

## NONSTATIONARY ETAS MODELS FOR NONSTANDARD EARTHQUAKES

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The conditional intensity function of a point process is a useful tool for generating probability forecasts of earthquakes. The epidemic-type aftershock sequence (ETAS) model is defined by a conditional intensity function, and the corresponding point process is equivalent to a branching process, assuming that an earthquake generates a cluster of offspring earthquakes (triggered earthquakes or so-called aftershocks). Further, the size of the first-generation cluster depends on the magnitude of the triggering (parent) earthquake. The ETAS model provides a good fit to standard earthquake occurrences. However, there are nonstandard earthquake series that appear under transient stress changes caused by aseismic forces such as volcanic magma or fluid intrusions. These events trigger transient nonstandard earthquake swarms, and they are poorly fitted by the stationary ETAS model. In this study, we examine nonstationary extensions of the ETAS model that cover nonstandard cases. These models allow the parameters to be time-dependent and can be estimated by the empirical Bayes method. The best model is selected among the competing models to provide the inversion solutions of nonstationary changes. To address issues of the uniqueness and robustness of the inversion procedure, this method is demonstrated on an inland swarm activity induced by the 2011 Tohoku-Oki, Japan earthquake of magnitude 9.0.

**1. Introduction.** The epidemic-type aftershock sequence (ETAS) model [Ogata (1985, 1986, 1988, 1989)] is one of the earliest point-process models created for clustered events. It is defined in terms of a conditional intensity [Hawkes (1971), Hawkes and Adamopoulos (1973), Ogata (1978, 1981)], is equivalent to epidemic branching processes [Kendall (1949), Hawkes and Oakes (1974)], and allows each earthquake to generate (or trigger) offspring earthquakes. Besides being used in seismology, the ETAS model has been applied to various fields in the social and natural sciences [e.g., Balderama et al. (2012), Chavez-Demoulina and McGillb (2012), Hassan Zadeh and Sharda (2012), Herrera and Schipp (2009),

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Mohler et al. (2011), Peng, Schoenberg and Woods (2005), Schoenberg, Peng and Woods (2003)].

Similar magnitude-dependent point-process models have been applied to seismological studies [Vere-Jones and Davies (1966), Lomnitz (1974), Kagan and Knopoff (1987)] and statistical studies [Vere-Jones (1970)]. The ETAS model is stationary if the immigration rate (background seismicity rate) of an earthquake remains constant and the branching ratio is subcritical [Hawkes (1971), Hawkes and Oakes (1974), Zhuang and Ogata (2006)].

The history-dependent form of the ETAS model on occurrence times and sizes (magnitudes) lends itself to the accumulated empirical studies by Utsu (1961, 1962, 1969, 1970, 1971, 1972) and Utsu and Seki (1955), and its establishing history is detailed by Utsu, Ogata and Matsu'ura (1995). ETAS model parameters can be estimated from earthquake occurrence data by maximizing the log-likelihood function to provide estimates for predicting seismic activity (i.e., number of earthquakes per unit time). The model has been frequently used and cited in seismological studies, especially to compare the features of simulated seismicity with those of real seismicity data. The model is also recommended for use in short-term predictions [Jordan, Chen and Gasparini (2012)] in the report of the International Commission on Earthquake Forecasting for Civil Protection. It is planned to be adopted for operational forecasts of earthquakes in California (The Uniform California Earthquake Rupture Forecast, Version 3, URL: [http://www.wgcep.org/sites/wgcep.org/files/UCERF3\\_Project\\_Plan\\_v55.pdf](http://www.wgcep.org/sites/wgcep.org/files/UCERF3_Project_Plan_v55.pdf)).

The ETAS model has also been used to detect anomalies such as quiescence in seismicity. Methods and applications are detailed in Ogata (1988, 1989, 1992, 1999, 2005, 2006a, 2007, 2010, 2011a, 2012), Ogata, Jones and Toda (2003), Kumazawa, Ogata and Toda (2010), and Bansal and Ogata (2013). A change-point analysis examines a simple hypothesis that specific parameters change after a certain time. The misfit of occurrence rate prediction after a change point is then preliminarily shown by the deviation of the empirical cumulative counts of the earthquake occurrences from the predicted cumulative function. The predicted function is the extrapolation of the model fitted before the change point. A downward and upward deviation corresponds to relative quiescence and activation, respectively.

This study considers a number of nonstationary extensions of the ETAS model to examine more detailed nonstandard transient features of earthquake series. The extended models take various forms for comparison with the reference ETAS model, which represents the preceding normal activity in a given focal region. Because changing stresses in the crust are not directly observable, it is necessary to infer relevant quantitative characteristics from seismic activity data. For example, Hainzl and Ogata (2005) and Lombardi, Cocco and Marzocchi (2010) estimated time-dependent background rates (immigration rates) in a moving time window by removing the triggering effect in the ETAS model.

In Section 2, time-dependent parameters for both background rates and productive rates are simultaneously estimated. There, the penalized log-likelihood is considered for the trade-off between a better fit of the nonstationary models and the roughness penalties against overfitting. Then, not only is an optimal strength adjusted for each penalty but also a better penalty function form is selected using the Akaike Bayesian Information Criterion (*ABIC*) [Akaike (1980)]. These parameter constraints together with the existence of a change point are further examined to determine if they improve the model fit. One benefit of this model is that it allows varying parameters to have sharp changes or discontinuous jumps at the change point while sustaining the smoothness constraints in the rest of the period.

In Section 3, the methods are demonstrated by applying the model to a swarm activity. The target activity started after the March 11, 2011 Tohoku-Oki earthquake of magnitude (M) 9.0, induced at a distance from the M9.0 rupture source. Section 4 concludes and discusses the models and methods. The reproducibility of the inversion results is demonstrated in the Appendix by synthesizing the data and re-estimating it using the same procedure.

## 2. Methods.

2.1. *The ETAS model.* A conditional intensity function characterizes a point (or counting) process  $N(t)$  [Daley and Vere-Jones (2003)]. The conditional intensity  $\lambda(t|H_t)$  is defined as follows:

$$(1) \quad \Pr\{N(t, t + dt) = 1|H_t\} = \lambda(t|H_t) dt + o(dt),$$

where  $H_t$  represents the history of occurrence times of marked events up to time  $t$ . The conditional intensity function is useful for the probability forecasting of earthquakes, which is obtained by integrating over a time interval.

The ETAS model, developed by Ogata (1985, 1986, 1988, 1989), is a special case of the marked Hawkes-type self-exciting process, and has the following specific expression for conditional intensity:

$$(2) \quad \lambda_\theta(t|H_t) = \mu + \sum_{\{i: S < t_i < t\}} \frac{K_0 e^{\alpha(M_i - M_z)}}{(t - t_i + c)^p},$$

where  $S$  is the starting time of earthquake observation and  $M_z$  represents the smallest magnitude (threshold magnitude) of earthquakes to be treated in the data set.  $M_i$  and  $t_i$  represent the magnitude and the occurrence time of the  $i$ th earthquake, respectively, and  $H_t$  represents the occurrence series of the set  $(t_i, M_i)$  before time  $t$ . The parameter set  $\theta$  thus consists of five elements  $(\mu, K_0, c, \alpha, p)$ . In fact, the second term of equation (2) is a weighted superposition of the Omori–Utsu empirical function [Utsu (1961)] for aftershock decay rates,

$$(3) \quad \lambda_\theta(t) = \frac{K}{(t + c)^p},$$

where  $t$  is the elapsed time since the main shock. It is important to note that, while the concept of a main shock and its aftershocks is intuitively classified by seismologists sometime after the largest earthquake occurs, there is no clear discrimination between them in equation (2). That is, each earthquake can trigger aftershocks, and the expected cluster size depends on the magnitude of the triggering earthquake with the parameter  $\alpha$ .

The parameter  $K_0$  (earthquakes/day) is sometimes called the “aftershock productivity.” As the name explains, the parameter controls the overall triggering intensity. The factor  $c$  (day) is a scaling time to establish the power-law decay rate and allows a finite number of aftershocks at the origin time of a triggering earthquake (a main shock). In practice, the fitted values for  $c$  are more likely to be caused by the under-reporting of small earthquakes hidden in the overlapping wave trains of large earthquakes [Utsu, Ogata and Matsu’ura (1995)]. The exponent  $p$  is the power-law decay rate of the earthquake rate in equation (3). The magnitude sensitivity parameter  $\alpha$  (magnitude<sup>-1</sup>) accounts for the efficiency of an earthquake of a given magnitude in generating aftershocks. A small  $\alpha$  value allows a small earthquake to trigger a larger earthquake more often. Finally, the background (spontaneous) seismicity rate  $\mu$  represents sustaining external effects and superposed occurrence rates of long-range decays from unobserved past large earthquakes. It also accounts for the triggering effects by external earthquakes.

