

Research Article

Singular Value Decomposition-Based Method for Sliding Mode Control and Optimization of Nonlinear Neutral Systems

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Received 26 December 2012; Revised 28 February 2013; Accepted 14 March 2013

Academic Editor: Hak-Keung Lam

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The sliding mode control and optimization are investigated for a class of nonlinear neutral systems with the unmatched nonlinear term. In the framework of Lyapunov stability theory, the existence conditions for the designed sliding surface and the stability bound α^* are derived via twice transformations. The further results are to develop an efficient sliding mode control law with tuned parameters to attract the state trajectories onto the sliding surface in finite time and remain there for all the subsequent time. Finally, some comparisons are made to show the advantages of our proposed method.

1. Introduction

Time delays often arise in the processing state, input, or related variables of dynamic systems. In particular, when the state derivative also contains time-delay, the considered systems are called as neutral systems [1]. The outstanding characteristic of neutral systems is the fact that such systems contain the same highest order derivatives for the state vector $x(t)$, at both time t and past time(s) $t_s \leq t$. Many engineering systems can be represented as neutral equation [2–8], such as population ecology [9], distributed networks containing lossless transmission lines [10], heat exchangers, and robots in contact with rigid environments [11]. The delay-dependent stability criteria for neutral stochastic systems with time delay in state are studied in [4, 5]. The difference is that the former is about exponential stability; the other is on robust stochastic stability, stabilization, and H_∞ control. Furthermore, a robust H_∞ reduced-order filter and a memory state feedback control are developed to establish the improved stability criteria for neutral systems in [6, 7], respectively. In [12], a periodic output feedback is studied in the context of infinite-dimensional linear systems modeled by neutral functional differential equations, and the main work only focuses on stabilization

of neutral systems with delayed control. The stability and H_∞ performance analysis, the reliable stabilization, and the finite-time H_∞ control for uncertain switched neutral systems are investigated in [13–15], respectively.

On the other hand, as an important robust control approach, sliding mode control strategy has many advantages such as fast response, insensitiveness to parametric uncertainties and external disturbances, and unnecessariness for online identification. Hence, the sliding mode control for dynamic systems has received attention extensively [16–24]. Xia et al. propose a novel approach combining SMC and ESO and utilize the backstepping technique to control the attitude of a nonlinear missile system in [16]. In [17], a stable integral type fractional-order sliding surface is introduced to stabilize and synchronize a class of fractional-order chaotic systems. In [18, 19], Xia et al. investigate the robust sliding control for uncertain continuous systems and discrete-time systems with constant delays, respectively. In [20, 21], the PSMC approach and the PDMSMO approach are utilized, respectively, to deal with the fault-tolerant control question for uncertain systems. It is worth pointing out that the nonlinear terms are subject to the matched condition in [20]. The same requirement also appears in [25, 26] for neutral systems. In addition, the

matched condition is required for the parametric uncertainty in the input in [27].

To the best of the authors' knowledge, the sliding control and optimization for uncertain neutral systems have not yet been investigated, which motivates the present study. One contribution of this paper is the optimization for the upper bound of the unmatched nonlinear term. The other contribution lies in the introduction of the scalars K_X and K_{Q_2} to prevent the unacceptably high gains. In this paper, the state transformations based on singular value decomposition and the descriptor system model are utilized to obtain the existence conditions for the designed sliding surface and the upper bound α^* . In particular, the achieved bound α^* secures the quadratic stability of the sliding motion with all α satisfying $\bar{\alpha} < \alpha < \alpha^*$. In addition, an efficient control law is designed to attract the state trajectories onto the sliding surface in finite time and remain there for all the subsequent time. In order to reduce the chattering on the sliding surface, the tuned parameters are introduced into the developed control law. One example under three cases is given to illustrate our proposed method and make some comparisons. Compared with [27], our results have the advantages of the more rapid convergent rate and the less chattering phenomena.

The sliding mode control problem formulation is described in Section 2. In Section 3, the sliding surface design and the reaching motion control design are developed. An example under three cases and some compared results are provided in Section 4. Finally, conclusion is given in Section 5.

2. Problem Formulation

Consider the following class of nonlinear neutral systems:

$$\begin{aligned} \dot{x}(t) - A_\eta \dot{x}(t - \eta(t)) \\ = [A + \Delta A(t)] x(t) + [A_\sigma + \Delta A_\sigma(t)] x(t - \sigma(t)) \\ + Bu(t) + h(x), \quad x(t) = \phi(t), \quad t \in [-l_0, 0], \end{aligned} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the state of the system and $u(t) \in \mathcal{R}^m$ is the input vector. $A \in \mathcal{R}^{n \times n}$, $A_\sigma \in \mathcal{R}^{n \times n}$, $A_\eta \in \mathcal{R}^{n \times n}$, and $B \in \mathcal{R}^{n \times m}$ are known constant matrices, and $h : \mathcal{R}^n \rightarrow \mathcal{R}^n$ represents a piecewise-continuous nonlinear function satisfying $h(0) = 0$. It is assumed that the nonlinear term $h(x)$ can be bounded by a quadratic inequality

$$h^T(x) h(x) \leq \alpha^2 x^T H^T H x, \quad (2)$$

where H is a constant matrix and $\alpha > 0$ is a scalar parameter. $\phi(t)$ is the initial condition. $\sigma(t)$ and $\eta(t)$ are the time-varying delays. Assume that there exist constants f_0 , g_0 , l_0 , f , and g satisfying

$$\begin{aligned} 0 \leq \sigma(t) \leq f_0, \quad 0 \leq \eta(t) \leq g_0, \quad \dot{\sigma}(t) \leq f < 1, \\ \dot{\eta}(t) \leq g < 1, \quad \max\{f_0, g_0\} = l_0. \end{aligned} \quad (3)$$

Time-varying parametric uncertainties $\Delta A(t)$ and $\Delta A_\sigma(t)$ are assumed to be of the following form:

$$\Delta A(t) = EF(t)D, \quad \Delta A_\sigma(t) = E_\sigma F_\sigma(t)D_\sigma, \quad (4)$$

where E , D , E_σ , and D_σ are constant matrices of appropriate dimensions and $F(t)$ and $F_\sigma(t)$ are the unknown matrix function satisfying $F^T(t)F(t) \leq I$ and $F_\sigma^T(t)F_\sigma(t) \leq I$ for all $t \geq 0$.

Without loss of the generality, assume that $\text{rank}(B) = m$. One can easily get the singular value decomposition of B

$$B = [U_1 \ U_2] \begin{bmatrix} \Sigma \\ 0_{(n-m) \times m} \end{bmatrix} V^T, \quad (5)$$

where $\Sigma \in \mathcal{R}^{m \times m}$ is a diagonal positive definite matrix; $U_1 \in \mathcal{R}^{n \times m}$, $U_2 \in \mathcal{R}^{n \times (n-m)}$, and $V \in \mathcal{R}^{m \times m}$ are unitary matrices.

