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TENSOR PRODUCTS OF HYPERRIGID OPERATOR SYSTEMS

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Abstract. In this article, we prove that the tensor product of two hyperrigid operator systems is hyperrigid in the spatial tensor product of C^*-algebras. We deduce this by establishing that the unique extension property for unital completely positive maps on operator systems carry over to tensor products such maps defined on the tensor product operator systems. Hopenwasser's result about the tensor product of boundary representations follows as a special case. We also provide examples to illustrate the hyperrigidity property of tensor products of operator systems.

1. Introduction and preliminaries

The notion of *boundary representation* of a C^*-algebra for an *operator system* introduced by Arveson [1] greatly influenced the theory of noncommutative approximation theory and other related areas such as Korovkin type properties for completely positive maps, peaking phenomena for operator systems, and noncommutative convexity, and so on. Arveson [2] also introduced the notion of *hyperrigid set* as a noncommutative analogue of the classical Korovkin set and studied extensively the relation between hyperrigid operator systems and boundary representations.

In this article, we study hyperrigidity of operator systems in C^*-algebras in the context of tensor products of C^*-algebras. One interesting area to investigate is whether tensor product of hyperrigid operator systems are hyperrigid. By a result of Hopenwasser [7], a tensor product of boundary representations of

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C^*-algebras for operator systems is a boundary representation if one of the constituent C^*-algebras is a GCR algebra. Since hyperrigidity implies that all irreducible representations are boundary representations, we will be able to deduce Hopenwasser's result as a spatial case if we can prove a similar result for hyperrigidity. We achieve this by establishing first that unique extension property for unital completely positive maps on operator systems carry over to the tensor product of those maps defined on the tensor product of operator systems in the spatial tensor product of C^*-algebras.

To fix our notation and terminology, we first recall the fundamental notions. Let H be a complex Hilbert space, and let $B(H)$ be the set of all bounded linear operators on H. An *operator system* S in a C^*-algebra A is a self-adjoint linear subspace of A containing the identity of A. Given a linear map ϕ from a C^*-algebra A into a C^*-algebra B, we can define a family of maps $\phi_n : M_n(A) \to M_n(B)$ given by $\phi_n([a_{ij}]) = [\phi(a_{ij})]$, $n \in \mathbb{N}$. We say that ϕ is *completely bounded* (CB) if $\|\phi\|_{\mathrm{CB}} = \sup_{n\geq 1} \|\phi_n\| < \infty$. We say that ϕ is *completely contractive* (CC) if $\|\phi\|_{\mathrm{CB}} \leq 1$, and we say that ϕ is *completely isometric* if ϕ_n is isometric for all $n \geq 1$. We say that ϕ is *completely positive* (CP) if ϕ_n is positive for all $n \geq 1$, and we say that ϕ is *unital completely positive* (UCP) if in addition $\phi(1) = 1$.

Definition 1.1. Let S be an operator system generating the C^*-algebra $C^*(S)$. A UCP map $\pi : S \to B(H)$ is said to have *unique extension property* (UEP) for S if

(i) π has a unique completely positive extension $\widetilde{\pi} : C^*(S) \to B(H)$, and
(ii) $\widetilde{\pi}$ is a representation of $C^*(S)$ on H.

If the extension $\widetilde{\pi}$ of such a map π to $C^*(S)$ is an irreducible representation, then the extension is called a *boundary representation* for S.

The noncommutative approximation theory initiated by Arveson [2] benefited remarkably from the theory of boundary representations. The noncommutative analogue of classical Korovkin sets introduced by Arveson in [2] is as follows.

Definition 1.2. Let A be a C^*-algebra, and let $G \subseteq A$ (finite or countably infinite) be a set of generators of A (i.e., $A = C^*(G)$). Then G is said to be hyperrigid if, for every faithful representation $A \subseteq B(H)$ of A on a Hilbert space H and every sequence of unital completely positive maps $\phi_n : B(H) \to B(H)$, $n = 1, 2, \ldots,$

$$\lim_{n\to\infty} \big\|\phi_n(g) - g\big\| = 0, \quad \forall g \in G \quad \Longrightarrow \quad \lim_{n\to\infty} \big\|\phi_n(a) - a\big\| = 0, \quad \forall a \in A.$$

The following characterization of hyperrigid operator systems due to Arveson [2] is more of a workable definition of hyperrigidity of operator systems.