The FORTRAN program package associated with manuals regarding ETAS analysis is available to calculate the maximum likelihood estimates (MLEs) of  $\theta$  and to visualize model performances [Ogata (2006b)]. See also [http://www.ism.ac.jp/~ogata/Ssg/ssg\\_softwareE.html](http://www.ism.ac.jp/~ogata/Ssg/ssg_softwareE.html).

*2.2. Theoretical cumulative intensity function and time transformation.* Suppose that the parameter values  $\theta = (\mu, K, c, \alpha, p)$  of the ETAS, equation (2), are given. The integral of the conditional intensity function,

$$(4) \quad \Lambda_\theta(t|H_t) = \int_S^t \lambda_\theta(u|H_u) du,$$

provides the expected cumulative number of earthquakes in the time interval  $[0, t]$ . The time transformation from  $t$  to  $\tau$  is based on the cumulative intensity,

$$(5) \quad \tau = \Lambda(t|H_t),$$

which transforms the original earthquake occurrence time  $(t_1, t_2, \dots, t_N)$  into the sequence  $(\tau_1, \tau_2, \dots, \tau_N)$  in the time interval  $[0, \Lambda(T)]$ . If the model represents a good approximation of the real seismicity, it is expected that the integrated function [equation (4)] and the empirical cumulative counts  $N(t)$  of the observed earthquakes are similar. This implies that the transformed sequence appears to be a stationary Poisson process (uniformly distributed occurrence times) if the model is sufficiently correct, and appears to be heterogeneous otherwise.

2.3. *Two-stage ETAS model and the change-point problem.* In change-point analysis, the whole period is divided into two disjointed periods to fit the ETAS models separately, and is therefore called a two-stage ETAS model. This is one of the easiest ways to treat nonstationary data, and is best applied to cases in which parameters are suspected to change at a specific time. Such a change point is observed when a notably large earthquake or slow slip event (regardless of observed or unobserved) occurs in or near a focal region. Many preceding studies [e.g., Ogata, Jones and Toda (2003), Ogata (2005, 2006a, 2007, 2010), Kumazawa, Ogata and Toda (2010)] have adopted this method to their case studies, and details can be found therein.

The question of whether the seismicity changes at some time  $T_0$  in a given period  $[S, T]$  is reduced to a problem of model selection. In this analysis, the ETAS models are separately fitted to the divided periods  $[S, T_0]$  and  $[T_0, T]$ , and their total performance is compared to an ETAS model fitted over the whole period  $[S, T]$  by the Akaike Information Criterion (AIC) [Akaike (1973, 1974, 1977)]. The AIC is described as follows:

$$(6) \quad AIC = -2 \max \log L(\theta) + 2k,$$

where  $\ln L(\theta)$  represents the log-likelihood of the ETAS model,

$$(7) \quad \log L(\theta) = \sum_{\{i: S < t_i < T\}} \log \lambda_{\theta}(t_i | H_{t_i}) - \int_S^T \log \lambda_{\theta}(t | H_t) dt,$$

and  $k$  is the number of parameters to be estimated. The variables  $t_i$  and  $H_{t_i}$  are the same as those in equation (2). Under this criterion, the model with a smaller AIC value performs better. It is useful to keep in mind that  $\exp\{-\Delta AIC/2\}$  can be interpreted as the relative probability of how a model with a smaller AIC value is superior to others [e.g., Akaike (1980)].

Let  $AIC_0$  be the AIC of the ETAS model estimated for the whole period  $[S, T]$ ,  $AIC_1$  be that of the first period  $[S, T_0]$ , and  $AIC_2$  be that of the second period  $[T_0, T]$ , therefore,

$$(8) \quad \begin{aligned} AIC_0 &= -2 \max_{\theta_0} \log L(\theta_0; S, T) + 2k_0, \\ AIC_1 &= -2 \max_{\theta_1} \log L(\theta_1; S, T_0) + 2k_1, \\ AIC_2 &= -2 \max_{\theta_2} \log L(\theta_2; T_0, T) + 2k_2. \end{aligned}$$

Let  $AIC_{12}$  represent the total AIC from the divided periods, such that

$$(9) \quad AIC_{12} = AIC_1 + AIC_2 + 2q,$$

with  $q$  being the degrees of freedom to search for the best change-point candidate  $T_0$ . Next,  $AIC_{12}$  is compared against  $AIC_0$ . If  $AIC_{12}$  is smaller, the two-stage ETAS model with the change point  $T_0$  fits better than the ETAS model applied to

the whole interval. The quantity  $q$  monotonically depends on sample size (number of earthquakes in the whole period  $[S, T]$ ) when searching for the maximum likelihood estimate of the change point [Ogata (1992, 1999), Kumazawa, Ogata and Toda (2010), Bansal and Ogata (2013)]. This penalty term  $q$ , as well as an increased number of estimated parameters, imposes a hurdle for a change point to be significant, and it is usually rejected when the one-stage ETAS model fits sufficiently well. If the change point  $T_0$  is predetermined from some information other than the data, then  $q = 0$ . This is often the case when a conspicuously large earthquake occurs within swarm activity, and will be discussed below. Also, even in this case, the overfitting by the change point is avoided by the  $AIC_{12}$  of a two-stage ETAS model, which has two times as many parameters of a single stationary ETAS model throughout the whole period.

**2.4. Anomaly factor functions for nonstationary ETAS models.** Assume that the ETAS model fits the data well for a period of ordinary seismic activity. Then, the concern is whether this model shows a good fit to the seismicity in a forward extended period. If there are misfits, time-dependent compensating factors are introduced to the parameters to be made time-dependent. These factors are termed “anomaly factor functions” and, thus, the transient changes in parameters are tracked. If earthquake activity is very low in and near a target region preceding the transient activity, data from a wider region, such as the polygonal region in Figure 1, is used to obtain a reference stationary ETAS model (Figure 2). Such a model is stable against small local anomalies, and is therefore a good reference model. The reference ETAS model, coupled with the corresponding anomaly factor functions, becomes the nonstationary ETAS model in this study.

Among the parameters of the ETAS model, the background rate  $\mu$  and the after-shock productivity  $K_0$  are sensitive to nonstationarity. We therefore introduce the anomaly factor functions as the nonstationary components to modify the reference stationary ETAS model in such a way that

$$(10) \quad \lambda_{\theta}(t|H_t) = \mu q_{\mu}(t) + \sum_{\{i: S < t_i < t\}} \frac{K_0 q_K(t_i) e^{\alpha(M_i - M_z)}}{(t - t_i + c)^p}.$$

Here  $q_{\mu}(t)$  and  $q_K(t)$  are referred to as anomaly factor functions of the parameters  $\mu$  and  $K_0$ , respectively. Because of technical reasons to avoid further model complexity, we did not consider the case in which the other three parameters  $c$ ,  $\alpha$  and  $p$  in equation (2) also are time-varying. One structural problem of the ETAS model is that  $K_0$  is correlated with the parameter  $\alpha$ . The trade-off is not negligible, especially when the range of magnitudes in the data set is small. See Section 4 for additional discussion of this issue.

We use the first-order spline function of the ordinary time  $t$ . This is a broken line interpolated by the coordinates  $\{(t_i, q_i); i = 0, 1, 2, \dots, N + 1\}$ , where  $t_i$  is the

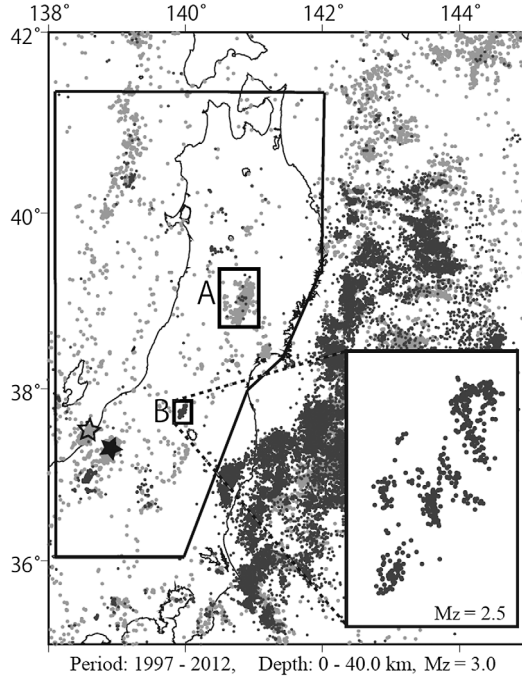


FIG. 1. Epicenters of earthquakes of magnitude ( $M$ )  $\geq 3.0$  in the Northern Honshu region, Japan, with depths shallower than 40 km, from 1997 to 2012, selected from the JMA Hypocenter catalog. The gray and black dots represent the earthquakes that occurred before and after the M9.0 Tohoku-Oki earthquake, respectively. The rectangular regions A and B include the aftershocks of the 2008 Iwate-Miyagi Prefectures Inland Earthquake of M7.2 and the swarm near Lake Inawashiro, respectively. Their inset panels magnify the epicenter distribution with  $M \geq 2.0$  and  $M \geq 2.5$ , respectively. The polygonal region indicates the Tohoku inland and its western offshore region; the earthquakes in this region are used in the reference stationary ETAS model. The closed star represents the epicenter of the 2004 Chuetsu earthquake of M6.8, and the open star represents the 2007 Chuetsu-Oki earthquake of M6.8.

