The Annals of Statistics
2015, Vol. 43, No. 5, 2326–2329
DOI: 10.1214/15-AOS1326

CORRECTION

INVERSE REGRESSION FOR LONGITUDINAL DATA

Ann. Statist. **42** (2014) 563–591

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It has been pointed out to us by L. Forzani (Universidad Nacional del Litoral) that a gap exists in Lemma 2.4 in Jiang, Yu and Wang (2014). In this note we replace it with Lemma 2.4 and Theorem 2.1, based on an additional condition on the decay rate of ξ_i and an additional assumption on the coefficients $a_{k,i} = \langle \phi_i, \eta_k \rangle$. Following are the specific changes.

In the abstract, the sentence "We develop asymptotic theory... rate of convergence." is replaced by "We develop asymptotic theory for the new procedure."

Page 565: "Asymptotic results for the new procedure are presented in Section 2.3, where... densely sampled longitudinal data (or functional data)." is replaced by "Asymptotic results for the new procedure are presented in Section 2.3."

Page 579: "In particular, we achieve the optimal rate of convergence for e.d.r. directions" is removed.

The text from Lemma 2.4 (page 573) to the end of page 574 is replaced by the following:

The following condition about ξ_i is similar to what has been assumed in Hall and Horowitz (2007).

CONDITION 1. The covariance operator satisfies $\Gamma(s,t) = \sum_{i=1}^{\infty} \xi_i \phi_i(s)\phi_i(t)$, where $\xi_i - \xi_{i+1} > C_0 i^{-\alpha_1 - 1}$ for some constant $C_0 > 0$, $i \geq 1$ and $\alpha_1 > 1$; hence $\xi_i \geq C_1 i^{-\alpha_1}$ for some constant $C_1 > 0$.

ASSUMPTION A.9. $|a_{k,j}| < C_2 j^{-\alpha_2}$ for some constant $C_2 > 0$.

Because $\eta_k = \sum_{i=1}^{\infty} a_{k,i}\phi_i(t)$ and $\eta_k \in R_{\Gamma^{1/2}}$, $\sum_{i=1}^{\infty} a_{k,i}^2 \xi_i^{-1} < \infty$. Assumption A.9 and $\eta_k \in R_{\Gamma^{1/2}}$ imply that $2\alpha_2 - \alpha_1 > 1$, which means that the space spanned by $\{\eta_k\}_{k=1}^{K}$ is smoother than Γ. The following two results in Hall and Horowitz (2007) will be used in the proof.

Received February 2015.
[1]Supported in part by NSC Grant NSC 101-2118-M-001-013-MY2, Taiwan.
[2]Supported in part by NSF Grants DMS-04-04630, DMS-09-06813 and DMS-12-28369.

Let $\Sigma_1 = \sum_i \tau_{1,i}\psi_{1,i}(s)\psi_{1,i}(t)$ and $\Sigma_2 = \sum_i \tau_{2,i}\psi_{2,i}(s)\psi_{2,i}(t)$ be two general covariance operators in L_2, then

$$\sup_{j\geq 1}|\tau_{1,j} - \tau_{2,j}| \leq \left(\iint (\Sigma_1 - \Sigma_2)^2\right)^{1/2}; \tag{1}$$

$$\sup_{j\geq 1}\delta_j \|\psi_{1,j} - \psi_{2,j}\| \leq 8^{1/2}\left(\iint (\Sigma_1 - \Sigma_2)^2\right)^{1/2}, \tag{2}$$

where $\delta_j = \min_{1\leq k\leq j}(\tau_{1,k} - \tau_{1,k+1})$ and $\int \psi_{1,j}(t)\psi_{2,j}(t) \geq 0$.

Let $\Gamma_L^{-1/2} = \sum_{i=1}^{L} \xi_i^{-1/2}\phi_i(s)\phi_i(t)$ be the truncated version of $\Gamma^{-1/2}$. Denote $r_{n1} = (nh^2EN)^{-1} + h^4$ and $r_{n2} = (nh_\mu^2 EN)^{-1} + (nh_\phi^2 EN)^{-1} + (h_\mu + h_\phi)^4$. In order to show the convergence rate of $\|\hat{\beta}_k - \beta_k\|^2$, we need the following lemma.