Theorem 1.3. *Let S be a separable operator system generating the C^*-algebra $A = C^*(S)$. Then S is hyperrigid if and only if every unital representation $\pi : A \to B(H)$ on a separable Hilbert space $\pi_{|S}$ has the unique extension property.*

In this context it is relevant to mention the "hyperrigidity conjecture" posed by Arveson [2]. The hyperrigidity conjecture states that if every irreducible representation of a C^*-algebra A is a boundary representation for a separable operator system $S \subseteq A$ and $A = C^*(S)$, then S is hyperrigid. Arveson [2] proved the conjecture for C^*-algebras having a countable spectrum, while Kleski [8] established the conjecture for all type-I C^*-algebras with some additional assumptions. Recently Davidson and Kennedy [4] proved the conjecture for function systems.

Using the obvious correspondence between representations and modules, one can translate many aspects of the above notions in terms of Hilbert modules. Muhly and Solel [9] gave an algebraic characterization of boundary representations in terms of Hilbert modules. Following Muhly and Solel, the present authors in [10, Lemma 0] established a Hilbert module characterization for hyperrigidity of certain operator systems in a C^*-algebra.

We will consider tensor products of C^*-algebras in this article. Let $A_1 \otimes A_2$ denote the algebraic tensor product of A_1 and A_2. Let $A_1 \otimes_s A_2$ denote the closure of $A_1 \otimes A_2$ provided with the spatial norm which is the minimal C^*-norm on the tensor product of C^*-algebras. In what follows we will be considering spatial norm for tensor product of C^*-algebras. We know that if representations π_1 is nondegenerate on A_1 and π_2 is nondegenerate on A_2, then the representation $\pi_1 \otimes \pi_2$ is nondegenerate on $A_1 \otimes A_2$. Conversely, from [3, Theorem II.9.2.1] we can see that if π is a nondegenerate representation of $A_1 \otimes A_2$, then there are unique nondegenerate representations π_1 of A_1 and π_2 of A_2 such that $\pi = \pi_1 \otimes \pi_2$.

Tensor products of operator spaces (linear subspaces) of C^*-algebras and operator spaces of tensor products of C^*-algebras were explored by Hopenwasser earlier in [6] and [7] in order to study boundary representations. In [6] it was shown that, under certain conditions, boundary representations of an operator subspace of a C^*-algebra $A \otimes M_n(\mathbb{C})$ are parameterized by the boundary representations of an operator subspace of the C^*-algebra A which is given by the operator subspace in $A \otimes M_n(\mathbb{C})$. In [7] it was proved that if one of the C^*-algebras of the tensor product is a GCR algebra, then the boundary representations of the tensor product of C^*-algebras correspond to products of boundary representations. It is this later result by Hopenwasser which motivated our work and influenced us to use similar techniques.

2. Main results

In the following result, we investigate the relation between the hyperrigidity of the tensor product of two operator system in the tensor product C^*-algebra and the hyperrigidity of the individual operator systems in the respective C^*-algebras. The following result shows that the unique extension property of completely positive maps on operator systems carries over to the tensor products of those maps defined on the tensor products of operator systems.

Theorem 2.1. *Let S_1 and S_2 be operator systems generating C^*-algebras A_1 and A_2, respectively. Let $\pi_i : S_i \to B(H_i), i = 1, 2$ be unital completely positive maps. Then π_1 and π_2 have unique extension property if and only if the unital completely positive map $\pi_1 \otimes \pi_2 : S_1 \otimes S_2 \to B(H_1 \otimes H_2)$ has unique extension property for $S_1 \otimes S_2 \subseteq A_1 \otimes_s A_2$.*