occurrence time of the  $i$ th earthquake, and  $t_0$  and  $t_{N+1}$  are the start and end of the period, respectively. Then, the spline functions are defined as follows:

$$(11) \quad q_{\mu}(t) = \sum_{i=1}^N I_{(t_i, t_{i+1})}(t) \left\{ \frac{q_{\mu, i+1} - q_{\mu, i}}{t_{i+1} - t_i} (t - t_i) + q_{\mu, i} \right\} = \sum_{i=1}^N q_{\mu, i} F_i(t)$$

and

$$(12) \quad q_K(t) = \sum_{i=1}^N I_{(t_i, t_{i+1})}(t) \left\{ \frac{q_{K, i+1} - q_{K, i}}{t_{i+1} - t_i} (t - t_i) + q_{K, i} \right\} = \sum_{i=1}^N q_{K, i} F_i(t),$$

where  $I_{(t_i, t_{i+1})}(t)$  is the indicator function, with the explicit form of  $F_i(t)$  given as

$$(13) \quad F_i(t) = \frac{t - t_{i-1}}{t_i - t_{i-1}} I_{(t_{i-1}, t_i)}(t) + \frac{t_{i+1} - t}{t_{i+1} - t_i} I_{(t_i, t_{i+1})}(t).$$

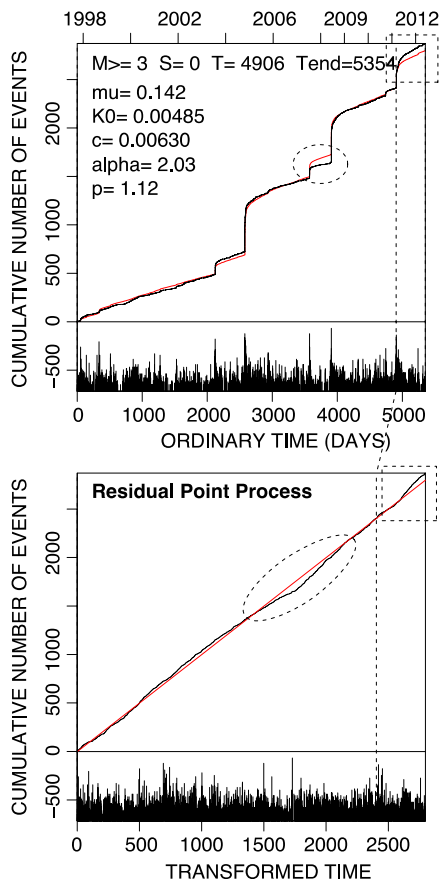


FIG. 2. Cumulative number and magnitude of earthquakes of  $M \geq 3$  against the ordinary time and transformed time by the ETAS model from the polygonal region in Figure 1. The fitted period of the model is from October 1997 to the M9.0 March 2011 Tohoku-Oki earthquake (indicated by vertical dashed lines). Red curves in the top and bottom panels represent the theoretical cumulative numbers against the ordinary time (4) and the transformed time, respectively. The dashed black ellipses and dashed rectangles highlight the anomalies around 2008 and after the Tohoku-Oki earthquake, respectively.

The log-likelihood function of the nonstationary point process can be written as follows:

$$(14) \quad \log L(q) = \sum_{\{i; S < t_i < T\}} \log \lambda_q(t_i | H_{t_i}) - \int_S^T \lambda_q(t | H_t) dt,$$

where  $q = (q_\mu, q_K)$ .

**2.5. Penalties against rough anomaly factor functions.** Since these anomaly functions have many coefficients representing flexible variations, coefficients are



estimated under an imposed smoothness constraint to avoid their overfitting. This study uses the penalized log-likelihood [Good and Gaskins (1971)] described below. With the roughness penalty functions,

$$\begin{aligned} \Phi_{\mu} &= \sum_{i=0}^N \left( \frac{q_{\mu,i+1} - q_{\mu,i}}{t_{i+1} - t_i} \right)^2 (t_{i+1} - t_i) \quad \text{and} \\ \Phi_K &= \sum_{i=0}^N \left( \frac{q_{K,i+1} - q_{K,i}}{t_{i+1} - t_i} \right)^2 (t_{i+1} - t_i), \end{aligned} \quad (15)$$

and the penalized log-likelihood against the roughness becomes

$$\mathcal{Q}(q|w_{\mu}, w_K) = \log L(q) - w_{\mu} \Phi_{\mu} - w_K \Phi_K, \quad (16)$$

where each “ $w$ ” represents weight parameters that tune the smoothness constraints of the anomaly factors. The roughness penalty, equation (15), imposes penalties to the log-likelihood according to parameter differentials at successive event occurrence times.

Furthermore, the degree of the smoothness constraints may not be homogeneous in ordinary time because earthquake series are often highly clustered. In other words, it is expected that more detailed or rapid changes of the anomaly factors appear during dense event periods rather than during sparse periods [Ogata (1989), Adelfio and Ogata (2010)]. Hence, for the same model, alternative constraints are considered by replacing  $\{t_i\}$  in equation (15) with  $\{\tau_i\}$  on the transformed time  $\tau$  in equation (5) of the reference ETAS model.

The following restricted cases of the nonstationary model in equation (10), together with different types of the aforementioned parameter constraints, are examined and summarized in Table 1. Model 1 restricts the parameter  $K_0$  to be constant and unchanged from the reference model, leaving  $q_{\mu}(t)$  to be unrestricted. Model 2 restricts the parameters  $\mu$  and  $K_0$  to have the same factor. In other words, model 2 estimates the anomaly factor for the total intensity  $\lambda_{\theta}(t|H_t)$  in equation (10). This restriction is assumed in Adelfio and Ogata (2010). Model 3 has no restriction.

Here, from a statistical modeling viewpoint, it should be noted that  $\mu$  and  $K_0$  are linearly parameterized regarding the conditional intensity [equation (2)], and

TABLE 1

*Summary of the competing nonstationary ETAS models. The numbers index the models. The row headers explain the model restrictions of anomaly factors  $q_{\mu}(t)$  and  $q_K(t)$ . The first column (a) uses smoothing on ordinary time, the second column (b) on the transformed time*

Restrictions	(a) Smoothing on ordinary time	(b) Smoothing on transformed time
$q_K(t) = 1$	Model 1(a)	Model 1(b)
$q_{\mu}(t) = q_K(t)$	Model 2(a)	Model 2(b)
No restriction	Model 3(a)	Model 3(b)

likewise the linearly parameterized coefficients of the functions  $q_\mu$  and  $q_K$  in equation (10). Together, they force the penalized log-likelihood function [equation (16)] to be strictly concave regardless of the dimensions of the coefficients' space [Ogata (1978, 2001), Ogata and Katsura (1993)]. Therefore, the maximizing solutions of the penalized log-likelihood function can be obtained uniquely and stably under a suitable numerical optimization algorithm [e.g., appendices of Ogata (2004, 2011b)]. The reproducibility of the inversion results of  $\mu(t)$  and  $K_0(t)$  are demonstrated in the [Appendix](#).