The following state transformation is similar to [18, 20, 28]. Choose $z = \Gamma x$, where $\Gamma = \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix}$, another form of the system (1) can be obtained

$$\begin{aligned} \dot{z}(t) - \bar{A}_\eta \dot{z}(t - \eta(t)) \\ = [\bar{A} + \Delta \bar{A}(t)] z(t) + [\bar{A}_\sigma + \Delta \bar{A}_\sigma(t)] z(t - \sigma(t)) \\ + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u(t) + \bar{h}(\Gamma^{-1}z), \\ z(t) = \bar{\phi}(t), \quad t \in [-l_0, 0], \end{aligned} \quad (6)$$

where $\bar{A}_\eta = \Gamma A_\eta \Gamma^{-1}$, $\bar{A} = \Gamma A \Gamma^{-1}$, $\Delta \bar{A} = \Gamma \Delta A \Gamma^{-1}$, $\bar{A}_\sigma = \Gamma A_\sigma \Gamma^{-1}$, $\Delta \bar{A}_\sigma = \Gamma \Delta A_\sigma \Gamma^{-1}$, $B_1 = \Sigma V^T$, $\bar{h}(\Gamma^{-1}z) = \Gamma h(x) = \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix} h(x) = \begin{bmatrix} \bar{h}_1(\Gamma^{-1}z) \\ \bar{h}_2(\Gamma^{-1}z) \end{bmatrix} = \begin{bmatrix} U_1^T h(\Gamma^{-1}z) \\ U_2^T h(\Gamma^{-1}z) \end{bmatrix}$, and $\bar{\phi}(t) = \Gamma \phi(t)$.

Furthermore, the system (6) can be rewritten as

$$\begin{aligned} \dot{z}_1(t) - \bar{A}_{\eta 11} \dot{z}_1(t - \eta(t)) - \bar{A}_{\eta 12} \dot{z}_2(t - \eta(t)) \\ = [\bar{A}_{11} + \Delta \bar{A}_{11}(t)] z_1(t) + [\bar{A}_{12} + \Delta \bar{A}_{12}(t)] z_2(t) \\ + [\bar{A}_{\sigma 11} + \Delta \bar{A}_{\sigma 11}(t)] z_1(t - \sigma(t)) \\ + [\bar{A}_{\sigma 12} + \Delta \bar{A}_{\sigma 12}(t)] z_2(t - \sigma(t)) \\ + B_1 u(t) + \bar{h}_1(\Gamma^{-1}z), \\ \dot{z}_2(t) - \bar{A}_{\eta 21} \dot{z}_1(t - \eta(t)) - \bar{A}_{\eta 22} \dot{z}_2(t - \eta(t)) \\ = [\bar{A}_{21} + \Delta \bar{A}_{21}(t)] z_1(t) + [\bar{A}_{22} + \Delta \bar{A}_{22}(t)] z_2(t) \\ + [\bar{A}_{\sigma 21} + \Delta \bar{A}_{\sigma 21}(t)] z_1(t - \sigma(t)) \\ + [\bar{A}_{\sigma 22} + \Delta \bar{A}_{\sigma 22}(t)] z_2(t - \sigma(t)) + \bar{h}_2(\Gamma^{-1}z), \\ z_1(t) = \bar{\phi}_1(t), \quad t \in [-l_0, 0], \\ z_2(t) = \bar{\phi}_2(t), \quad t \in [-l_0, 0], \end{aligned} \quad (7)$$

where $z_1(t) \in \mathfrak{R}^m$, $z_2(t) \in \mathfrak{R}^{n-m}$, $\bar{A}_{11} = U_1^T A U_1$, $\bar{A}_{12} = U_1^T A U_2$, $\bar{A}_{21} = U_2^T A U_1$, $\bar{A}_{22} = U_2^T A U_2$, $\bar{h}_1(\Gamma^{-1}z) = U_1^T h(\Gamma^{-1}z)$ and $\bar{h}_2(\Gamma^{-1}z) = U_2^T h(\Gamma^{-1}z)$ and the others can be obtained easily according to the same way.

On account of the second equation of (7) that represents the sliding motion dynamics of (6), one can construct the following sliding surface:

$$S(t) = [I \ C] z(t) = z_1(t) + C z_2(t) = 0, \quad (8)$$

where $C \in \mathfrak{R}^{m \times (n-m)}$ is the gain to be designed.

Based on (8) and the second equation of (7), one can get the following sliding motion:

$$\begin{aligned} \dot{z}_2(t) - [\bar{A}_{\eta 22} - \bar{A}_{\eta 21} C] \dot{z}_2(t - \eta(t)) \\ = [\bar{A}_{22} - \bar{A}_{21} C + \Delta \bar{A}_{22}(t) + \Delta \bar{A}_{21}(t) C] z_2(t) \\ + [\bar{A}_{\sigma 22} - \bar{A}_{\sigma 21} C + \Delta \bar{A}_{\sigma 22}(t) + \Delta \bar{A}_{\sigma 21}(t) C] \\ \times z_2(t - \sigma(t)) + \bar{h}_2(\Gamma^{-1}z), \\ z_2(t) = \bar{\phi}_2(t), \quad t \in [-l_0, 0]. \end{aligned} \quad (9)$$

In view of (2), (8), and $\bar{h}_2(\Gamma^{-1}z)$ in (9), one can obtain that

$$\begin{aligned} \bar{h}_2^T(\Gamma^{-1}z) \bar{h}_2(\Gamma^{-1}z) \\ = h^T(\Gamma^{-1}z) U_2 U_2^T h(\Gamma^{-1}z) \leq \alpha^2 z^T(t) \Gamma^{-T} H^T H \Gamma^{-1} z(t) \\ \leq \alpha^2 z_2^T(t) [-C^T \ I] \bar{H}^T \bar{H} \begin{bmatrix} -C \\ I \end{bmatrix} z_2(t), \end{aligned} \quad (10)$$

where $\bar{H} = H \Gamma^{-1}$.

Our purpose is to design a sliding surface $S(t)$ and a reaching motion control law $u(t)$ such that the following holds:

- (i) sliding motion (9) is quadratically stable with upper bound α^* ; that is, for any given $\bar{\alpha}$, find out the upper bound α^* so that for all α satisfying $\bar{\alpha} < \alpha < \alpha^*$, sliding motion (9) is quadratically stable;
- (ii) the system (7) is asymptotically stable under the reaching control law $u(t)$.

Lemma 1 (see [4]). For any constant matrix $M \in \mathfrak{R}^{q \times l}$, inequality

$$2u^T M v \leq r u^T M G M^T u + \frac{1}{r} v^T G^{-1} v, \quad u \in \mathfrak{R}^q, \quad v \in \mathfrak{R}^l \quad (11)$$

holds for any pair of symmetric positive definite matrix $G \in \mathfrak{R}^{l \times l}$ and positive number $r > 0$.

Lemma 2 (see [29]). Let Z , X , S , and Y be matrices of appropriate dimensions. Assume that Z is symmetric and $S^T S \leq I$, then $Z + XSY + Y^T S^T X^T < 0$ if and only if there exists a scalar $\varepsilon > 0$ satisfying

$$Z + \varepsilon X X^T + \varepsilon^{-1} Y^T Y = Z + \varepsilon^{-1} (\varepsilon X) (\varepsilon X)^T + \varepsilon^{-1} Y^T Y < 0. \quad (12)$$

