are nonempty closed subsets of X and $A_0 = \{(-1,0)\}$ and $B_0 = \{(1, 3/2)\}$. Note that $d(A,B) = 5/2$. Let $T : A \to B$ be defined as

$$T(x,0) = \left(1, 1 - \frac{x}{2}\right), \quad \forall (x,0) \in A. \tag{33}$$

Suppose that $(u_1, 0)$, $(u_2, 0)$, $(x_1, 0)$, $(x_2, 0)$ are elements in A such that

$$d((u_1, 0), T(x_1, 0)) = d((u_2, 0), T(x_2, 0)) = d(A,B). \tag{34}$$

Then, $(u_1, 0)$ and $(u_2, 0)$ become the members of A_0. Consequently, we have

$$d((u_1, 0), (u_2, 0)) = d(T(u_1, 0), T(u_2, 0)) = 0. \tag{35}$$

By assuming that $\psi, \phi : [0, \infty) \to [0, \infty)$ such that $\psi(t) = 2t$ and $\phi(t) = t/2$, we get

$$\begin{aligned} &\psi(d((u_1,0),(u_2,0))) = \psi(d(T(u_1,0), T(u_2,0))) = 0, \\ &\psi(m((x_1,0),(x_2,0))) - \phi(m((x_1,0),(x_2,0))) \\ &\quad = \frac{3}{2} m((x_1,0),(x_2,0)), \\ &\psi(m(T(x_1,0), T(x_2,0))) - \phi(m(T(x_1,0), T(x_2,0))) \\ &\quad = \frac{3}{2} m(T(x_1,0), T(x_2,0)). \end{aligned} \tag{36}$$

Therefore, T is both generalized proximal weak contraction on A and generalized proximal weak contraction on B. Hence T is generalized proximal weak contraction such that $T(A_0) \subseteq B_0$. So, all the hypotheses of Theorem 9 are satisfied. Further, it is easy to see that $(-1, 0)$ is the unique element satisfying the conclusion of Theorem 9.

It is easy to see that a self-mapping that is a generalized proximal weak contraction reduces to a generalized (ψ, ϕ)-weak contraction. Hence the above Theorem 9 gives rise to the following fixed point theorem, due to Dorić [9], which in turn extends the famous contraction principle.

Corollary 11. *Let (X, d) be a nonempty complete metric space and let $T : X \to X$ be a generalized proximal weak contraction map. Then T has a unique fixed point.*

Theorem 12. *Let (A, B) be a pair of two nonempty closed subsets of a complete metric space X such that B is approximatively compact with respect to A and A_0 is nonempty. Let $T : A \to B$ be a map satisfying the following conditions:*

(i) *T is a generalized proximal weak contraction on A,*

(ii) *$T(A_0) \subseteq B_0$.*

Then, there exists a unique $x \in A$ such that $d(x, Tx) = d(A, B)$. Further, for any fixed element $x_0 \in A_0$, the sequence $\{x_n\}$, defined by $d(x_{n+1}, Tx_n) = d(A, B)$, converges to the element x.

Proof. Following the proof of Theorem 9, there exists a sequence $\{x_n\}$ in A satisfying the following conditions:

$$d(x_{n+1}, Tx_n) = d(A, B), \quad \forall n \in \mathbb{N} \tag{37}$$

and x_n converges to x in A. Let us now prove that x is best proximity point for T.

Note that from (37), we have

$$\begin{aligned} d(x, B) &\le d(x, Tx_n) \le d(x, x_{n+1}) + d(x_{n+1}, Tx_n) \\ &\le d(x, x_{n+1}) + d(A, B) \\ &\le d(x, x_{n+1}) + d(x, B). \end{aligned} \tag{38}$$

Since $x_n \to x$, we get $d(x, Tx_n) \to d(x, B)$. Since B is approximatively compact with respect to the set A, it follows that the sequence $\{Tx_n\}$ has a subsequence converging to some y in B. Now arguing like Step 3 and Step 4 of Theorem 9, we get the required result. □

In what follows we prove that Theorem 12 is still valid for B not necessarily approximatively compact with respect to A, assuming that A_0 is closed.

Theorem 13. *Assume that "A_0 is closed" instead of "the statement B is approximatively compact with respect to A" in Theorem 12.*

Proof. Following the proof of Theorem 9, there exists a sequence $\{x_n\}$ in A satisfying the following condition:

$$d(x_{n+1}, Tx_n) = d(A, B), \tag{39}$$

and x_n converges to x in A. Note that the sequence $\{x_n\}$ in A_0 and A_0 is closed. Therefore, $x \in A_0$. Since $T(A_0) \subseteq B_0$, we get $Tx \in B_0$. Since $Tx \in B_0$, there exists $z \in A$ such that $d(z, Tx) = d(A, B)$. Since T is a generalized proximal weak contraction on A, we have

$$\psi(d(x_{n+1}, z)) \le \psi(m(x, x_n)) - \phi(m(x, x_n)), \tag{40}$$

where $m(x, x_n) = \max\{d(x_n, x), d(x_n, x_{n+1}), d(x, z), (d(x_n, z) + d(x, x_{n+1}))/2\}$.

Now arguing like Step 3 and Step 4 of Theorem 9, we get the required result. □

Theorem 14. *Assume that "T is continuous" instead of the statement "B is approximatively compact with respect to A" in the Theorem 12.*

Proof. Following the proof of Theorem 9, there exists a sequence $\{x_n\}$ in A satisfying the following condition:

$$d(x_{n+1}, Tx_n) = d(A, B), \tag{41}$$

and x_n converges to x in A. Since T are continuous, we have

$$d(x, Tx) = \lim_{n\to\infty} d(x_{n+1}, Tx_n) = d(A, B). \tag{42}$$

Uniqueness follows the same as in Theorem 9. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' Contribution

All authors contributed equally and significantly in writing this paper. All authors read and approved the final paper.

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