Research Article

New Exact Jacobi Elliptic Function Solutions for the Coupled Schrödinger-Boussinesq Equations

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A general algebraic method based on the generalized Jacobi elliptic functions expansion method, the improved general mapping deformation method, and the extended auxiliary function method with computerized symbolic computation is proposed to construct more new exact solutions for coupled Schrödinger-Boussinesq equations. As a result, several families of new generalized Jacobi elliptic function wave solutions are obtained by using this method, some of them are degenerated to solitary wave solutions and trigonometric function solutions in the limited cases, which shows that the general method is more powerful than plenty of traditional methods and will be used in further works to establish more entirely new solutions for other kinds of nonlinear partial differential equations arising in mathematical physics.

1. Introduction

Nonlinear partial differential equations (NLPDEs) are widely used to describe complex physical phenomena arising in the world around us and various fields of science. The investigation of exact solutions of NLPDEs plays an important role in the study of these phenomena such as the nonlinear dynamics and the mechanism behind the phenomena. With the development of soliton theory, many powerful methods for obtaining exact solutions of NLPDEs have been presented, such as homotopy perturbation method [1], nonperturbative method [2], homogeneous balance method [3], Bäcklund transformation [4], Darboux transformation [5], extended tanh-function method [6], extended F-expansion method [7], G'/G method [8], exp-function method [9], sine-cosine method [10], Jacobi elliptic function method [11], extended Riccati equation rational expansion method [12], extended auxiliary function method [13], and other methods [14, 15].

In [16, 17], Hong proposed a generalized Jacobi elliptic functions expansion method to obtain generalized exact solutions of NLPDEs. In [18], Hong and Lu proposed an improved general mapping deformation method which is more general than many other algebraic expansion methods

[19, 20]. The solution procedure of this method, by the help of Matlab or Mathematica, is of utmost simplicity, and this method can be easily extended to all kinds of NLPDEs. In this work, we will propose the general algebraic method which contained the two methods [16–18] to obtain several new families of exact solutions for the coupled Schrödinger-Boussinesq equations.

2. Summary of the General Algebraic Method

Consider a given nonlinear evolution equation with one physical field $u(x, t)$ in two variables x and t

$$P(u, u_t, u_x, u_{xx}, \ldots) = 0. \tag{1}$$

We seek the following formal solutions of the given system by a new intermediate transformation:

$$u(\xi) = \sum_{i=0}^{n} A_i \phi^i(\xi), \tag{2}$$

where A_i are constants to be determined later. $\xi = \xi(x, t)$ are arbitrary functions with the variables x and t. The parameter

n can be determined by balancing the highest order derivative terms with the nonlinear terms in (1). And $\phi(\xi)$ is a solution of the following ordinary differential equation:

$$\phi'^2(\xi) = \sum_{i=0}^{4} a_i \phi^i(\xi). \tag{3}$$

Substituting (3) and (2) into (1) and setting the coefficients of $\phi^i(\xi)$ ($i = 0, 1, 2, \ldots$) and $\phi^j(\xi)\sqrt{\sum_{i=0}^{4} a_i\phi^i(\xi)}$ ($j = \ldots, -2, -1, 0, 1, 2, \ldots$) to zero yield a set of algebraic equations for A_i, a_i, and ξ. Using the Mathematica to solve the algebraic equations and substituting each of the solutions of the set, that is, each of the expressions of $\phi(\xi)$ into (2), we can get the solutions of (1). In order to obtain some new general solutions of (3), we assume that (3) has the following solutions:

$$\phi(\xi) = c_0 + c_1 e(\xi) + c_2 f(\xi) + c_3 g(\xi) + c_4 h(\xi), \tag{4}$$

where $c_i = c_i(t)$ ($i = 0, \ldots, 4$) are functions of t to be determined later and the four functions $e = e(\xi)$, $f = f(\xi)$, $g = g(\xi)$, and $h = h(\xi)$ are expressed as follows:

$$e = \frac{1}{p + qF + rF^2 + lF'}, \qquad f = \frac{F}{p + qF + rF^2 + lF'},$$
$$g = \frac{F^2}{p + qF + rF^2 + lF'}, \qquad h = \frac{F'}{p + qF + rF^2 + lF'}, \tag{5}$$