3. Main Result

3.1. Sliding Surface Design for Nonlinear Neutral Systems

Theorem 3. The sliding motion (9) is quadratically stable with upper bound α^* if the following optimization problem

$$\min \quad \gamma + K_X + K_{Q_2} \quad (13)$$

subject to

$$\begin{aligned} \Omega \\ = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} & 0 & \Omega_{19} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & 0 & \Omega_{27} & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 & \Omega_{38} & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{88} & 0 \\ * & * & * & * & * & * & * & * & \Omega_{99} \end{bmatrix} < 0 \end{aligned} \quad (14)$$

$$\begin{bmatrix} -Q_2 & \bar{A}_{\eta 22} Q_2 - \bar{A}_{\eta 21} X \\ * & -Q_2 \end{bmatrix} < 0 \quad (15)$$

$$\gamma - \frac{1}{\bar{\alpha}^2} < 0 \quad (16)$$

$$\begin{bmatrix} -K_X I & X^T \\ * & -I \end{bmatrix} < 0 \quad (17)$$

$$\begin{bmatrix} Q_2 & I \\ * & K_{Q_2} I \end{bmatrix} > 0 \quad (18)$$

has a solution set (positive scalars $\gamma, \varepsilon_1, \varepsilon_2, K_{Q_2}$, and K_X , positive definite matrices Q_1, Q_2, Q_3 , and Q_4 , and matrix X). Moreover, the gain C of the sliding surface (8) and the upper bound α^* are obtained as follows:

$$C = X Q_2^{-1}, \quad \alpha^* = \sqrt{\frac{1}{\gamma}}, \quad (19)$$

where $\bar{\alpha}$ is a given scalar parameter in (2) and $\Omega_{11} = \bar{A}_{22} Q_2 - \bar{A}_{21} X + Q_2 \bar{A}_{22}^T - X^T \bar{A}_{21}^T + Q_3$, $\Omega_{12} =$

$$\begin{aligned} Q_1 - Q_2 + Q_2 \bar{A}_{22}^T - X^T \bar{A}_{21}^T, \Omega_{13} &= \bar{A}_{\sigma 22} Q_2 - \bar{A}_{\sigma 21} X, \Omega_{14} = \bar{A}_{\eta 22} Q_2 - \bar{A}_{\sigma 21} X, \Omega_{15} = \varepsilon_1 U_2^T E, \Omega_{16} = \\ Q_2 U_2^T D^T - X^T U_1^T D^T, \Omega_{17} &= \varepsilon_2 U_2^T E_\sigma, \Omega_{19} = [-X^T Q_2] \bar{H}^T, \Omega_{22} = -Q_2 - Q_2 + Q_4 + I, \Omega_{23} = \\ \Omega_{13}, \Omega_{24} &= \Omega_{14}, \Omega_{25} = \Omega_{15}, \Omega_{27} = \Omega_{17}, \Omega_{33} = \\ -(1-f)Q_3, \Omega_{38} &= Q_2 U_2^T D_\sigma^T - X^T U_1^T D_\sigma^T, \Omega_{44} = -(1-g)Q_4, \Omega_{55} = -\varepsilon_1 I, \Omega_{66} = -\varepsilon_1 I, \Omega_{77} = -\varepsilon_2 I, \Omega_{88} = -\varepsilon_2 I, \\ \text{and } \Omega_{99} &= -\gamma I. \end{aligned}$$

Proof. First, transform the sliding motion (9) to the equivalent descriptor system form as follows:

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z}_2(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ \xi(t) \end{bmatrix}, \quad (20)$$

where

$$\begin{aligned} \xi(t) &= -y(t) + [\bar{A}_{\eta 22} - \bar{A}_{\eta 21} C] y(t - \eta(t)) \\ &+ [\bar{A}_{22} - \bar{A}_{21} C + \Delta \bar{A}_{22}(t) + \Delta \bar{A}_{21}(t) C] z_2(t) \\ &+ [\bar{A}_{\sigma 22} - \bar{A}_{\sigma 21} C + \Delta \bar{A}_{\sigma 22}(t) + \Delta \bar{A}_{\sigma 21}(t) C] \\ &\times z_2(t - \sigma(t)) + \bar{h}_2(\Gamma^{-1} z). \end{aligned} \quad (21)$$

Choose $P_2 = Q_2^{-1}$, $P_1 = Q_2^{-1} Q_1 Q_2^{-1}$, $P_3 = Q_2^{-1} Q_3 Q_2^{-1}$, and $P_4 = Q_2^{-1} Q_4 Q_2^{-1}$, and construct the following Lyapunov functional

$$V(z_2(t), y(t), t)$$

$$\begin{aligned} &= \begin{bmatrix} z_2^T(t) & y^T(t) \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_2 & P_2 \end{bmatrix} \begin{bmatrix} z_2(t) \\ y(t) \end{bmatrix} \\ &+ \int_{t-\sigma(t)}^t z_2^T(s) P_3 z_2(s) ds + \int_{t-\eta(t)}^t y(s) P_4 y(s) ds. \end{aligned} \quad (22)$$

Obviously, $V(z_2(t), y(t), t) > 0$ for all $[z_2^T(t) \ y^T(t)] \neq 0$. Applying (3), (4), Lemma 1, and the decomposition techniques of matrix to the derivative of $V(z_2(t), y(t), t)$ along the trajectories of the sliding motion (20), yield

$$\dot{V}(z_2(t), y(t), t) \leq \zeta^T Y \zeta, \quad (23)$$

where

$$\begin{aligned} \zeta^T &= [z_2^T(t) \ y^T(t) \ z_2^T(t - \sigma(t)) \ y^T(t - \eta(t))], \\ Y &= \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ * & Y_{22} & Y_{23} & Y_{24} \\ * & * & Y_{33} & Y_{34} \\ * & * & * & Y_{44} \end{bmatrix} + \begin{bmatrix} P_2^T U_2^T E \\ P_2^T U_2^T E_\sigma \\ 0 \\ 0 \end{bmatrix} \\ &\times F(t) [DU_2 - DU_1 C \ 0 \ 0 \ 0] \\ &+ \begin{bmatrix} P_2^T U_2^T E_\sigma \\ P_2^T U_2^T E_\sigma \\ 0 \\ 0 \end{bmatrix} F_\sigma(t) [0 \ 0 \ D_\sigma U_2 - D_\sigma U_1 C \ 0] \\ &+ \begin{bmatrix} U_2^T D^T - C^T U_1^T D^T \\ 0 \\ 0 \\ 0 \end{bmatrix} F^T(t) [E^T U_2 P_2 \ E^T U_2 P_2 \ 0 \ 0] \\ &+ \begin{bmatrix} 0 \\ 0 \\ U_2^T D_\sigma^T - C^T U_1^T D_\sigma^T \\ 0 \end{bmatrix} F_\sigma^T(t) [E_\sigma^T U_2 P_2 \ E_\sigma^T U_2 P_2 \ 0 \ 0], \end{aligned} \quad (24)$$

$$\begin{aligned} Y_{11} &= P_2(\bar{A}_{22} - \bar{A}_{21} C) + (\bar{A}_{22} - \bar{A}_{21} C)^T P_2 + P_3 + (1/\gamma)[-C^T \ I] \bar{H}^T \bar{H} \begin{bmatrix} -C \\ I \end{bmatrix}, Y_{12} = P_1 - P_2 + (\bar{A}_{22} - \bar{A}_{21} C)^T P_2, Y_{13} = P_2(\bar{A}_{\sigma 22} - \bar{A}_{\sigma 21} C), Y_{14} = P_2(\bar{A}_{\eta 22} - \bar{A}_{\eta 21} C), Y_{22} = -P_2 - P_2 + P_4 + P_2 P_2, Y_{23} = Y_{13}, Y_{24} = Y_{14}, Y_{33} = -(1-f)P_3, Y_{44} = -(1-g)P_4, \gamma = 1/\alpha^2. \end{aligned}$$