*2.6. Tuning smoothness constraints, model selection and error evaluation.* In a Bayesian context, given the weights, the solution of the parameters  $q$  that minimize the penalized log-likelihood  $Q$  in (16) is termed the maximum a posteriori (MAP) estimate. In the following section, we describe how to determine the optimal MAP (OMAP) estimate. To obtain the optimal weights in the penalty functions in equation (16), this study uses a Bayesian interpretation of penalized log-likelihood as suggested by Akaike (1980). Specifically, the exponential of each penalty function is proportional to a prior Gaussian distribution of the forms

$$(17) \quad \pi(q_\mu | w_\mu) \propto e^{-w_\mu q_\mu \Sigma_\mu q_\mu^t / 2} \quad \text{and} \quad \pi(q_K | w_K) \propto e^{-w_K q_K \Sigma_K q_K^t / 2},$$

since the coefficients of the function  $q(\cdot)$  in the penalty term  $\Phi$  take a quadratic form with a symmetric  $(N+1) \times (N+1)$  nonnegative definite matrix  $\Sigma$ . Since each matrix  $\Sigma$  is degenerate and has  $\text{rank}(\Sigma) = N$ , above each prior distribution becomes improper [Ogata and Katsura (1993)]. To avoid such improper priors, we divide each of the vectors  $q$  into  $(q^c, q^{(N+1)})$  so that each of the priors becomes a probability density function with respect to  $q^c$ :

$$(18) \quad \pi(q^c | w, q_{N+1}) = \frac{(w^N \det \Sigma^c)^{1/2}}{\sqrt{2\pi}^N} \exp\left(-\frac{1}{2} w^N q^c \Sigma^c q^c\right),$$

where  $\Sigma^c$  is the cofactor of the last diagonal element of  $\Sigma$ , and  $w$  and  $q^{(N+1)}$  are considered hyperparameters to maximize the integral of the posterior distribution with respect to  $q^c$ ,

$$(19) \quad \begin{aligned} & \Psi(w_\mu, w_K; q_\mu^{(N+1)}, q_K^{(N+1)}) \\ &= \int L(q_\mu, q_K) \pi(q_\mu | w_\mu) \pi(q_K | w_K) dq_\mu^c dq_K^c, \end{aligned}$$

which refers to the likelihood of a Bayesian model. Good (1965) suggests the maximization of equation (19) with respect to the hyperparameters and termed this the Type II maximum likelihood procedure.

By applying Laplace's method [Laplace (1774), pages 366–367], the posterior distribution is approximated by a Gaussian distribution, by which the integral in

equation (19) becomes

$$\begin{aligned}
 (20) \quad & \Psi(w_\mu, w_K; q_\mu^{(N+1)}, q_K^{(N+1)}) \\
 &= Q(\hat{q}_\mu^c, \hat{q}_K^c | w_\mu, w_K; q_\mu^{(N+1)}, q_K^{(N+1)}) \\
 &\quad - \frac{1}{2} \log(\det H_\mu) - \frac{1}{2} \log(\det H_K) + MN \log 2\pi,
 \end{aligned}$$

where  $\hat{q}$  is the maximum of the penalized log-likelihood  $Q$  in equation (16) and

$$(21) \quad H(\hat{q}^c | w, q^{(N+1)}) = \frac{\partial^2 \log L(\hat{q}^c | w, q^{(N+1)})}{\partial q^c \partial (q^c)^t} - \Sigma^c(w, q^{(N+1)}),$$

for a fixed weight  $w$  for either  $w_\mu$  or  $w_K$ .

Thus, maximizing equation (16) with respect to  $q^c$  and equation (20) with respect to  $(w_\mu, w_K; q_\mu^{(N+1)}, q_K^{(N+1)})$ , in turn, achieves our objective. In the former maximization, a quasi-Newton method using the gradients  $\partial \log L(q)/\partial q$  and the Newton method making use of the Hessian matrices, equation (21), endure a fast convergence regardless of high dimensions. For the latter maximization, a direct search such as the simplex method is used. A flowchart of numerical algorithms is described in the appendices of Ogata (2004, 2011b).

Anomaly factor functions under the optimal roughness penalty result in suitably smooth curves throughout the period. Furthermore, there may be a change point that results in sudden changes in parameters  $\mu$  or  $K$ . To examine such a discontinuity, a sufficiently small weight is put into the interval that includes a change point (e.g.,  $w = 10^{-5}$ ), and the goodness-of-fit by *ABIC* is compared with that of the smooth model with the optimal weights for all intervals.

It is useful to obtain the estimation error bounds of the MAP estimate  $\hat{q}$  at each time of an observed earthquake. The joint error distribution of the parameters at  $\hat{q}$  is nearly a  $2N$ -dimensional normal distribution  $N(0, H^{-1})$ , where  $H^{-1} = (h^{i,j})$ , and  $H = (h_{i,j})$  is the Hessian matrix in equation (21). Hence, the covariance function of the error process becomes

$$(22) \quad c(u, v) = \sum_{i=1}^{2N} \sum_{j=1}^{2N} F_i(u) h^{i,j} F_j(v),$$

where  $F_i = F_{N+i}$  for  $i = 1, 2, \dots, N$ , which is defined in equation (13). Thus, the standard error of  $q$  is provided by

$$(23) \quad \varepsilon(t) = [\varepsilon_\mu(t), \varepsilon_K(t)] = \sqrt{C(t, t)}.$$

**2.7. Bayesian model comparison.** It is necessary to compare the goodness of fit among the competing models. From equation (20), the *ABIC* [Akaike (1980)] can be obtained as

$$\begin{aligned}
 (24) \quad & ABIC = (-2) \max_{w_\mu, w_K; q_\mu^{(N+1)}, q_K^{(N+1)}} \log \Psi(w_\mu, w_K; q_\mu^{(N+1)}, q_K^{(N+1)}) \\
 & + 2 \times (\# \text{hyperparameter}).
 \end{aligned}$$

Specifically, models 1 and 2 [1(a) and (b), 2(a) and (b) in Table 1] have four hyperparameters, and model 3 [3(a) and (b)] has eight. A Bayesian model with the smallest  $ABIC$  value provides the best fit to the data.

Since there are various constraints in the different setups, the resulting  $ABIC$  values cannot be simply compared because of unknown different constants, mainly due to the approximations in equation (20). Alternatively, the difference of  $ABIC$  values relative to those corresponding to the reference model are used. In other words, the reduction amount of the  $ABIC$  value from a very heavily constrained case,

$$(25) \quad \Delta ABIC = ABIC - ABIC_0,$$

where  $ABIC$  is that of equation (24) and  $ABIC_0$  is the  $ABIC$  value with very heavy fixed weights, which constrain the function to be almost constant. Therefore, the  $\Delta ABIC$  approximates the  $ABIC$  improvement from the flat anomaly functions [ $q(t) = 1$  for all  $t$ ] to the optimal functions.

Likewise in  $AIC$ , it is useful to keep in mind that  $\exp\{-\Delta ABIC/2\}$  can be interpreted as the relative probability of how the model with the smallest  $ABIC$  value is superior to others [e.g., Akaike (1980)].

### 3. Applications.

3.1. *The stationary ETAS model versus the two-stage ETAS model.* First, we estimate the stationary ETAS model that has been applied to a series of earthquakes of magnitude ( $M$ ) 3.0 and larger contained in the polygonal region highlighted in Figure 1, from October 1997 to the M9.0 Tohoku-Oki earthquake on March 11, 2011. Specifically, the MLE has been obtained for the stationary ETAS model [equation (2)] by applying a normal activity for earthquakes of M3.0 and larger from October 1997 to March 10, 2011 (Figure 2). According to the estimated theoretical cumulative curve in ordinary time [equation (4)] and transformed time [equation(5)] in Figure 2, the ETAS model appears to fit very well except for a period near 2008 and a period after the Tohoku-Oki earthquake, which is in good accordance with Ogata (2012). These anomalies are highlighted by dashed ellipses and dashed rectangles in Figure 2.

The former is the apparent lowering due to substantially small productivity in the aftershock activity of the 2007 Chuetsu-Oki earthquake (open star in Figure 1). Interestingly enough, the 2004 Chuetsu earthquake (closed star) and the 2007 Chuetsu-Oki earthquake, which are about 40 km apart, have the same magnitude (M6.8), but the number of aftershocks of  $M \geq 4.0$  differs by 6–7 times [Japan Meteorological Agency (2009)].

The latter is due to the activation relative to the predicted ETAS model. The March 11, 2011 M9.0 Tohoku-Oki earthquake induces this activation. On the other hand, a series of aftershocks (located in region A, Figure 1) of the 2008 M7.2 Iwate-Miyagi Prefecture inland earthquake is quiet relative to the occurrence rate

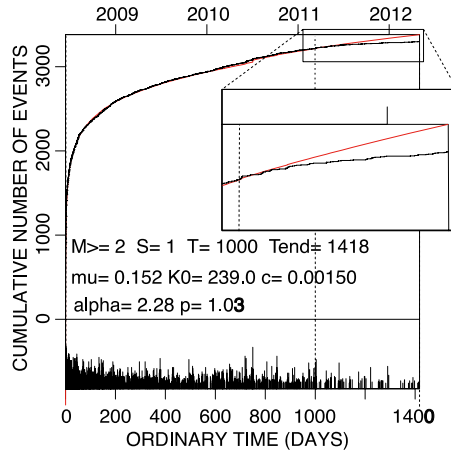


FIG. 3. Cumulative number and magnitude of the aftershock sequence with  $M \geq 1.5$ , following the 2008 Iwate-Miyagi earthquake of  $M7.2$ , from the region A against ordinary time. The ETAS model is fitted to the sequence for the period from one day after the main shock ( $S = 1.0$  day) to the Tohoku-Oki earthquake (March 11, 2011; dashed line). The almost overlapping red curve indicates the theoretical ETAS cumulative function, equation (4), and the extension to the rest of the period until April 2012. The inset rectangle magnifies the cumulative curve for the extrapolated period.

predicted by the ETAS model estimated from the aftershock data before the  $M9.0$  earthquake.