LEMMA 2.4. *Under Condition* 1 *and Assumptions* A.1–A.9, *we have*

$$\begin{aligned}
&\|\hat{\Gamma}_L^{-1}\hat{\Gamma}_e\hat{\Gamma}_L^{-1/2} - \Gamma^{-1}\Gamma_e\Gamma^{-1/2}\|^2 \\
&\quad = O_p\big(L^{(-2\alpha_2+\alpha_1+1)} + L^{(3\alpha_1+2)}r_{n1} + L^{(3\alpha_1-2\alpha_2+4)}r_{n2}\big).
\end{aligned}$$

THEOREM 2.1. *Under Condition* 1 *and Assumptions* A.1–A.9, *we have*

$$\|\hat{\beta}_k - \beta_k\|^2 = O_p\big(L^{(-2\alpha_2+\alpha_1+1)} + L^{(3\alpha_1+2)}r_{n1} + L^{(3\alpha_1-2\alpha_2+4)}r_{n2}\big).$$

Theorem 2.1 indicates that when $\alpha_2 \gg \alpha_1 > 1$ and all the bandwidths are of the same order, the best convergence rates of $\hat{\beta}_k$ are close to $(1/\sqrt{nh} + h^2)$ and $(1/\sqrt{nh^2} + h^2)$ for functional data (i.e., $0 < ENh < \infty$) and for longitudinal data (i.e., $EN < \infty$), respectively.

Further, the proof for Theorem 2.1 in the Appendix from page 586 to page 590 is replaced by the following two proofs.

PROOF OF LEMMA 2.4. Observe that

$$\|\hat{\Gamma}_L^{-1}\hat{\Gamma}_e\hat{\Gamma}_L^{-1/2} - \Gamma^{-1}\Gamma_e\Gamma^{-1/2}\|^2 \leq 3(T_1 + T_2 + T_3),$$

where $T_1 = \|\hat{\Gamma}_L^{-1}\hat{\Gamma}_e\hat{\Gamma}_L^{-1/2} - \hat{\Gamma}_L^{-1}\Gamma_e\hat{\Gamma}_L^{-1/2}\|^2$, $T_2 = \|\hat{\Gamma}_L^{-1}\Gamma_e\hat{\Gamma}_L^{-1/2} - \Gamma_L^{-1}\Gamma_e\Gamma_L^{-1/2}\|^2$ and $T_3 = \|\Gamma_L^{-1}\Gamma_e\Gamma_L^{-1/2} - \Gamma^{-1}\Gamma_e\Gamma^{-1/2}\|^2$. To complete the proof, we simply need the convergence rates of T_1, T_2 and T_3. First,

$$\begin{aligned}
T_1 &\leq \|\hat{\Gamma}_L^{-1}\|^2\|\hat{\Gamma}_e - \Gamma_e\|^2\|\hat{\Gamma}_L^{-1/2}\|^2 \\
&\leq O_p\left(\left(\sum_{j=1}^{L} j^{2\alpha_1}\right) \times r_{n1} \times \left(\sum_{j=1}^{L} j^{\alpha_1}\right)\right) \leq O_p\big(L^{(3\alpha_1+2)}r_{n1}\big).
\end{aligned}$$

For the convergence rate of T_2, it suffices to show the order of $\|(\hat{\Gamma}_L^{-1} - \Gamma_L^{-1})\Gamma_e\Gamma_L^{-1/2}\|^2$, as the remainder terms are of smaller orders. Observe that

$$\begin{aligned}
&\|(\hat{\Gamma}_L^{-1} - \Gamma_L^{-1})\Gamma_e\Gamma_L^{-1/2}\|^2 \\
&\quad \le C\left(\left\|\sum_{i=1}^{L}\left(\frac{1}{\hat{\xi}_i} - \frac{1}{\xi_i}\right)\hat{\phi}_i(s)\hat{\phi}_i(t)\right\|^2 + \left\|\sum_{i=1}^{L}\frac{1}{\xi_i}(\hat{\phi}_i(s)\hat{\phi}_i(t) - \phi_i(s)\phi_i(t))\right\|^2\right) \\
&\qquad \times \|\Gamma_e\Gamma_L^{-1/2}\|^2,
\end{aligned}$$

for some positive constant C. Direct calculations lead to

$$\left\|\sum_{i=1}^{L}\left(\frac{1}{\hat{\xi}_i} - \frac{1}{\xi_i}\right)\hat{\phi}_i(s)\hat{\phi}_i(t)\right\|^2 \le \sum_{i=1}^{L}\left|\left(\frac{\xi_i - \hat{\xi}_i}{\hat{\xi}_i\xi_i}\right)\right|^2 \le O_p(L^{(4\alpha_1+1)}r_{n2}), \tag{3}$$

$$\begin{aligned}
\left\|\sum_{i=1}^{L}\frac{\hat{\phi}_i(s)\hat{\phi}_i(t) - \phi_i(s)\phi_i(t)}{\xi_i}\right\|^2 &\le O_p\left(\sum_{i=1}^{L}\frac{\|\hat{\phi}_i - \phi_i\|^2}{\xi_i^2}\right) \\
&\le O_p(L^{(4\alpha_1+3)}r_{n2})
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\|\Gamma_e\Gamma_L^{-1/2}\|^2 = \|\Gamma_L^{1/2}\Gamma_L^{-1/2}\Gamma_e\Gamma_L^{-1/2}\|^2 &\le O_p\left(\sum_{k=1}^{K}\sum_{i=1}^{L}a_{k,i}^2\xi_i\right) \\
&= O_p(L^{(-2\alpha_2-\alpha_1+1)}).
\end{aligned} \tag{5}$$