Pre- and postmultiplying Y in (23) by Y^T and Y , where $Y^T = \text{diag}\{P_2^{-1}, P_2^{-1}, P_2^{-1}, P_2^{-1}\}$, we obtain the following matrix:

$$\begin{aligned} \bar{Y} &= \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \bar{Y}_{13} & \bar{Y}_{14} \\ * & \bar{Y}_{22} & \bar{Y}_{23} & \bar{Y}_{24} \\ * & * & \bar{Y}_{33} & \bar{Y}_{34} \\ * & * & * & \bar{Y}_{44} \end{bmatrix} \\ &+ \begin{bmatrix} U_2^T E \\ U_2^T E_\sigma \\ 0 \\ 0 \end{bmatrix} F(t) [DU_2 Q_2 - DU_1 X \ 0 \ 0 \ 0] \\ &+ \begin{bmatrix} U_2^T E_\sigma \\ U_2^T E_\sigma \\ 0 \\ 0 \end{bmatrix} F_\sigma(t) [0 \ 0 \ D_\sigma U_2 Q_2 - D_\sigma U_1 X \ 0] \end{aligned}$$

$$\begin{aligned}
& + \begin{bmatrix} Q_2 U_2^T D^T - X^T U_1^T D^T \\ 0 \\ 0 \\ 0 \end{bmatrix} F^T(t) \begin{bmatrix} E^T U_2 & E^T U_2 & 0 & 0 \end{bmatrix} \\
& + \begin{bmatrix} 0 \\ 0 \\ Q_2 U_2^T D_\sigma^T - X^T U_1^T D_\sigma^T \\ 0 \end{bmatrix} F_\sigma^T(t) \begin{bmatrix} E_\sigma^T U_2 & E_\sigma^T U_2 & 0 & 0 \end{bmatrix},
\end{aligned} \quad (25)$$

where $\bar{Y}_{11} = \bar{A}_{22}Q_2 - \bar{A}_{21}X + Q_2\bar{A}_{22} - X^T\bar{A}_{21}^T + Q_3 + (1/\gamma)[-X^T Q_2]\bar{H}^T\bar{H}[-X/Q_2]$, $\bar{Y}_{12} = Q_1 - Q_2 + Q_2\bar{A}_{22}^T - X^T\bar{A}_{21}^T$, $\bar{Y}_{13} = \bar{A}_{\sigma 22}Q_2 - \bar{A}_{\sigma 21}X$, $\bar{Y}_{14} = \bar{A}_{\eta 22}Q_2 - \bar{A}_{\eta 21}X$, $\bar{Y}_{22} = -Q_2 - Q_2 + Q_4 + I$, $\bar{Y}_{23} = \bar{Y}_{13}$, $\bar{Y}_{24} = \bar{Y}_{14}$, $\bar{Y}_{33} = -(1 - f)Q_3$, $\bar{Y}_{44} = -(1 - g)Q_4$, and $\gamma = 1/\alpha^2$.

By Schur complement lemma and Lemma 2, the condition $\Omega < 0$ in (14) is equivalent to $\bar{Y} < 0$ in (25). Obviously, $\bar{Y} < 0$ means $Y < 0$. From the condition $Y < 0$ in (23), there exists a constant $\rho > 0$, such that

$$\dot{V}(z_2(t), y(t), t) \leq -\rho \|z_2(t)\|^2. \quad (26)$$

According to Schur complement lemma, LMI (15) is equivalent to the following inequality:

$$-Q_2 + (\bar{A}_{\eta 22}Q_2 - \bar{A}_{\eta 21}X)Q_2^{-1}(\bar{A}_{\eta 22}Q_2 - \bar{A}_{\eta 21}X)^T < 0 \quad (27)$$

that is,

$$-Q_2 + (\bar{A}_{\eta 22} - \bar{A}_{\eta 21}C)Q_2(\bar{A}_{\eta 22}^T - C^T\bar{A}_{\eta 21}^T) < 0. \quad (28)$$

Thus, one can obtain that

$$\rho(\bar{A}_{\eta 22} - \bar{A}_{\eta 21}C) = \|\bar{A}_{\eta 22} - \bar{A}_{\eta 21}C\| < 1. \quad (29)$$

This implies that the sliding motion (9) is Lipschitzian in the term $\bar{A}_{\eta 22} - \bar{A}_{\eta 21}C$ with Lipschitz constant less than 1 [29].

By conditions (22) and (26) and $\|\bar{A}_{\eta 22} - \bar{A}_{\eta 21}C\| < 1$, one can conclude that the sliding motion (9) with (3) and (4) is quadratically stable.

Furthermore, for some desired value $\bar{\alpha}$ satisfying (2), the design of gain C can be formulated as an LMI problem in $\gamma, \varepsilon_1, \varepsilon_2, K_{Q_2}, K_X, Q_1, Q_2, Q_3, Q_4$, and X in (13). Also, one can obtain the upper bound α^* which guarantees the quadratic stability for the sliding motion (9) with all α satisfying $\bar{\alpha} < \alpha \leq \alpha^*$ in (2) if the LMI optimization is feasible. This completes the proof. \square

Remark 4. It should be pointed out that the norm of the gain matrix is implicitly bounded by (17) and (18), which imply that $\|C\| \leq \sqrt{K_X K_{Q_2}}$. This is necessary in order to prevent the unacceptably high gains.

Remark 5. It is obvious that (14), (15), (16), (17), and (18) are a group of LMIs with respect to solution variables; various efficient convex algorithms can be used to ascertain the LMI

solutions. In this paper, we utilize Matlab's LMI Toolbox [30] to solve the convex optimization problem to obtain directly the gain C as $C = XQ_2^{-1}$.

Remark 6. If the convex optimization problem is solvable, the bound α^* guarantees the quadratic stability for the sliding motion (9) with all α satisfying $\bar{\alpha} < \alpha < \alpha^*$ in (2).

3.2. Reaching Motion Control Design for Nonlinear Neutral Systems

Theorem 7. Suppose that the optimization problem (13) has solutions: positive scalars $\gamma, \varepsilon_1, \varepsilon_2, K_{Q_2}$, and K_X , positive definite matrices Q_1, Q_2, Q_3 , and Q_4 , and matrix X , and the sliding surface is given by (8). Then, the trajectory of the closed-loop system (7) can be driven onto the sliding surface in finite time with the control

$$\begin{aligned}
u(t) = & -B_1^{-1} \left[KS + \theta \operatorname{sign}(S) + \bar{C}\bar{A}z(t) + \bar{C}\bar{A}_\sigma z(t - \sigma(t)) \right. \\
& \left. + \bar{C}\bar{A}_\eta \dot{z}(t - \eta(t)) + \operatorname{diag}(\operatorname{sign}(S)) \right. \\
& \left. \times (N_1 + N_2 + N_3) \right],
\end{aligned} \quad (30)$$

where

$$\operatorname{diag}(\operatorname{sign}(S)) = \operatorname{diag}\{\operatorname{sign}(s_1), \operatorname{sign}(s_2), \dots, \operatorname{sign}(s_m)\},$$

$$\bar{C} = [I \ C] = \begin{bmatrix} \bar{c}_1 \\ \bar{c}_2 \\ \vdots \\ \bar{c}_m \end{bmatrix},$$

$$N_1 = |\bar{C}\Gamma E D \Gamma^{-1} z(t)| = \begin{bmatrix} N_{11} \\ N_{21} \\ \vdots \\ N_{m1} \end{bmatrix} = \begin{bmatrix} |\bar{c}_1 \Gamma E D \Gamma^{-1} z(t)| \\ |\bar{c}_2 \Gamma E D \Gamma^{-1} z(t)| \\ \vdots \\ |\bar{c}_m \Gamma E D \Gamma^{-1} z(t)| \end{bmatrix},$$

$$N_2 = |\bar{C}\Gamma E_\sigma D_\sigma \Gamma^{-1} z(t - \sigma(t))| = \begin{bmatrix} N_{12} \\ N_{22} \\ \vdots \\ N_{m2} \end{bmatrix}$$