where p, q, r, and l are arbitrary constants which ensure the denominator unequal to zero, and $F = F(\xi)$ is a solution of the following equations:

$$F'^2 = A + BF^2 + CF^4 + 2DF + 2EF^3,$$
$$F'' = BF + 2CF^3 + D + 3EF^2, \tag{6}$$

where "$'$" denotes $d/d\xi$, "$''$" denotes $d^2/d\xi^2$, A, B, C, D, and E are arbitrary constants, and the four functions e, f, g, and h satisfy the following relations:

$$e' = -qeh - 2rfh - l\left(De^2 + Bef + 2Cfg + 3Ef^2\right),$$
$$f' = peh - rgh + l\left(Ae^2 + Def - Cg^2 - Efg\right),$$
$$g' = qgh + 2pfh + l\left(2Aef + 3Df^2 + Bfg + Eg^2\right),$$
$$h' = (Dp - Aq)e^2 + (Bp - Dq - 2Ar)ef$$
$$\quad + (2Cp + Eq - Br)fg + 3(Ep - Dr)f^2 + (Cq - Er)g^2,$$
$$f^2 = eg, \qquad h^2 = Ae^2 + Bf^2 + Cg^2 + 2Def + 2Efg,$$
$$pe + qf + rg + lh = 1. \tag{7}$$

And e, f, g, and h satisfy one of the following relations at the same time.

Family 1. When $p = 0$

$$\left(Cl^2 - r^2\right)h^2 = -C + 2Clh - Br\left(1 - lh - qf\right)e$$
$$\quad - Ae^2r^2 - 2Dr^2ef + (2Cq - 2Er)f$$
$$\quad + (2lEr - 2Clq)fh + \left(2Eqr - Cq^2\right)f^2. \tag{8a}$$

Family 2. When $q = 0$

$$\left(Cl^2 - r^2\right)h^2 = 2C\left(lh + pe - pleh\right) + 2Er\left(lh - 1\right)f$$
$$\quad - Br\left(1 - lh - pe\right)e - \left(Cp^2 + Ar^2\right)e^2 \tag{8b}$$
$$\quad + \left(2Epr - 2Dr^2\right)ef - C.$$

Family 3. When $r = 0$

$$Cl^2g^2 = 1 - 2El^2fg - 2pe + \left(p^2 - Al^2\right)e^2$$
$$\quad - 2qf + 2\left(pq - Dl^2\right)ef + \left(q^2 - Bl^2\right)eg. \tag{8c}$$

Family 4. When $l = 0$

$$r^2h^2 = C - 2Cpe + (2Er - 2Cq)f + \left(Cp^2 + Ar^2\right)e^2$$
$$\quad + \left(Cq^2 - 2Eqr + Br^2\right)eg \tag{8d}$$
$$\quad + \left(2Cpq - 2Epr + 2Dr^2\right)ef.$$

Substituting (4), (7) along with (8a)–(8d) into (3) separately yields four families of polynomial equations for e, f, g, and h. Setting the coefficients of $e^i, e^i f, e^i g, e^i h, e^i fg, e^i fh$, and $e^i gh$ ($i = 0, 1, 2, \ldots$) to zero yields a set of overdetermined differential equations (ODEs) in p, q, r, l, a_i, and c_i ($i = 0, 1, 2, 3, 4$), A, B, C, D, E, and $\xi(x, t)$, solving the ODEs by Mathematica and Wu elimination, we can obtain many exact solutions of (3) according to (4), (5), (6). If we let $c_0 = c_1 = c_3 = c_4 = 0$, $c_2 = 1$, $p = 1$, $q = r = l = 0$, $a_0 = A$, $a_1 = 2D$, $a_2 = B, a_3 = 2E$, and $a_4 = C$, we have $\varphi(\xi) = F(\xi)$; our method contains the improved general mapping deformation method [18].

Remark 1. Our method proposed here is more general than the G'/G method [8], the extended Riccati equation rational expansion method [12], the extended auxiliary function method [13], the generalized Jacobi elliptic functions expansion method [16, 17], and many other algebra expansion methods [6, 7, 11, 18–21].