An analysis of the 2008 earthquake aftershock sequence is shown in Figure 3. Here the ETAS model is fitted to the period from one day after the main shock until the  $M9.0$  earthquake. The estimated intensity is then extrapolated to span an additional year. The change point at the  $M9.0$  earthquake is substantial, decreasing the total  $AIC$  by 28.5, showing a relative quiescence afterward. The penalty quantity  $q$  in the  $AIC_{12}$  of equation (9) equals zero because the change point is given by the information outside of the aftershock data, hence,  $\Delta AIC = -28.5$ . Therefore, the occurrence of the Tohoku-Oki earthquake is a significant change point.

Hereafter, the data set becomes very difficult for conventional ETAS analysis. The earthquake swarm near Lake Inawashiro began March 18, 2011, a week after the  $M9.0$  earthquake in region B (Figure 1). Seismic activity in this area was very low before the  $M9.0$  event. The swarm mostly consisted of small earthquakes with magnitudes less than 3.0. The largest earthquake in this cluster, an earthquake of  $M4.6$ , occurred 50 days after the  $M9.0$  earthquake, and its aftershock sequence seemed to decay normally.

First, the stationary ETAS model is applied to the whole period. The theoretical cumulative function (solid light blue curves, Figure 4) is biased below from the empirical cumulative function, indicating a substantial misfit. Hence, the two-stage ETAS model is applied to the data to search the MLE for a change point. Table 2 lists the estimated parameters and  $AIC$  values. The change-point analysis (cf. Section 2.2) implies that the MLE of the change point is at  $t = 49.8$  days from the

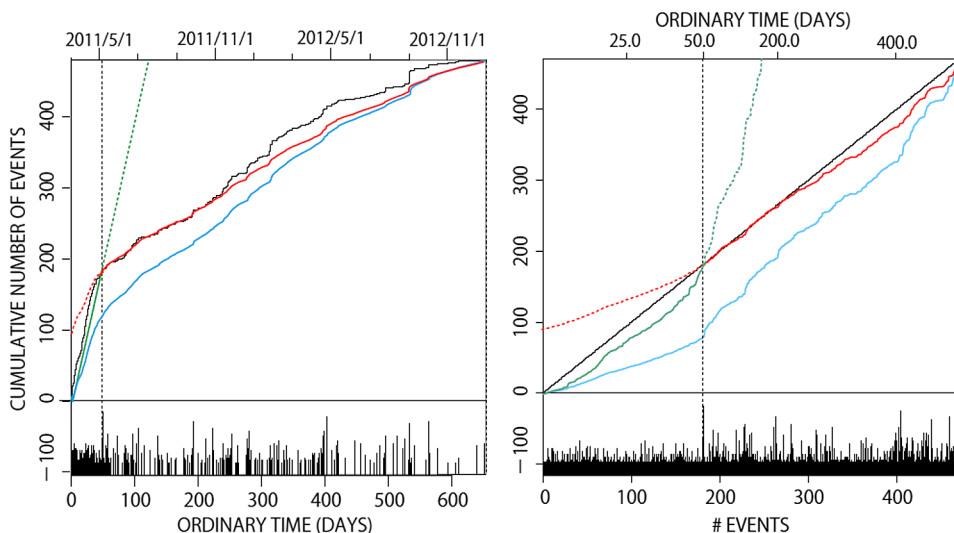


FIG. 4. Stationary and two-stage ETAS models fitted to region B. The ETAS model is fitted to the entire period from March 18, 2011, to the end of 2012 with the preliminary period of the first 0.1 days (blue line), the period before the M4.6 event ( $t = 49.8$  days) (green solid line) then extrapolated forward (green dashed line), and the period after the M4.6 event (red solid line) then extrapolated backward (red dashed line). The black curve shows the cumulative number of observed earthquakes. The left panel plots these against ordinary time, whereas the right panel plots these against the number of earthquakes.

beginning of this cluster, which coincides with the time just before the M4.6 earthquake occurred. The two-stage ETAS model with this change point improves the  $AIC$  by 138.2 (see Table 2). The first-stage ETAS model before the change point, with a fixed parameter  $p = 1.0$ , still displays a large deviation from the ideal fit (cf. the solid green curve in Figure 4). The magnitude sensitivity parameter  $\alpha$  becomes very small relative to that of the second-stage ETAS model. Such a small value implies that almost all earthquakes in the first stage occurred independently to preceding magnitudes (i.e., close to a Poisson process), and can be mostly attributed to the average  $\mu$  rate of the background seismicity. The first stage  $\mu$  rate is two orders of magnitude higher than the second stage rate.

If  $p$  is not fixed, the estimated  $K_0$ ,  $c$  and  $p$  have extremely large values for a normal earthquake sequence while  $\alpha$  approaches zero. Consequently, the model is again approximate to a nonstationary Poisson process, characterizing the sequence as a swarm, with an  $AIC$  smaller than that of the  $p = 1.0$  scenario. The large discrepancies between the estimated parameter values between (b) and (c) in Table 2 suggest that the stationary ETAS model is not well defined for this particular earthquake sequence in the first period before the change point. The standard errors for the parameter  $\alpha$  are multiple orders of magnitude greater than those of the estimates themselves. The narrow magnitude range makes it difficult for the model

TABLE 2

The ETAS parameters of region B fitted to (a) the entire period, (b) and (c) before the change point, and (d) after the change point. Their standard errors are in parentheses. The improvement of the two-stage ETAS model relative to the stationary ETAS model is

$\Delta AIC = (422.9 - 118.3) - 442.8 = -138.2$ . The MLE for the change point is  $t = 49.8$ , which coincides with the time just before the M4.6. The threshold magnitude is  $M_z = 2.5$ . Numbers are rounded to three significant digits

Period	$\mu$	$K_0$	$c$	$\alpha$	$p$	AIC
(a) The whole period	$9.77 \times 10^{-2}$ ( $7.81 \times 10^{-2}$ )	$6.54 \times 10^{-2}$ ( $2.37 \times 10^{-2}$ )	$9.64 \times 10^{-4}$ ( $6.35 \times 10^{-4}$ )	0.215 ( $9.77 \times 10^{-2}$ )	0.900 ( $9.84 \times 10^{-3}$ )	442.8
(b) Before change point with fixed $p = 1.0$	1.41 ( $3.39 \times 10^{-1}$ )	$1.05 \times 10^{-1}$ ( $6.99 \times 10^{-2}$ )	$8.52 \times 10^{-2}$ ( $1.09 \times 10^{-1}$ )	$3.06 \times 10^{-15}$ ( $9.35 \times 10^{-1}$ )	1.00	-103.0
(c) Before change point without fixed $p$	1.27 ( $5.52 \times 10^{-1}$ )	$2.12 \times 10^{+11}$ (4.71)	$1.04 \times 10^{+1}$ ( $3.81 \times 10^{-1}$ )	$2.25 \times 10^{-12}$ (1.03)	$1.13 \times 10^{+1}$ ( $2.31 \times 10^{-1}$ )	-118.3
(d) After change point	$6.58 \times 10^{-2}$ ( $1.43 \times 10^{-1}$ )	$3.58 \times 10^{-2}$ ( $1.90 \times 10^{-2}$ )	$7.11 \times 10^{-5}$ ( $1.01 \times 10^{-3}$ )	0.912 ( $1.10 \times 10^{-1}$ )	0.945 ( $1.87 \times 10^{-1}$ )	422.9

to distinguish the effects of  $K_0$  and  $\alpha$ , causing a trade-off between these two parameters, thus providing inaccurate estimations. For the case without a fixed  $p$ , the aftershock productivity  $K_0$  becomes extremely small in compensation for the small  $\alpha$  estimate.

After the change-point time of the M4.6 earthquake, the ETAS model fits considerably well for several months. Then, a deviation becomes noticeable relative to the solid red cumulative curve in Figure 4. From these observations, it is concluded that the M4.6 earthquake has reduced swarm activity and that decaying normal aftershock type activity has dominated.