$$= \begin{bmatrix} |\bar{c}_1 \Gamma E_\sigma D_\sigma \Gamma^{-1} z(t - \sigma(t))| \\ |\bar{c}_2 \Gamma E_\sigma D_\sigma \Gamma^{-1} z(t - \sigma(t))| \\ \vdots \\ |\bar{c}_m \Gamma E_\sigma D_\sigma \Gamma^{-1} z(t - \sigma(t))| \end{bmatrix},$$

$$\begin{aligned}
N_3 &= \sqrt{\frac{1}{\gamma}} |\bar{C}\Gamma H\Gamma^{-1}z(t)| = \begin{bmatrix} N_{13} \\ N_{23} \\ \vdots \\ N_{m3} \end{bmatrix} \\
&= \begin{bmatrix} \sqrt{\frac{1}{\gamma}} |\bar{c}_1\Gamma H\Gamma^{-1}z(t)| \\ \sqrt{\frac{1}{\gamma}} |\bar{c}_2\Gamma H\Gamma^{-1}z(t)| \\ \vdots \\ \sqrt{\frac{1}{\gamma}} |\bar{c}_m\Gamma H\Gamma^{-1}z(t)| \end{bmatrix}, \\
K &= \text{diag}\{k_1, k_2, \dots, k_m\},
\end{aligned} \tag{31}$$

$\theta = \text{diag}\{\theta_1, \theta_2, \dots, \theta_m\}$, K_i , and θ_i are positive constants.

Proof. From the sliding surface (8), one can compute the derivative of S as follows:

$$\begin{aligned}
\dot{S} &= [I \ C] \dot{z}(t) = \bar{C}\bar{A}_\eta \dot{z}(t - \eta(t)) + \bar{C}[\bar{A} + \Delta\bar{A}(t)]z(t) \\
&\quad + \bar{C}[\bar{A}_\sigma + \Delta\bar{A}_\sigma(t)]z(t - \sigma(t)) + B_1u(t) + \bar{C}h(\Gamma^{-1}z).
\end{aligned} \tag{32}$$

Replacing $u(t)$ in (32) with $u(t)$ in (30) yields that

$$\begin{aligned}
\dot{S} &= -KS - \theta \text{sign}(S) \\
&\quad - [\text{diag}(\text{sign}(S)) N_1 - \bar{C}\Gamma EF(t) D\Gamma^{-1}z(t)] \\
&\quad - [\text{diag}(\text{sign}(S)) N_2 - \bar{C}\Gamma E_\sigma F_\sigma(t) D_\sigma \Gamma^{-1}z(t - \sigma(t))] \\
&\quad - [\text{diag}(\text{sign}(S)) N_3 - \bar{C}h(\Gamma^{-1}z)].
\end{aligned} \tag{33}$$

That is, for each one s_i in S , we have

$$\begin{aligned}
\dot{s}_i &= -k_i s_i - \theta_i \text{sign}(s_i) \\
&\quad - [\text{sign}(s_i) N_{i1} - \bar{c}_i \Gamma EF(t) D\Gamma^{-1}z(t)] \\
&\quad - [\text{sign}(s_i) N_{i2} - \bar{c}_i \Gamma E_\sigma F_\sigma(t) D_\sigma \Gamma^{-1}z(t - \sigma(t))] \\
&\quad - [\text{sign}(s_i) N_{i3} - \bar{c}_i h(\Gamma^{-1}z)].
\end{aligned} \tag{34}$$

In view of

$$\begin{aligned}
\bar{c}_i \Gamma EF(t) D\Gamma^{-1}z(t) &\leq |\bar{c}_i \Gamma ED\Gamma^{-1}z(t)| = N_{i1}, \\
\bar{c}_i \Gamma E_\sigma F_\sigma(t) D_\sigma \Gamma^{-1}z(t) &\leq |\bar{c}_i \Gamma E_\sigma D_\sigma \Gamma^{-1}z(t)| = N_{i2}, \\
\bar{c}_i h(\Gamma^{-1}z) &\leq |\bar{c}_i \Gamma H\Gamma^{-1}z(t)| = N_{i3},
\end{aligned} \tag{35}$$

thus one can conclude that

$$\begin{aligned}
\dot{s}_i &< 0, \quad s_i > 0, \\
\dot{s}_i &> 0, \quad s_i < 0
\end{aligned} \tag{36}$$

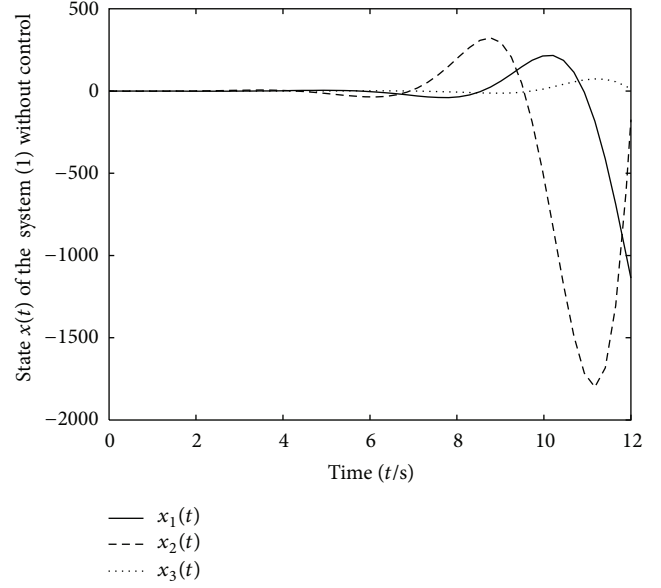


FIGURE 1: State response of the open-loop system (1) in Case 1.

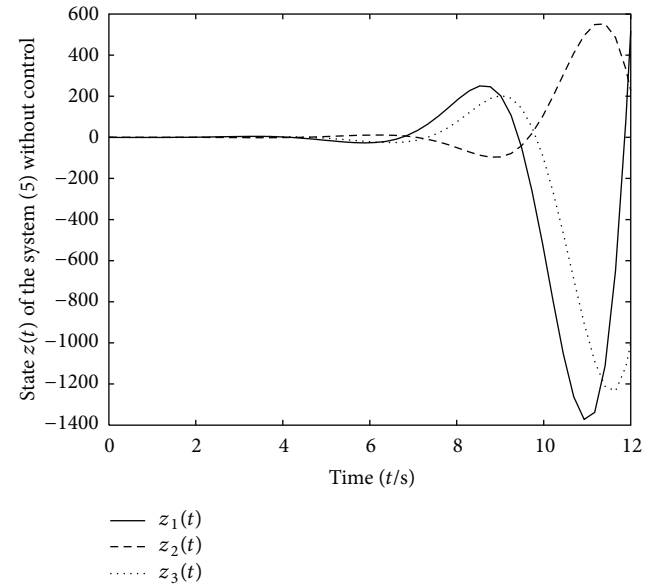


FIGURE 2: State response of the open-loop system (6) in Case 1.

which means that the trajectory of the system (7) with the control law (30) can be driven onto the sliding surface in finite time and remain there in the subsequent time. This completes the proof. \square

4. Numerical Example

In this section, three examples are presented to illustrate the design approach of sliding surface and reaching motion control and show their advantages.