Remark 2. Equations (2) and (3) can be extended to the following forms:

$$u(\xi) = \sum_{i=0}^{n} A_i(t)\phi^i(\xi) + \sum_{i=-n}^{-1} A_{n-i}(t)\phi^i(\xi)$$

$$+ \sum_{i=-n}^{n} B_i(t)\phi^i(\xi)\phi'(\xi), \qquad (9)$$

$$\phi'^2(\xi) = \sum_{i=0}^{N} a_i(t)\phi^i(\xi),$$

where N can be an arbitrary positive integer. n is usually a positive integer. If n is a fraction or a negative integer, we make the following transformation:

   (a) when $n = d/c$ is a fraction, we let $u(\xi) = v^{d/c}(\xi)$, then return to determine the balance constant n again;

   (b) when n is a negative integer, we suppose $u(\xi) = v^n(\xi)$, then return to determine the balance constant n again.

Remark 3. Notice that

$$F_1(\xi)\big|_{(A,B,C,D,E)_1} \longrightarrow \varphi_1(\xi)\big|_{(a_0,a_1,a_2,a_3,a_4)_1}$$

$$= F_2(\xi)\big|_{(A,B,C,D,E)_2} \longrightarrow \varphi_2(\xi)\big|_{(a_0,a_1,a_2,a_3,a_4)_2} \qquad (10)$$

$$= F_3(\xi)\big|_{(A,B,C,D,E)_3} \longrightarrow \cdots.$$

We find a meanful conclusion that this general method implies a BT of (1) with the compatible conditions (4), (5), (6), (7), and (8a)–(8d).

In the following, we will use this method to solve the Schrödinger-Boussinesq equations.

3. Exact Solutions to the Coupled Schrödinger-Boussinesq Equations

We consider the coupled Schrödinger-Boussinesq equations [21–28]

$$iE_t + E_{xx} + \alpha E - NE = 0,$$

$$3N_{tt} - N_{xxxx} + 3(N^2)_{xx} + \beta N_{xx} - (|E|^2)_{xx} = 0. \qquad (11)$$

These equations are known to describe various physical processes in laser and plasma, such as formation, Langmuir field amplitude, intense electromagnetic waves, and modulational instabilities [22–25]. The problem of the complete integrability of this system has been studied by Chowdhury et al. from the point of view of Painlevé analysis [25]. The solitary wave solutions for system (11) have been obtained in [26, 27]. The Jacobi doubly periodic wave solutions and a range of other solutions for this system have been investigated in [21, 28]. We are interested in searching new generalized Jacobi elliptic function solutions for (11) by using our method.

We consider the following transformations:

$$E(x,t) = E(\xi) = u(\xi)e^{i(kx+\omega t+\xi_{00})},$$

$$N(x,t) = N(\xi) = v(\xi), \qquad (12)$$

$$\xi = ax - 2akt + \xi_0, \qquad (13)$$

where a and k are constants to be determined later and ξ_0 and ξ_{00} are arbitrary constants.

Substituting (12) and (13) into (11) and integrating the second equation of system (11) twice, we have

$$a^2 u'' + (\alpha - \omega - k^2)u - uv = 0,$$

$$a^2 v'' + u^2 - (12k^2 + \beta)v - 3v^2 = 0. \qquad (14)$$

By balancing the highest derivative term with the nonlinear terms in (14), we obtain $n = 2$. Therefore, we assume that (14) have the following solutions:

$$u(\xi) = A_0 + A_1\varphi(\xi) + A_2\varphi^2(\xi),$$

$$v(\xi) = B_0 + B_1\varphi(\xi) + B_2\varphi^2(\xi). \qquad (15)$$

Substituting (3), (13), and (15) into (14) and setting the coefficients of $\phi^i(\xi)$ ($i = 0, 1, 2, \ldots$) and $\varphi^j(\xi)\sqrt{\sum_{i=0}^{4} a_i\varphi^i(\xi)}$ ($j = \ldots, -2, -1, 0, 1, 2, \ldots$) to zero yield a set of over-determined equations (ODEs) for A_i, k, ω, and a_i. After solving the ODEs by Mathematica, we could determine the following solutions.