**3.2. Comparison of the nonstationary models.** In this section the proposed nonstationary models and methods outlined in Sections 2.4–2.6 are applied to the same data from region B near Lake Inawashiro. To replicate the transient nonstationary activities in this particular region, we use the seismic activity in the larger polygonal region in Figure 1 for the period before the M9.0 earthquake (MLEs are shown in Figure 2). Such a reference model represents a typical seismicity pattern over a wide region throughout the period, and therefore represents a robust estimate against the inclusion of local and transient anomalies.

By fixing the reference parameters  $c$ ,  $\alpha$  and  $p$ , both in the stationary and two-stage ETAS models,  $\mu$  and  $K_0$  are estimated for events from region B after the

TABLE 3

*Reference parameters adjusted to the data from region B and the parameters of the present two-stage ETAS model (standard errors in parentheses) with fixed  $c$ ,  $\alpha$  and  $p$  of the reference model (standard errors in brackets), with their AIC values. The improvement of the two-stage ETAS model relative to the present stationary ETAS model is  $\Delta AIC = 434.7 - 95.4 - 465.5 = -126.2$ . Also, the improvement of the present two-stage ETAS model relative to the stationary ETAS model in Table 2 is as follows:  $\Delta AIC = 434.7 - 95.4 - 442.8 = -103.5$ . The change point is at  $t = 49.8$ , corresponding to the time just before the M4.6 earthquake. The threshold magnitude  $M_z = 2.5$ . Numbers are rounded to three significant digits*

Period	$\mu$	$K_0$	$c$	$\alpha$	$p$	AIC
(a) The whole period	$1.92 \times 10^{-1}$ ( $3.58 \times 10^{-2}$ )	$2.49 \times 10^{-2}$ ( $5.82 \times 10^{-3}$ )	$6.02 \times 10^{-3}$ [ $2.50 \times 10^{-3}$ ]	2.03 [ $1.27 \times 10^{-2}$ ]	1.11 [ $5.44 \times 10^{-3}$ ]	465.5
(b) Before the change point	3.31 ( $1.04 \times 10^{-1}$ )	$6.77 \times 10^{-3}$ ( $3.27 \times 10^{-3}$ )	$602 \times 10^{-3}$	2.03	1.11	-95.4
(c) After the change point	$1.95 \times 10^{-1}$ ( $2.99 \times 10^{-2}$ )	$1.56 \times 10^{-2}$ ( $6.41 \times 10^{-3}$ )	$6.02 \times 10^{-3}$	2.03	1.11	434.7

M9.0 event, with a magnitude  $M \geq 2.5$ . Table 3 summarizes the re-estimated parameters, together with the corresponding AIC values. The AIC improvement of the two-stage ETAS model is 126.2.

Next, we have applied the nonstationary ETAS models listed in Table 1, with and without a change point taken into consideration, using the reference parameters in the first row of Table 3. Here, if a change point of M4.6 at the time  $t = 49.8$  days occurs, we propose a very small fixed value such as that described in Section 2.5.

Figure 5 shows all of the inversion results (maximum posterior estimates) for a total of 12 models. The  $\Delta ABIC$  values of the corresponding models are given in Table 4. Models with the change point outperform corresponding models without the change point. This highlights the significance of jumps at the change point. Such improvements via jumps are smaller between corresponding models with constraints on the transformed time. This is because those models already present jumps or sharp changes to some extent in the target parameters even without setting change points, due to the expanded transformed time during the dense event period after the M4.6 event in ordinary time. Results also show that models with constraints on ordinary time yield better results than those with the transformed time. This is probably because the data set only contains gradually changing parameters except at the change point.

The smallest  $\Delta ABIC$  is achieved by model 3(a') in which both  $q_\mu(t)$  and  $q_K(t)$  are nonstationary on the smoothness constraints under ordinary time, with a jump at the time of the M4.6 earthquake. Figure 6 shows variations of the background and productivity rates in the selected nonstationary model. These variations suggest that the intensity of aftershock productivity  $K_0(t)(= K_0 q_K(t))$  is extremely



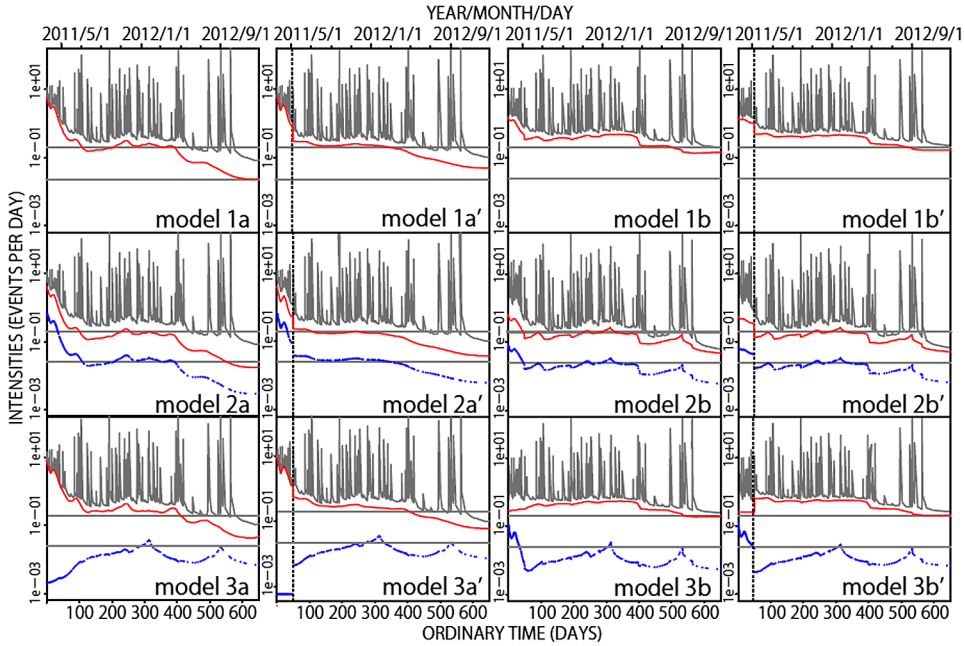


FIG. 5. Various inversion results of all considered models for the data from region B. The model numbers correspond to those of Table 1, and the models with prime (') correspond to those that include a change point. The background rates  $\mu(t)$  are shown in red connected lines, and the productivity  $K_0(t)$  is shown in blue dots at earthquake occurrence times. The gray spiky curves represent the conditional intensity rates  $\lambda(t|H_t)$ . The above three functions are plotted on a logarithmic scale. The upper and lower gray horizontal lines represent the reference parameters  $\mu$  and  $K_0$ , respectively (see Table 2). The vertical dashed line shows the change-point time,  $t = 49.8$  days elapsed from March 18, 2011. The horizontal axis indicates days elapsed.

low during early periods of earthquake swarms until the M4.6 earthquake occurs; meanwhile, the background seismicity  $\mu(t)(= \mu q_\mu(t))$  changes at a high rate. Therefore, the total seismicity  $\lambda_\theta(t|H_t)$  in that period is similar to a nonstationary Poisson process with intensity rates  $\mu(t)$  of the background activity. Af-

TABLE 4

$\Delta ABIC$  value of each model defined in equation (25). The underlined model has the smallest value. The prime (') indicates the models that further assume a change point at  $t = 49.8$ , the time when the M4.6 earthquake occurred

Models	a	a'	b	b'
1	-170.0	-177.2	-132.4	-134.1
2	-175.3	-180.1	-136.1	-137.2
3	-250.1	<u>-260.8</u>	-148.1	-151.5

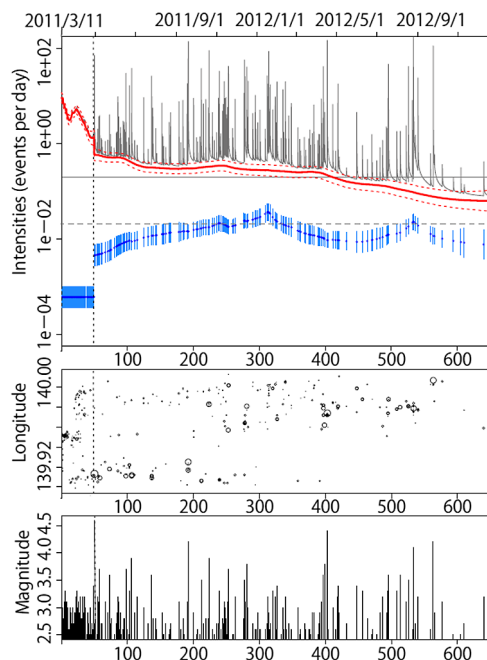


FIG. 6. The selected best-fitted model 3(a') and errors of the inversion solutions. The background rate  $\mu(t)$  is shown in solid red, with one  $-\sigma$  error bounds in red dashed lines.  $K_0(t)$  is shown in blue dots with one  $-\sigma$  error bars at the occurrence times. The gray spiky curve represents the variation of the intensity rates  $\lambda(t|H_t)$ . All of the above estimates are plotted on a logarithmic scale. The solid gray horizontal line represents the reference  $\mu$  value, and the horizontal dashed line represents the reference  $K_0$  value (see Table 2). The horizontal axis is the elapsed days from March 18, 2011. The vertical dashed line shows the change point  $t = 49.8$  elapsed days. The middle panel displays the longitudes versus the elapsed times of the earthquake occurrences in region B. The diameters of the circles are proportional to the earthquake magnitudes. The bottom panel shows magnitudes of earthquakes versus the ordinary elapsed times in days.

ter the M4.6 earthquake occurred, the  $\mu(t)$  rate gradually decreased while  $K_0(t)$  increased. These changes are roughly approximated by the estimated two-stage ETAS model in Table 3, in which  $\mu$  before the change point is higher, while  $K_0$  is lower than those after the change point.