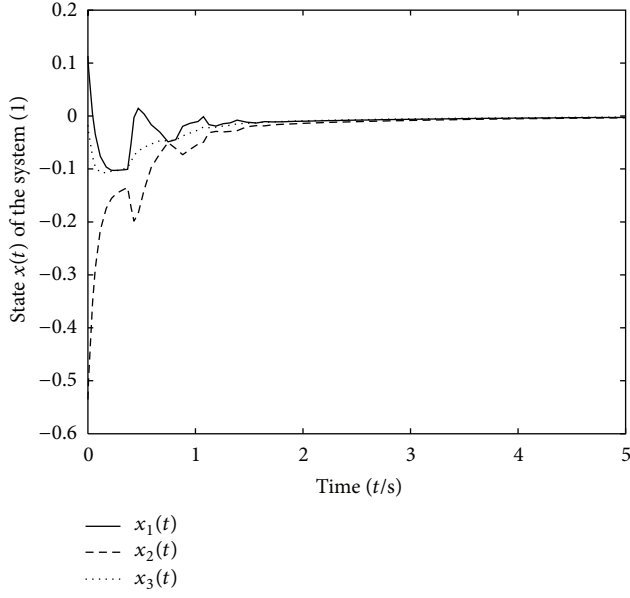


FIGURE 3: State response of the closed-loop system (1) in Case 1.

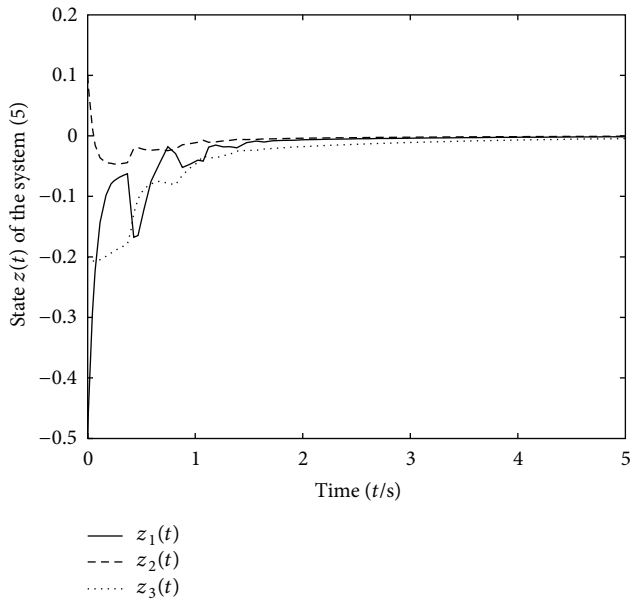


FIGURE 4: State response of the closed-loop system (6) in Case 1.

Consider parts of parameters in [27]

$$A = \begin{bmatrix} 1 & 0.3 & 0 \\ -3 & 0.1 & 0 \\ 0.1 & 0 & -2 \end{bmatrix}, \quad A_\sigma = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0 & 0 \\ 0.1 & -0.1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 5 & -7 \\ -9 & 8 \\ 3 & 5 \end{bmatrix}, \quad A_\eta = \begin{bmatrix} -0.1 & 0 & 0.1 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.02 \end{bmatrix},$$

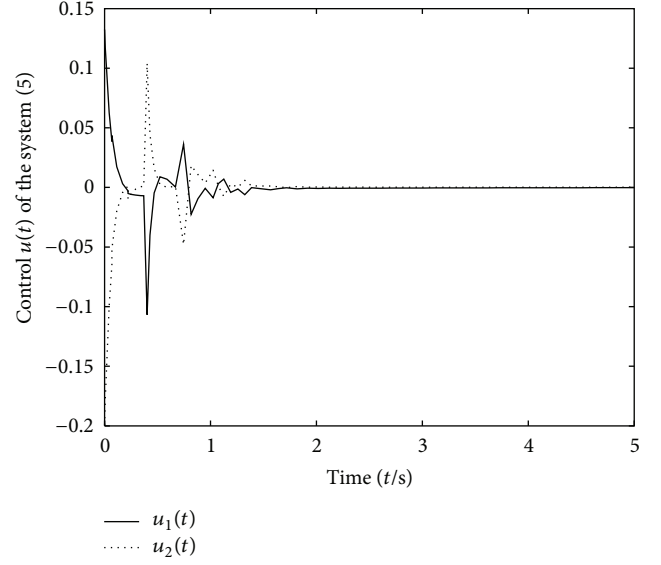


FIGURE 5: Control signal in (30) in Case 1.

$$E = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}, \quad E_\sigma = \begin{bmatrix} -0.2 \\ 0 \\ 0.1 \end{bmatrix},$$

$$D = [-0.02 \ 0.02 \ 0.10], \quad D_\sigma = [0.1 \ 0.01 \ 0.02]. \quad (37)$$

4.1. Case 1. For the system (1) in our paper, when

$$H = \begin{bmatrix} 0.3 & -0.3 & -0.1 \\ 0.1 & 0.5 & -0.1 \\ 0.2 & 0.1 & -0.5 \end{bmatrix}, \quad \sigma(t) = 0.1(2 + \sin(t)),$$

$$\eta(t) = 0.2(1 + \cos(t)) \quad (38)$$

and the desired parameter $\bar{\alpha} = 0.1$, by solving the optimization problem (13) subject to (14), (15), (16), (17), and (18), we can get following results:

$$\gamma = 0.2779, \quad \varepsilon_1 = 9.9589, \quad \varepsilon_2 = 1.8862,$$

$$K_{Q_2} = 0.3980, \quad K_X = 0.0574, \quad Q_1 = 3.5006,$$

$$Q_2 = 2.5130, \quad Q_3 = 0.6958, \quad Q_4 = 3.8877, \quad (39)$$

$$X = \begin{bmatrix} 0.1515 \\ -0.1847 \end{bmatrix}.$$

Thus, the gain C and the upper bound α^* are computed as

$$C = XQ_2^{-1} = \begin{bmatrix} 0.0603 \\ -0.0735 \end{bmatrix}, \quad \alpha^* = \sqrt{\frac{1}{\gamma}} = 1.8969. \quad (40)$$

Under the following initial condition

$$\phi(0) = [-0.5 \ 0.1 \ -0.2], \quad (41)$$

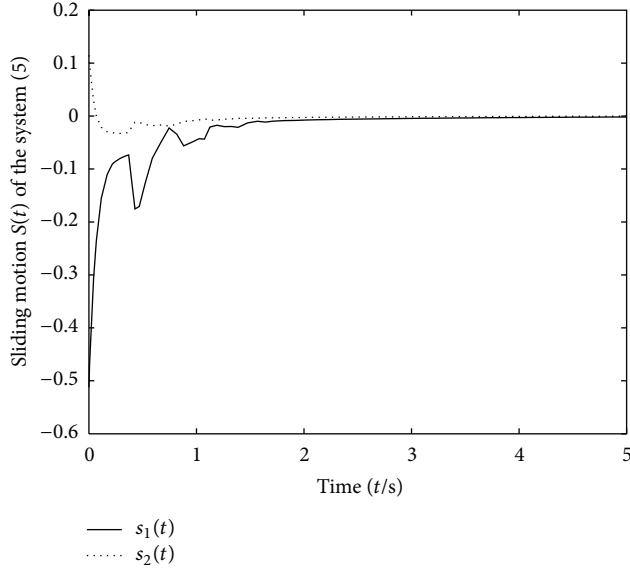


FIGURE 6: Trajectories of sliding variable in sliding surface (8) in Case 1.

when the tuned parameters K and θ in (30) are chosen as

$$K = \begin{bmatrix} 3.8974 & 0 \\ 0 & 5.7103 \end{bmatrix}, \quad \theta = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.002 \end{bmatrix}, \quad (42)$$

the simulation results are shown in Figures 1–6 based on the above parameters. From Figures 1 and 2, one can see that the nonlinear neutral systems (1) and (6) without control are divergent. From Figures 3 and 4, one can see that the nonlinear neutral system (6) with control (30) is indeed well stabilized. The control signal $u(t)$ of the system (6) and the sliding surface (8) are rather smooth in Figures 5 and 6.