Family 1.

$$A_0 = \pm a^2\sqrt{\frac{-a_1 a_3 - 8a_0 a_4}{2}},$$

$$A_1 = \pm\frac{2a^2 a_4}{a_3}\sqrt{-2a_1 a_3 - 16a_0 a_4}, \qquad A_2 = 0,$$

$$B_0 = 0, \qquad B_1 = a^2 a_3, \qquad B_2 = 2a^2 a_4,$$

$$\omega = -\frac{a^2(a_3^4 - 80a_1 a_3 a_4^2 - 128a_0 a_4^3)}{48 a_3^2 a_4} + \alpha + \frac{\beta}{12}, \qquad (16)$$

$$k = \frac{\varepsilon}{4}\sqrt{\frac{a^2(a_3^4 + 16a_1 a_3 a_4^2 - 128a_0 a_4^3) - 4a_3^2 a_4 \beta}{3a_3^2 a_4}},$$

$$\varepsilon^2 = 1, \qquad a_2 = \frac{a_3^2}{4a_4} + \frac{2a_1 a_4}{a_3}.$$

Family 2.

$$A_0 = \pm\sqrt{2}B_0, \qquad A_1 = 0, \qquad A_2 = \pm 6\sqrt{2}a^2 a_4,$$

$$B_0 = \text{const}, \qquad B_1 = 0, \qquad B_2 = 6a^2 a_4,$$

$$a_1 = a_3 = 0, \qquad \omega = \frac{1}{12}(44a^2 a_2 - 22B_0 + 12\alpha + \beta), \qquad (17)$$

$$k = \varepsilon\sqrt{\frac{4a^2 a_2 - 2B_0 - \beta}{12}}, \qquad \varepsilon^2 = 1.$$
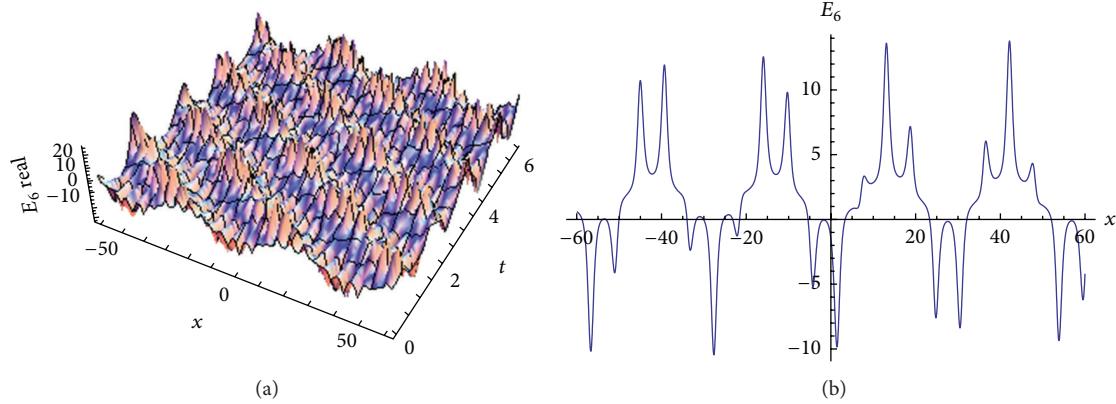
(a)

(b)

FIGURE 1: The real part of E_6 when $a = \alpha = 1$, $\beta = -1$, $\xi_0 = 0$, and $m = 0.9$ and a plane graph when $t = 0$.
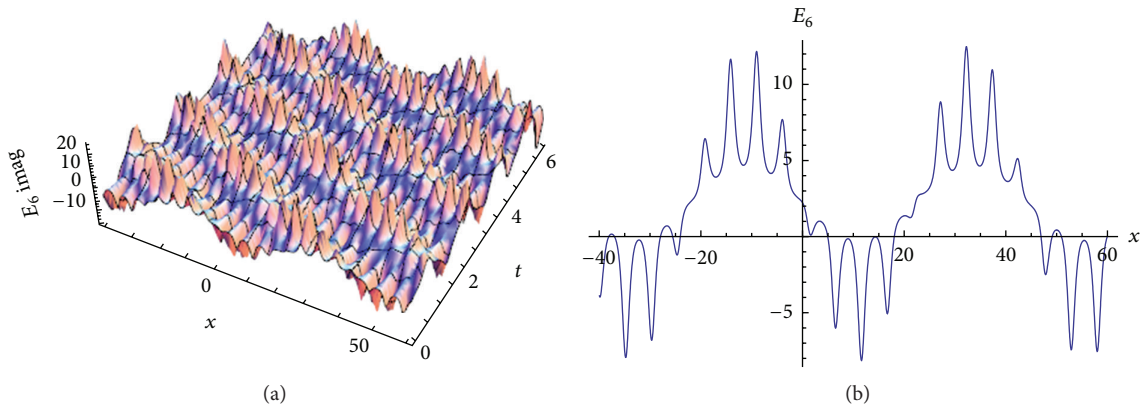