If the  $\Delta ABIC$  of model 3(a') in Table 4 and  $\Delta AIC$  of the two-stage ETAS models in Tables 2 and 3 are compared [Akaike (1985, 1987)], the former model displays a much better fit, with a difference of more than 130. This indicates that the specific details of transient variations in model 3(a') appear to be substantial. Model 3(a') further shows that the background  $\mu(t)$  rate decreased after about  $t = 400$  days, indicating that the swarm component of the seismicity decreased. To demonstrate the reproducibility of the detailed variations with the similar data sets, Figure 7 shows the re-estimated model 3(a') utilizing the same optimization procedure from

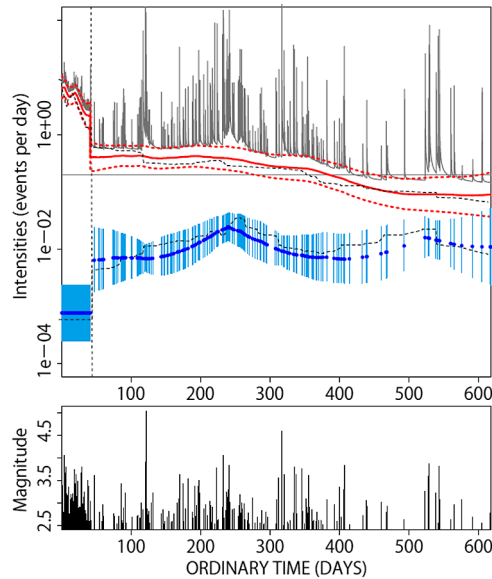


FIG. 7. The maximum a posteriori (MAP) solution of a synthesized data set by the estimated model 3(a') (shown in Figure 6) with the same reference parameters in Table 2. The re-estimated parameters  $\mu(t)$  and  $K_0(t)$  are shown in red and blue curves, respectively, with two-fold error bounds. The upper and lower dashed black curves represent the true  $\mu(t)$  and  $K_0(t)$  (same as those in Figure 6), respectively.

simulated data in the estimated model 3(a') in Figure 6. See the [Appendix](#) for more details.

The model's performance is graphically examined by plotting the estimated cumulative number of events (4) to compare with the observed events in Figure 8, which shows that the observed events become almost a stationary Poisson process, although a few clustering features remain.

It is worthwhile to discuss why model 3(b') with constraints under the transformed time has a poorer fit than model 3(a') with constraints under ordinary time. The MAP estimate of model 3(b') is shown in Figure 9, where the transformed time  $\tau$  in this case is defined in equations (4) and (5) using the reference ETAS model, the parameter value of which is listed in the first row of Table 3. Parameter variations in the period after the change point are similar to those of the overall best model 3(a'). Variations during the period before the change point are different with higher  $K_0(t)$  and lower  $\mu(t)$ . However, in this particular application, the performance of model 3(b') on the whole is inferior in terms of  $\Delta ABIC$  by a difference of greater than 100. This may be because the above mentioned reference ETAS-based transformed time of the former period worked poorly, unlike during the latter period.

Although the goodness of fit of model 3(b') over the whole period (particularly during the former period) is not quite satisfactory, it is worthwhile to examine

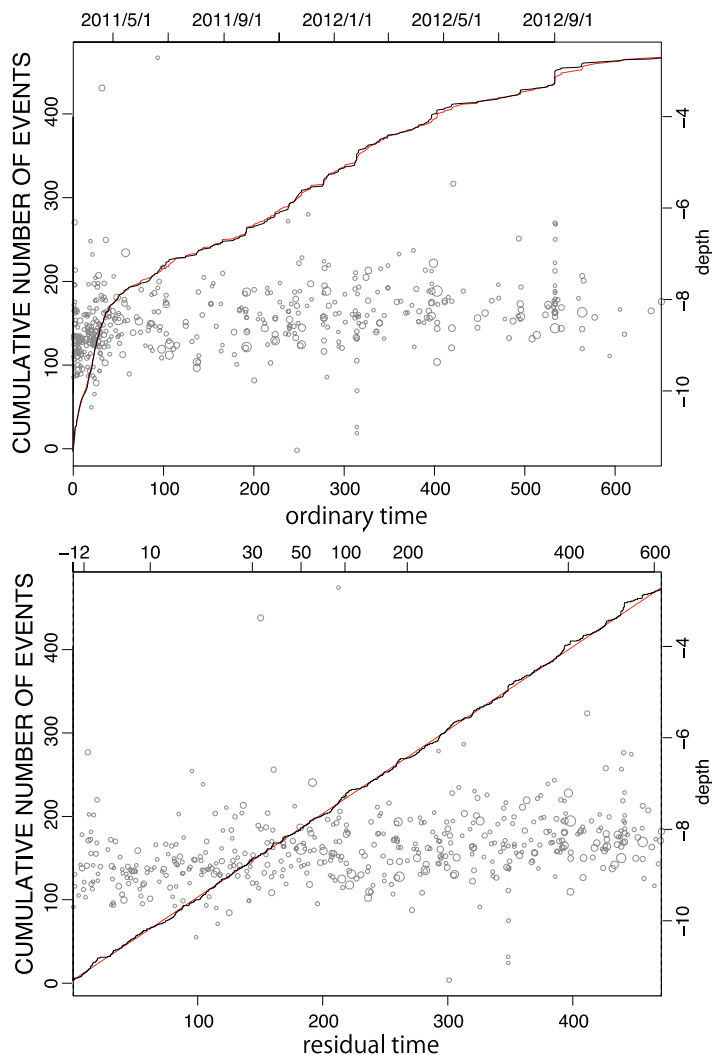


FIG. 8. Estimated cumulative number of events by model 3(a') (red curves) and the observed number of events (black curve) for the ordinary time (top panel) and residual time (bottom panel). Gray circles show the depths of the swarm events versus the corresponding time.

the changes of  $\mu(\tau)$  and  $K_0(\tau)$  during the latter period in Figure 9. The conditional intensity rate  $\lambda_\theta(t|H_t)$ , background rate  $\mu(\tau)$  and aftershock productivity rate  $K_0(\tau)$  rapidly decrease not only after the M4.6 earthquake but also after relatively large earthquakes. On such sharp drops, there is a technical but simple explanation. Models in Table 4 with smoothness constraints on the transformed time are sensitive to catalog incompleteness during small time intervals after large earthquakes. In other words, a substantial number of small earthquakes that oc-

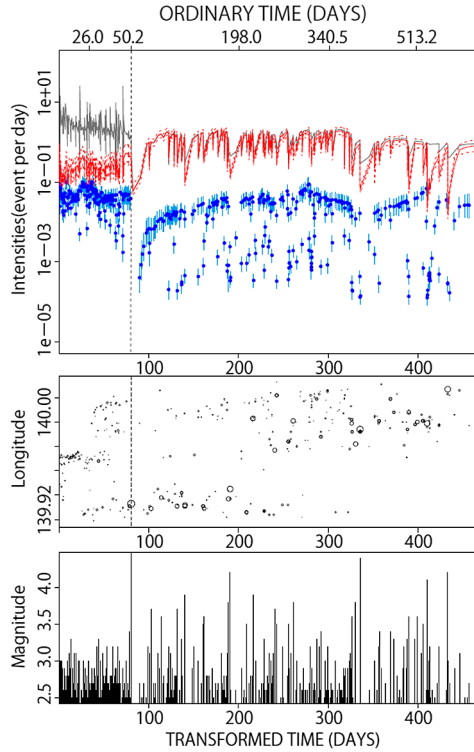


FIG. 9. Variations of conditional intensity rates  $\lambda(\tau|H_\tau)$ , background rate  $\mu(\tau)$  and aftershock productivity  $K_0(\tau)$  of model 3(b') versus the transformed time  $\tau$  of the reference ETAS model (the first row of Table 3). The other details are the same as those in Figure 6.

cur immediately after a large earthquake are missing in the earthquake catalog [e.g., Ogata and Katsura (2006); Omi et al. (2013)]. Present results suggest that the smoothing on the transformed time can be used as a supplemental tool to check catalog completeness. The time transformation stretches out ordinary time where the intensity rate is high and, hence, transforming the smoothed parameters back to ordinary time can result in sharp changes. This type of constraint can be useful for different applications in which occasional rapid changes are expected.