4.2. Case 2. In order to compare with [27], we need guarantee no difference for the considered system firstly. Because in [27] the system (1) contains the term $B + \Delta B(t)$, where $\Delta B(t) = B\delta(t)$, and does not have the term $h(t)$, and also the delays are constant, for the system (1) in our paper, matrix L is needed between B and $u(t)$, where $L = I + \delta(t)$. When L is contained in our system (1), the theoretical results should make some modification; that is, the reaching motion control $u(t)$ should have the following form:

$$\begin{aligned} u(t) = & -(B_1 L)^* \left[KS + \theta \operatorname{sign}(S) + \bar{C} \bar{A} z(t) + \bar{C} \bar{A}_\sigma z \right. \\ & \times (t - \sigma(t)) + \bar{C} \bar{A}_\eta \dot{z}(t - \eta(t)) \\ & \left. + \operatorname{diag}(\operatorname{sign}(S)) (N_1 + N_2 + N_3) \right], \end{aligned} \quad (43)$$

where $(\bullet)^*$ denotes the pseudoinverse of the argument \bullet .

In addition, to remove the nonlinear term $h(t)$, we chose $h(t) = [0 \ 0 \ 0]^T$, which means that the parameter α in (2) does not need to be optimized.

As for the constant delays, their derivatives, f and g , are equal to zero.

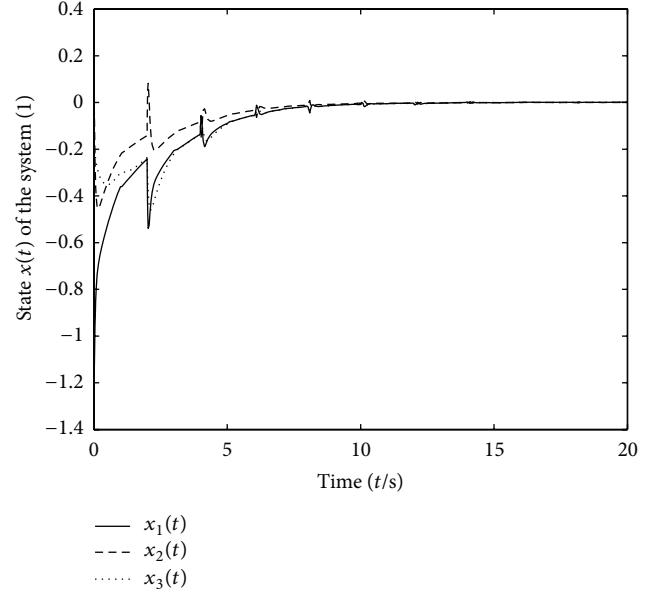


FIGURE 7: State response of the closed-loop system (1) in Case 2.

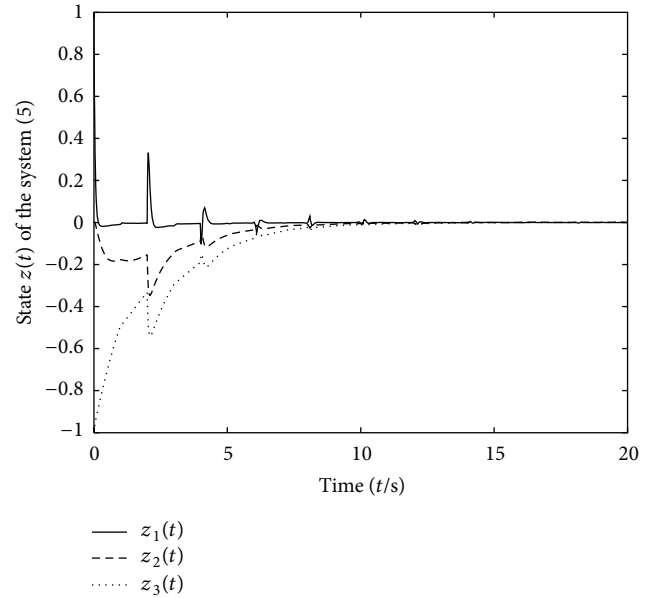


FIGURE 8: State response of the closed-loop system (6) in Case 2.

Utilizing the parameters in [27] to solve LMIs (14) and (15), one can get following results:

$$\begin{aligned} \varepsilon_1 &= 53.0488, & \varepsilon_2 &= 51.8644, & Q_1 &= 77.2200, \\ Q_2 &= 46.7611, & Q_3 &= 46.0459, & Q_4 &= 48.8738, \\ X &= \begin{bmatrix} 13.6121 \\ -12.1425 \end{bmatrix}. \end{aligned} \quad (44)$$

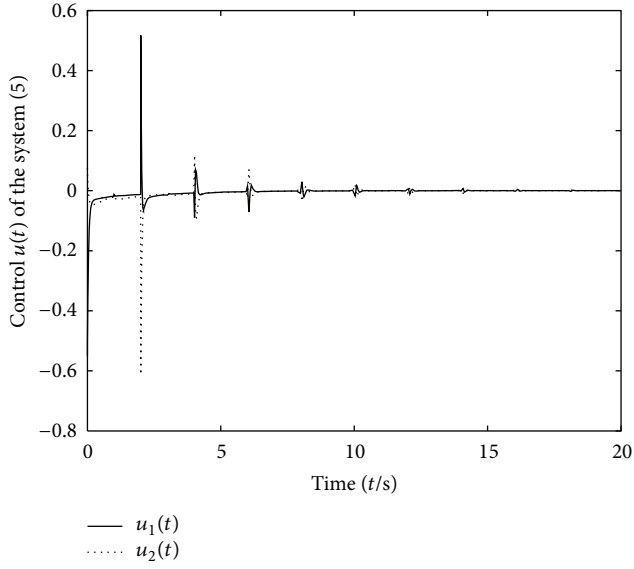


FIGURE 9: Control signal in (30) in Case 2.

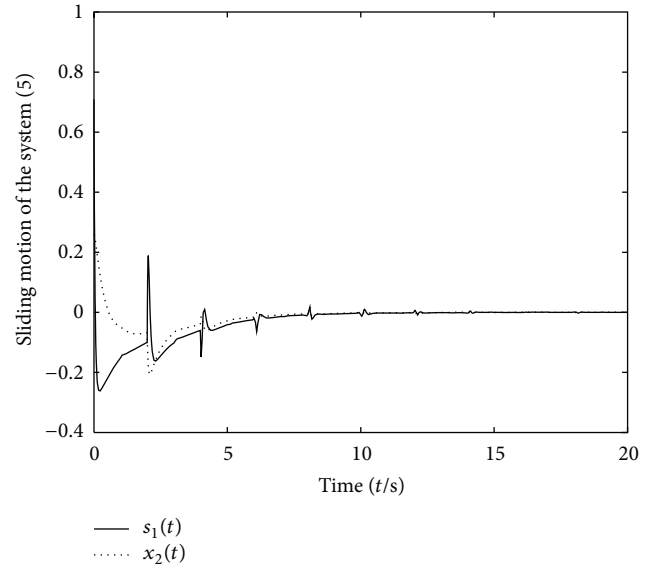


FIGURE 10: Trajectories of sliding variable in sliding surface (8) in Case 2.