(a)

(b)

FIGURE 2: The imaginary part of E_6 when $a = \alpha = 1$, $\beta = -1$, $\xi_0 = 0$, and $m = 0.9$ and a plane graph when $t = 0$.

Substituting (4), (5), (6), (7) along with (8a)–(8d) and (16) into (14) separately yields an ODEs; after solving the ODEs by Mathematica and Wu elimination, we can obtain the following solutions of (14) according to (4), (5), (6), (16).

Case 1.

$$A = a_{0_1} = \frac{1}{4}\left(1 - m^2\right), \qquad B = a_{2_1} = 2 - m^2,$$

$$C = a_{4_1} = 1, \qquad D = \frac{1}{2}a_{1_1} = \frac{1}{2}\left(m^2 - 1\right),$$

$$E = \frac{1}{2}a_{3_1} = -1, \quad 0 \le m \le 1,$$

$$p = 1, \qquad q = r = l = 0, \tag{18}$$

$$c_0 = c_1 = c_3 = c_4 = 0, \qquad c_2 = 1,$$

$$\varphi_1\left(\xi_1\right) = F_1\left(\xi_1\right) = \frac{\mathrm{cn}\xi_1}{\pm 1 \,{}^{+}_{\pm}\, \mathrm{sn}\xi_1 + \mathrm{cn}\xi_1},$$

$$\xi_1 = ax - 2ak_1 t + \xi_0.$$

Case 2.

$$A = a_{0_2} = \frac{1}{4}, \qquad B = a_{2_2} = 2 - m^2,$$

$$C = a_{4_2} = 1 - m^2, \qquad D = \frac{1}{2}a_{1_2} = \frac{1}{2},$$

$$E = \frac{1}{2}a_{3_2} = 1 - m^2, \quad 0 \le m \le 1,$$

$$p = 1, \qquad q = r = l = 0,$$

$$c_0 = c_1 = c_3 = c_4 = 0, \qquad c_2 = 1, \tag{19}$$

$$\varphi_2\left(\xi_2\right) = F_2\left(\xi_2\right) = \frac{\mathrm{sn}\xi_2}{\pm 1 - \mathrm{sn}\xi_2 \,{}^{+}_{\pm}\, \mathrm{cn}\xi_2},$$

$$\xi_2 = ax - 2ak_2 t + \xi_0.$$

Case 3.

$$A = a_{0_3} = -\left(c_0^2 - 1\right)\left(1 + \left(c_0^2 - 1\right)m^2\right),$$

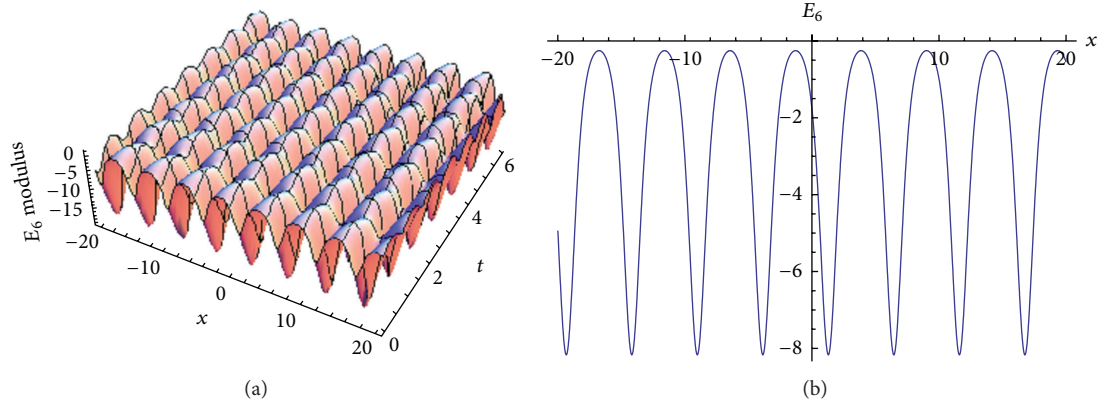$$B = a_{2_3} = \left(2 - 6c_0^2\right)m^2 - 1, \qquad C = a_{4_3} = -m^2,$$