**3.3. Seismological complements and implications of the results.** Used as a reference model, the polygonal region in Figure 1 is known to have a similar seismicity pattern with similar focal mechanisms under the west–east compressional tectonic field, as described in Terakawa and Matsu'ura (2010) and Toda, Lian and Ross (2011, 2011). For example, earthquakes have mostly north–south strike angles and west–east directional thrust faults in this region. This pattern can also be seen in the configurations of active fault systems on the surface.

In the above sections, the estimation procedures of the models presented here have been illustrated with a data set that includes a cluster of swarm earthquakes

triggered by the March 11, 2011 M9.0 Tohoku-Oki earthquake. Swarm activity in this region seems to be triggered by surface waves emitted from the M9.0 source, and has been studied by Terakawa, Hashimoto and Matsu'ura (2013) using the seismological theory and methods used in Terakawa, Miller and Deichmann (2012). Here they attribute swarm activity to the weakening of the fault via an increase of pore fluid pressure caused by the dynamic triggering effect due to surface waves of the Tohoku-Oki rupture. Thus, the initially very high and then decreasing rate of  $\mu(t)$  reflects changes in fault strength, probably due to the intrusion and decrease in pore fluid pressure. The analyses presented here support the quantitative, phenomenological evidence of fault weakening via the intrusion of water into the fault system in earlier periods [Terakawa, Miller and Deichmann (2012, 2013)]. Similarly, by monitoring swarm activity, this nonstationary model can be expected to make quantitative inferences of magma intrusions and draining during volcanic activity.

The background seismicity parameter in the ETAS model is sensitive to transient aseismic phenomena such as slow slips (quiet earthquakes) on and around tectonic plate boundaries [Llenos, McGuire and Ogata (2009), Okutani and Ide (2011)]. This could possibly link a given swarm activity to the weakening of interfaces. Changes in the pore fluid pressure, for example, alter the friction rate of fault interfaces, thereby changing the fault strength. Hence, monitoring the changes in background seismicity has the potential to detect such aseismic events.

Changes in the aftershock productivity  $K_0$ , on the other hand, appear to depend on the locations of earthquake clusters and appear to vary among clusters where secondary aftershocks are conspicuous. The aftershock productivity  $K_0$  therefore reflects the geology around faults rather than the changes in stress rate. The application of the space–time ETAS model with location-dependent parameters [e.g., Ogata, Katsura and Tanemura (2003), Ogata (2004, 2011b)] reveals that the  $K_0$  function varies (i.e., location sensitive) unlike other parameters. Still, the task remains to confirm the link between the changes in ETAS parameters and physical processes happening on and around faults.

**4. Conclusions and discussion.** There are many examples in seismology in which different authors have obtained differing inversion results for the same scientific phenomenon. These differences are attributed to the adoption of different priors for the parameters of a given model. Scenarios in this study have the same problem and are highlighted in Figure 5. Model parameters in this study are estimated by maximizing the penalized log likelihood, which is intrinsically nonlinear. Besides adjusting the weights in the penalty (namely, hyperparameters of a prior distribution), it is necessary to compare the adequacy of different penalties (prior distributions) associated with the same likelihood function. For these purposes, we have proposed the objective procedure using  $\Delta ABIC$  and  $\Delta AIC$ .

A suitable ETAS model [equation (2)] is first established with MLE as the reference predictive model to monitor future seismic activity and to detect anomalous

seismic activity. Sometimes, transient activity starts in a region with very low seismicity. In such a case, it is both practical and applicable to use a data set from a wider region to estimate the stable and robust parameter values of  $c$ ,  $\alpha$  and  $p$  in the ETAS model [equation (2)]. Then, the competing nonstationary ETAS models in equation (10) are fitted together with constraint functions in equations (11) and (12) using either ordinary time or transformed time to penalize the time-dependent parameters in the models. The corresponding Bayesian models include a different prior distribution of the anomaly factor coefficients  $q_\mu(\cdot)$  and  $q_K(\cdot)$ , which are functions of either the ordinary time  $t$  [models 1(a)–3(a) in Table 1] or the transformed time  $\tau$  in the reference ETAS model [models 1(b)–3(b)]. Furthermore, models in which the anomaly functions involve a discontinuity [models 1(a')–3(a') and 1(b')–3(b')] are considered. Using the  $\Delta ABIC$  value, the goodness-of-fit performances of all of the different models are summarized in Table 4. Among the competing models, model 3(a') attained the smallest  $\Delta ABIC$  value, and it is therefore concluded that this model provides the best inversion result for this particular data set.

Thus, changes in background seismicity  $\mu$  and/or aftershock productivity  $K_0$  of the ETAS model can be monitored. The background seismicity rate in the ETAS models represents a portion of the occurrence rate due to external effects that are not included in the observed earthquake occurrence history in the focal region of interest. Therefore, changes in the background rate have been attracting the interest of many researchers because such changes are sometimes precursors to large earthquakes. The declustering algorithms [e.g., Reasenberg (1985), Zhuang, Ogata and Vere-Jones (2002, 2004)] have been adopted to determine the background seismicity by stochastically removing the clustering components depending on the ratio of the background rate to the whole intensity at each occurrence time. The change-point analysis and nonstationary models presented in this study, however, objectively serve a more quantitatively explicit way to approach this task.

The case where the other three parameters  $c$ ,  $\alpha$  and  $p$  in equation (2) also vary with time was not examined in this study. For example, in Figure 8, we have seen that the best model in our framework does not capture all of the clustering events but misses a few small clusters, which suggests the time dependency of the parameters. For another example, we have seen the effect of missing earthquakes in Figure 7, suggesting that parameter  $c$  may depend on the magnitude of the earthquake, leading to a significant correlation between  $c$  and  $p$ . Furthermore, in Section 2.3, it is mentioned that  $K_0$  is correlated with the parameter  $\alpha$ . Unstable estimations of  $K_0$  and the  $\alpha$  value in the swarm period before the M4.6 earthquake can be seen in Table 2, during which period most of the magnitudes are between 2.5 and 3. This is another reason why the  $\alpha$  value is fixed by the corresponding reference parameter  $\alpha$  when the nonstationary models are applied. Owing to the linearly parameterized coefficients of the functions  $q_\mu$  and  $q_K$  in equation (10), the maximizing solutions of the penalized log-likelihood function [equation (16)], in spite of the high dimension, can be obtained uniquely and stably by fixing the three parameters  $c$ ,  $\alpha$  and  $p$ .



## APPENDIX: SYNTHETIC TEST OF REPRODUCIBILITY OF NONSTATIONARY PATTERNS

We tested our method with synthetic data sets to check if both  $\mu(t)$  and  $K_0(t)$  can be reproduced by simulated data sets that are similar to observed data sets. We used the reference parameter set (Table 2) with the best estimated  $\mu(t)$  and  $K_0(t)$  of model 3(a').

The magnitude sequence of the synthetic data was generated on the basis of the Gutenberg–Richter law with a  $b$ -value of the original data set ( $b = 1.273$ ). In other words, the magnitude of each earthquake will independently obey an exponential distribution such that  $f(M) = \beta \exp\{-\beta(M - M_c)\}$ ,  $M \geq M_c$ , where  $\beta = b \ln 10$ , and  $M_c = 2.5$  is the magnitude value above which all earthquakes are detected.

The thinning method [Ogata (1981, 1998)] is adopted for data simulation. A total of 470 events were simulated with a threshold magnitude of 2.5. Model 3(a') was then fitted to the simulated data sets, with a change point at the same time as the original data (between the 182nd and 183rd event). Results are shown in Figure 7; the estimated  $\mu(t)$  and  $K_0(t)$  appear to be similar to the original  $\mu(t)$  and  $K_0(t)$  in Figure 6, respectively, within a  $2\sigma$  error.

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