Combining (3), (4) and (5), we have

$$\|(\hat{\Gamma}_L^{-1} - \Gamma_L)\Gamma_e\Gamma_L^{-1/2}\|^2 = O_p(L^{(3\alpha_1-2\alpha_2+4)}r_{n2}).$$

Also, $T_3 \le O_p(\sum_{k=1}^{K}\sum_{i>L}a_{k,i}^2\xi_i^{-1}) = O_p(L^{(-2\alpha_2+\alpha_1+1)})$. The proof is thus complete. $\square$

PROOF OF THEOREM 2.1. Observe that

$$\begin{aligned}
\|\hat{\beta}_k - \beta_k\| &= \|\hat{\lambda}_k^{-1}\hat{\Gamma}_L^{-1}\hat{\Gamma}_e\hat{\Gamma}_L^{-1/2}\hat{\eta}_k - \lambda_k^{-1}\Gamma^{-1}\Gamma_e\Gamma^{-1/2}\eta_k\| \\
&\le \hat{\lambda}_k^{-1}\|\hat{\Gamma}_L^{-1}\hat{\Gamma}_e\hat{\Gamma}_L^{-1/2}\hat{\eta}_k - \Gamma^{-1}\Gamma_e\Gamma^{-1/2}\hat{\eta}_k\| \\
&\quad + \|\hat{\lambda}_k^{-1}\Gamma^{-1}\Gamma_e\Gamma^{-1/2}\hat{\eta}_k - \lambda_k^{-1}\Gamma^{-1}\Gamma_e\Gamma^{-1/2}\eta_k\|.
\end{aligned}$$

The convergence rate of the first term can be obtained by employing the result of Lemma 2.4. The second term satisfies

$$\begin{aligned}
&\|\hat{\lambda}_k^{-1}\Gamma^{-1}\Gamma_e\Gamma^{-1/2}\hat{\eta}_k - \lambda_k^{-1}\Gamma^{-1}\Gamma_e\Gamma^{-1/2}\eta_k\| \\
&\quad \le \|\hat{\lambda}_k^{-1}\Gamma^{-1}\Gamma_e\Gamma^{-1/2}(\hat{\eta}_k - \eta_k)\| + \|(\hat{\lambda}_k^{-1} - \lambda_k^{-1})\Gamma^{-1}\Gamma_e\Gamma^{-1/2}\eta_k\|,
\end{aligned}$$

where the first component satisfies

$$
\begin{aligned}
&\|\hat{\lambda}_k^{-1}\Gamma^{-1}\Gamma_e\Gamma^{-1/2}(\hat{\eta}_k-\eta_k)\|^2 \\
&\quad\le \hat{\lambda}_k^{-2}\|\Gamma^{-1}\Gamma_e\Gamma^{-1/2}\|^2\|(\hat{\eta}_k-\eta_k)\|^2 \\
&\quad\le O_p\Biggl(\Biggl(\sum_{k=1}^{K}\sum_{j=1}^{\infty} a_{k,j}^2\xi_j^{-1}\Biggr)\|(\hat{\eta}_k-\eta_k)\|^2\Biggr) \\
&\quad\le O_p\bigl(\|(\hat{\eta}_k-\eta_k)\|^2\bigr) \\
&\quad\le O_p\bigl(L^{(-2\alpha_2+\alpha_1+1)}+L^{(3\alpha_1+2)}r_{n1}+L^{(3\alpha_1-2\alpha_2+4)}r_{n2}\bigr).
\end{aligned}
$$

The third inequality above results from the fact that $\sum_{k=1}^{K}\sum_{j=1}^{\infty} a_{k,j}^2\xi_j^{-1}<\infty$, and the last inequality results from (2). The rest of the proof follows from the fact that

$$
\begin{aligned}
&\|(\hat{\lambda}_k^{-1}-\lambda_k^{-1})\Gamma^{-1}\Gamma_e\Gamma^{-1/2}\eta_k\|^2 \\
&\quad\le \left(\frac{\hat{\lambda}_k-\lambda_k}{\hat{\lambda}_k\lambda_k}\right)^2\|\lambda_k\beta_k\|^2 \\
&\quad\le O_p\bigl((\hat{\lambda}_k-\lambda_k)^2\bigr) \\
&\quad\le O_p\bigl(L^{(-2\alpha_2+\alpha_1+1)}+L^{(3\alpha_1+2)}r_{n1}+L^{(3\alpha_1-2\alpha_2+4)}r_{n2}\bigr). \qquad \square
\end{aligned}
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