$$D = \frac{1}{2}a_{1_3} = c_0 - 2c_0 m^2 + 2c_0^3 m^2,$$

$$E = \frac{1}{2}a_{3_3} = 2c_0 m^2, \quad 0 \le m \le 1,$$

$$p = 1, \qquad q = r = l = 0,$$

$$c_0 = \text{const}, \qquad c_1 = c_3 = c_4 = 0, \qquad c_2 = 1,$$

$$\varphi_3\left(\xi_3\right) = F_3\left(\xi_3\right) = c_0 + \mathrm{cn}^2\xi_3, \quad \xi_3 = ax - 2ak_3 t + \xi_0. \tag{20}$$

(a)

(b)

FIGURE 3: The modulus of E_6 when $a = \alpha = 1$, $\beta = -1$, $\xi_0 = 0$, and $m = 0.9$ and a plane graph when $t = 0$.

Case 4.

$$A = 1, \qquad B = -m^2 - 1, \qquad C = m^2,$$

$$D = E = 0, \qquad F = \mathrm{sn}\xi, \qquad 0 \le m \le 1,$$

$$a_{0_4} = 1, \qquad a_{1_4} = \mp 4\sqrt{2(1+m)\sqrt{m}},$$

$$a_{3_4} = \pm 2\sqrt{2(1+m)\sqrt{m}}\left(1 + 6m + m^2 - 4(1+m)\sqrt{m}\right),$$

$$a_{2_4} = 12(1+m)\sqrt{m} - 6m - m^2 - 1,$$

$$a_{4_4} = 8m(m+1)^2 - 2(1+m)\left(1 + m^2 + 6m\right)\sqrt{m},$$

$$p = 1, \qquad q = \pm\sqrt{2(1+m)\sqrt{m}}, \qquad r = m,$$

$$l = 0, \qquad c_0 = c_1 = c_3 = c_4 = 0, \qquad c_2 = 1,$$

$$\varphi_4(\xi_4) = \frac{\mathrm{sn}\xi_4}{1 \pm \sqrt{2(1+m)\sqrt{m}}\,\mathrm{sn}\xi_4 + m\,\mathrm{sn}^2\xi_4},$$

$$\xi_4 = ax - 2ak_4 t + \xi_0.$$

$$(21)$$

Case 5.

$$A = 1 - m^2, \qquad B = 2m^2 - 1,$$

$$C = -m^2, \quad D = E = 0, \qquad 0 \le m \le 1,$$

$$F = \mathrm{cn}\xi \qquad a_{0_5} = 1, \qquad a_{1_5} = -4,$$

$$a_{2_5} = 8 - 4m^2, \qquad a_{3_5} = 8m^2 - 8, \qquad a_{4_5} = 4 - 4m^2,$$

$$p = 0, \qquad q = 1, \qquad r = 0, \qquad l = \pm 1,$$

$$c_0 = c_1 = c_3 = c_4 = 0, \qquad c_2 = 1,$$

$$\varphi_5(\xi_5) = \frac{\mathrm{cn}\xi_5}{\mathrm{cn}\xi_5 \mp \mathrm{sn}\xi_5 \mathrm{dn}\xi_5}, \qquad \xi_5 = ax - 2ak_5 t + \xi_0.$$

$$(22)$$

Case 6.