Thus, the gain C is computed as

$$C = XQ_2^{-1} = \begin{bmatrix} 0.2911 \\ -0.2597 \end{bmatrix}. \quad (45)$$

Under the initial $\phi(0) = [1 \ 0 \ -1]$ in [27], when the tuned parameters K and θ in (30) are chosen as

$$K = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}, \quad \theta = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}, \quad (46)$$

the simulation results are presented in Figures 7, 8, 9, and 10. Seen from Figures 7–10 in our paper, the convergent time is about 10 seconds. However, the convergent time is about 16 seconds from Figures 2–4 in [27]. Compared with Figures 2–4 in [27], one can conclude that the convergent rate of our results is faster.

4.3. *Case 3.* Under the initial $\phi(0) = [0 \ -1 \ 1]$ in [27], when the tuned parameters K and θ in (30) are chosen as

$$K = \begin{bmatrix} 1.5 & 0 \\ 0 & 5 \end{bmatrix}, \quad \theta = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix} \quad (47)$$

and other parameters are similar to [27], the simulation results are presented in Figures 11, 12, 13, and 14.

Seen from Figures 11–14 in our paper, the convergent time is about 20 seconds. However, the convergent time is about 30 seconds from Figures 5–7 in [27]. Also, the amplitudes of the corresponding control signal $u(t)$ are 0.45 and 3.5, respectively. Therefore, one can conclude that the convergent rate of our results is faster and the amplitude of control signal $u(t)$ of our results is smaller.

5. Conclusion

The sliding mode control and optimization problem of nonlinear neutral systems with time-varying delays are complex

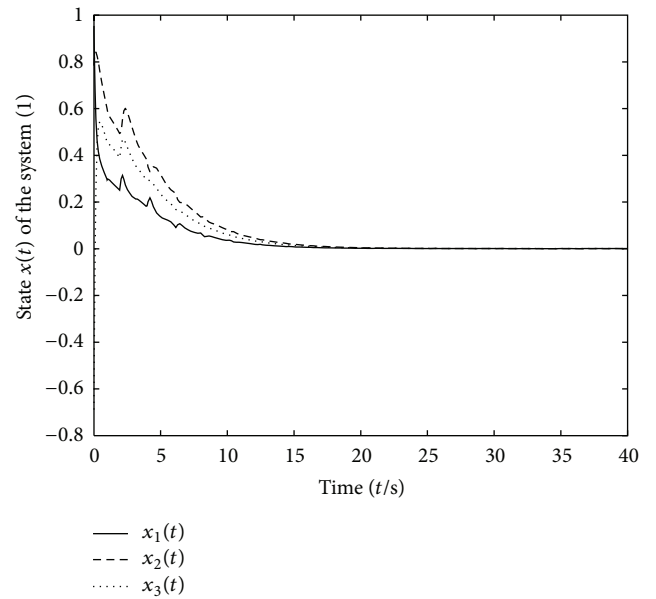


FIGURE 11: State response of the closed-loop system (1) in Case 3.

and challenging. In the framework of the Lyapunov stability theory, based on the methods of singular value decomposition and descriptor system model transformation, the sliding surface and the reaching control law are designed, which can be obtained by solving the optimization problem. Also, the upper bound α^* that guarantees the quadratic stability of sliding motion (9) for all $\bar{\alpha} < \alpha < \alpha^*$ is derived. The numerical example has shown the validity of the present design and the advantages of the schemes over the existing results in the literature.

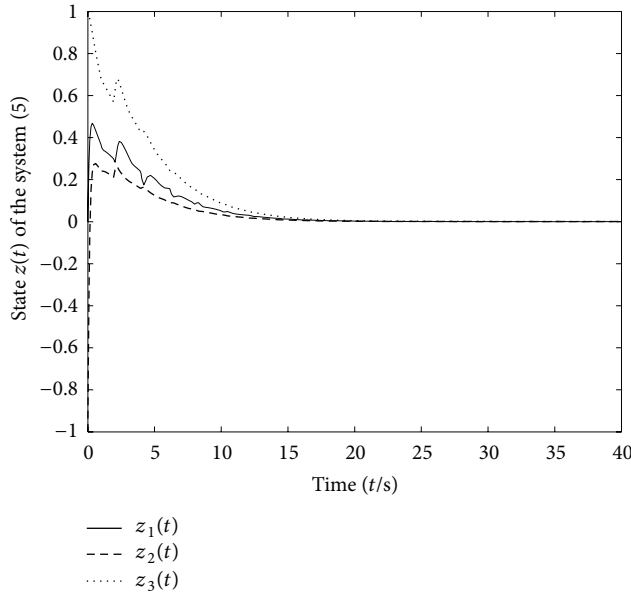


FIGURE 12: State response of the closed-loop system (6) in Case 3.

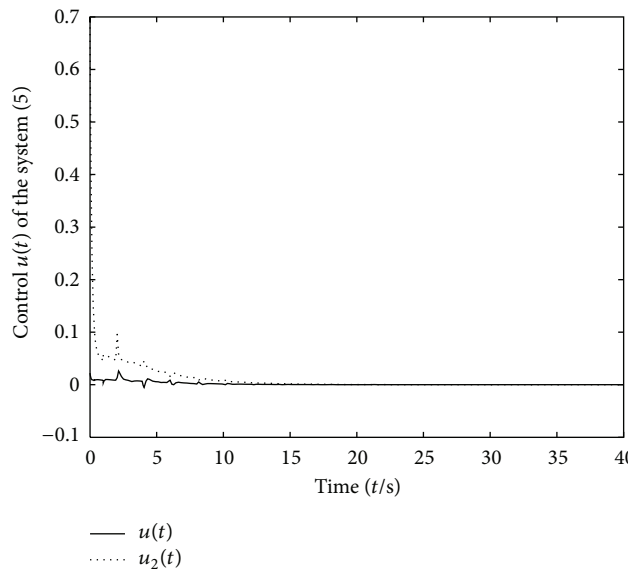


FIGURE 13: Control signal in (30) in Case 3.

In future research, it is expected to investigate the optimization of reaching motion control for a class of nonlinear neutral systems with time-varying delays.

Acknowledgments

This research is supported by Natural Science Foundation of China (no. 61104106), Science Foundation of Department of Education of Liaoning Province (no. L2012422), and Start-up Fund for Doctors of Shenyang University (no. 20212340).

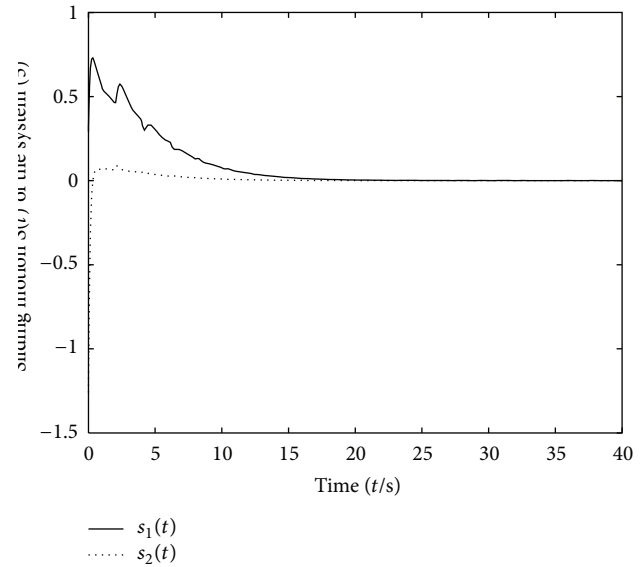


FIGURE 14: Trajectories of sliding variable in sliding surface (8) in Case 3.

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