$$A = m^2 - 1, \qquad B = 2 - m^2, \qquad C = -1,$$

$$D = E = 0, \qquad 0 \le m \le 1,$$

$$F = \mathrm{dn}\xi \qquad a_{0_6} = 1, \qquad a_{1_6} = -4m,$$

$$a_{2_6} = 8m^2 - 4, \qquad a_{3_6} = 8m - 8m^3, \qquad a_{4_6} = 4m^4 - 4m^2,$$

$$p = 0, \qquad q = m, \qquad r = 0, \qquad l = \pm 1,$$

$$c_0 = c_1 = c_3 = c_4 = 0, \qquad c_2 = 1,$$

$$\varphi_6(\xi_6) = \frac{\mathrm{dn}\xi_6}{\mathrm{dn}\xi_6 \mp m^2 \mathrm{sn}\xi_6 \mathrm{cn}\xi_6}, \qquad \xi_6 = ax - 2ak_6 t + \xi_0.$$

$$(23)$$

Case 7.

$$A = \frac{C_1 C_3 q - 5C_3^2 q^3 + \varepsilon\left(C_1 + 3C_3 q^2\right)\sqrt{C_3\left(3C_3 q^2 - 2C_1\right)}}{4C_3},$$

$$B = 0, \qquad C = 0, \qquad 2D = C_1, \qquad 2E = C_3,$$

$$F = \wp\left(\frac{\sqrt{C_3}}{2}\xi, \frac{-4C_1}{C_3}, \frac{-4M}{C_3}\right),$$

$$a_{0_7} = 0, \qquad a_{1_7} = C_3, \qquad a_{2_7} = -3C_3 q,$$

$$a_{3_7} = C_1 + 3C_3 q^2,$$

$$a_{4_7} = \left(-3C_1 C_3 q - 9C_3^2 q^3\right.$$

$$\left.+ \varepsilon\left(C_1 + 3C_3 q^2\right)\sqrt{C_3\left(3C_3 q^2 - 2C_1\right)}\right)$$

$$\times \left(4C_3\right)^{-1},$$

$$\varepsilon = \text{sgn}\left[C_1 + 3C_3 q^2\right], \qquad p = 0, \qquad q = \text{const},$$

$$r = 1, \qquad l = 0, \qquad c_0 = c_1 = c_3 = c_4 = 0, \qquad c_2 = 1,$$

$$\varphi_7\left(\xi_7\right) = \frac{1}{q + \wp\left(\left(\sqrt{C_3}/2\right)\xi_7, -4C_1/C_3, -4M/C_3\right)},$$

$$\xi_7 = ax - 2ak_7 t + \xi_0. \tag{24}$$

We obtain the following solutions of (11) according to (4), (5), (12), (13), (15), and (16) and Cases 1–7:

$$E_i\left(x,t\right) = \left[\pm a^2 \sqrt{\frac{-a_{1_i} a_{3_i} - 8a_{0_i} a_{4_i}}{2}}\right.$$

$$\left.\pm \frac{2a^2 a_{4_i}}{a_{3_i}} \sqrt{-2a_{1_i} a_{3_i} - 16a_{0_i} a_{4_i}}\, \varphi_i\left(\xi_i\right)\right] \tag{25}$$

$$\times e^{i\left(k_i x + \omega_i t + \xi_{00}\right)},$$

$$N_i\left(x,t\right) = a^2 a_{3_i} \varphi_i\left(\xi_i\right) + 2a^2 a_{4_i} \varphi_i^2\left(\xi_i\right),$$

$$\xi_i = ax - 2ak_i t + \xi_0, \qquad \left(i = 1,\ldots,7\right),$$

where a is an arbitrary constant, $a_{0_i}, a_{1_i}, a_{2_i}, a_{3_i},$ and a_{4_i} are defined as Cases 1–7, and ω_i and k_i are defined as follows:

$$\omega_i = -\frac{a^2\left(a_{3_i}^4 - 80a_{1_i} a_{3_i} a_{4_i}^2 - 128a_{0_i} a_{4_i}^3\right)}{48a_{3_i}^2 a_{4_i}} + \alpha + \frac{\beta}{12},$$

$$k_i = \frac{\varepsilon}{4} \sqrt{\frac{a^2\left(a_{3_i}^4 + 16a_{1_i} a_{3_i} a_{4_i}^2 - 128a_{0_i} a_{4_i}^3\right) - 4a_{3_i}^2 a_{4_i} \beta}{3a_{3_i}^2 a_{4_i}}}, \tag{26}$$

$$\varepsilon^2 = 1, \qquad \left(i = 1,\ldots,7\right).$$

With the similar process, substituting (4), (5), (6), and (7) along with (8a)–(8d) and (17) into (14) separately yields ODEs; after solving the ODEs by Mathematica and Wu elimination, we can obtain the following solutions of (11) according to (4), (5), (12), (13), (15), and (17) and the solutions of (3) mentioned in [29]:

$$E_j\left(x,t\right) = \left[\pm\sqrt{2}B_0 \pm 6\sqrt{2}a^2 a_{4_j} \varphi_j^2\left(\xi_j\right)\right] e^{i\left(k_j x + \omega_j t + \xi_{00}\right)},$$

$$\left(j = 0,1,2,\ldots\right),$$

$$N_j\left(x,t\right) = B_0 + 6a^2 a_{4_j} \varphi_j^2\left(\xi_j\right), \qquad \xi_j = ax - 2ak_j t + \xi_0,$$

$$\omega_j = \frac{1}{12}\left(44a^2 a_{2_j} - 22B_0 + 12\alpha + \beta\right),$$

$$k_j = \varepsilon \sqrt{\frac{4a^2 a_{2_j} - 2B_0 - \beta}{12}}, \qquad \varepsilon^2 = 1. \tag{27}$$

Remark 4. If we let $a_{0_{1.1}} = 1 - m^2$, $a_{2_{1.1}} = 2m^2 - 1$, $a_{4_{1.1}} = -m^2$, $a_{1_{1.1}} = a_{3_{1.1}} = 0$, and $\varphi_{1.1}(\xi_{1.1}) = \text{cn}\,\xi_{1.1}$, then we have

$$E_{1.1}\left(x,t\right) = \left[\pm\sqrt{2}B_0 \mp 6\sqrt{2}a^2 m^2 \text{cn}^2\left(\xi_{1.1}\right)\right] e^{i\left(k_1 x + \omega_1 t + \xi_{00}\right)},$$

$$N_{1.1}\left(x,t\right) = B_0 - 6a^2 m^2 \text{cn}^2\left(\xi_{1.1}\right), \qquad \xi_{1.1} = ax - 2ak_{1.1} t + \xi_0,$$

$$\omega_{1.1} = \frac{1}{12}\left(44a^2\left(2m^2 - 1\right) - 22B_0 + 12\alpha + \beta\right),$$

$$k_{1.1} = \varepsilon \sqrt{\frac{4a^2\left(2m^2 - 1\right) - 2B_0 - \beta}{12}}, \qquad \varepsilon^2 = 1. \tag{28}$$

If we let $a = (1/2)\sqrt{b_0}$, $m = 1$, or $a = (1/2)\sqrt{(1/2m^2 - 1)b_0}$, then solution $(E_{1.1}(x,t), N_{1.1}(x,t))$ is in full agreement with the solution E_1, N_1 and E_2, N_2 mentioned in [21].

Remark 5. The seven types of explicit solutions we obtained here to (11) are not shown in the previous literature to our knowledge. They are new exact solutions of (11). Notice that $\text{sn}\,\xi \to \tanh\xi$, $\text{cn}\,\xi \to \text{sech}\xi$, and $\text{dn}\,\xi \to \text{sech}\xi$ when the modulus $m \to 1$, and $\text{sn}\,\xi \to \sin\xi$, $\text{cn}\,\xi \to \cos\xi$, and $\text{dn}\,\xi \to 1$ when the modulus $m \to 0$. Solutions E_i, N_i ($i = 1,\ldots,6$) are degenerated to solitary wave solutions when the modulus $m \to 1$ and to triangular functions solutions when the modulus $m \to 0$.

We can give the numerical simulation of E_6 to show their physical properties (see Figures 1, 2, and 3).

4. Conclusion

In this paper, we succeed to propose a general algebraic method approach for finding new exact solutions of the nonlinear evolution equations. By using this method and computerized symbolic computation, we have found abundant new exact solutions for the coupled Schrödinger-Boussinesq equations (11). More importantly, our method is much simpler and powerful to find new solutions to various kinds of nonlinear evolution equations, such as KdV equation, Boussinesq equation, and zakharov equation. We believe that this method should play an important role for finding exact solutions in the mathematical physics.

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