Proof. Assume that $\pi_1 \otimes \pi_2$ has unique extension property, that is $\pi_1 \otimes \pi_2$ has unique completely positive extension $\widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2 : A_1 \otimes_s A_2 \to B(H_1 \otimes H_2)$ which is a representation of $A_1 \otimes_s A_2$. We will show that π_1 and π_2 have unique extension properties. On the contrary, assume that one of the factors, say π_1, does not have unique extension property. This means that there exist at least two extensions of π_1, a completely positive map $\phi_1 : A_1 \to B(H_1)$ and the representation $\widetilde{\pi}_1 : A_1 \to B(H_1)$ such that $\phi_1 \neq \widetilde{\pi}_1$ on A_1, but $\phi_1 = \widetilde{\pi}_1 = \pi_1$ on S_1. Using [3, Theorem II.9.7], we can see that the tensor product of two completely positive maps is completely positive. We have that $\phi_1 \otimes_s \widetilde{\pi}_2$ is a completely positive extension of $\pi_1 \otimes \pi_2$ on $S_1 \otimes S_2$, where $\widetilde{\pi}_2$ is a unique completely positive extension of π_2 on S_2. Hence $\phi_1 \otimes_s \widetilde{\pi}_2 \neq \widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2$ on $A_1 \otimes_s A_2$. This contradicts our assumption.

Conversely, assume that π_1 and π_2 have the unique extension property; that is, π_1 and π_2 have unique completely positive extensions $\widetilde{\pi}_1 : A_1 \to B(H_1)$ and $\widetilde{\pi}_2 : A_2 \to B(H_2)$, respectively, where $\widetilde{\pi}_1$ and $\widetilde{\pi}_2$ are representations of A_1 and A_2, respectively. We will show that $\pi_1 \otimes \pi_2$ has the unique extension property. We have that $\widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2 : A_1 \otimes_s A_2 \to B(H_1 \otimes H_2)$ is a representation and an extension of $\pi_1 \otimes \pi_2$ on $S_1 \otimes S_2$. It is enough to show that if $\phi : A_1 \otimes_s A_2 \to B(H_1 \otimes H_2)$ is a completely positive extension of $\pi_1 \otimes \pi_2$ on $S_1 \otimes S_2$, then $\phi = \widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2$ on $A_1 \otimes A_2$.

Let P be any rank 1 projection in $B(H_2)$. The map $a \to (1 \otimes P)\phi(a \otimes 1)(1 \otimes P)$ is completely positive on A_1, since the map is a composition of three completely positive maps. Let v be a unit vector in the range of P, and let K be the range of $1 \otimes P$. Define $U : H_1 \to K$ by $U(x) = x \otimes v$, $x \in H_1$, where U is a unitary map. Let $\hat{\pi} = U\widetilde{\pi}_1(a)U^*$, $a \in A_1$, and let $\hat{\pi}(a)$ be the restriction to K of $\widetilde{\pi}_1(a) \otimes P = (1 \otimes P)(\widetilde{\pi}_1(a) \otimes 1)(1 \otimes P)$. Since $\hat{\pi}$ is unitarily equivalent to $\widetilde{\pi}_1$, the representation $\hat{\pi}_{|S_1}$ has unique extension property. Let $\psi(a)$ be the restriction to K of $(1 \otimes P)\phi(a \otimes 1)(1 \otimes P)$, which implies that ψ is a completely positive map that agrees with $\hat{\pi}$ on S_1, and hence on all of A_1.

Let $x, y \in H_1$, and let $r \in H_2$. From the above paragraph we have, for any $a \in A_1$, $\langle \phi(a \otimes 1)(x \otimes r), y \otimes r\rangle = \langle (\widetilde{\pi}_1(a) \otimes 1)(x \otimes r), y \otimes r\rangle$. (We let P be the rank 1 projection on the subspace spanned by r.) Let $D = \phi(a \otimes 1) - \widetilde{\pi}_1 \otimes 1$. Then we have $\langle D(x \otimes r), y \otimes r\rangle = 0$, for all $x, y \in H_1$, $r \in H_2$, using polarization formula

$$
\begin{aligned}
4\big\langle D(x \otimes r), y \otimes s\big\rangle = {} & \big\langle D\big(x \otimes (r+s)\big), y \otimes (r+s)\big\rangle \\
& - \big\langle D\big(x \otimes (r-s)\big), y \otimes (r-s)\big\rangle \\
& + i\big\langle D\big(x \otimes (r+is)\big), y \otimes (r+is)\big\rangle \\
& - i\big\langle D\big(x \otimes (r-is)\big), y \otimes (r-is)\big\rangle.
\end{aligned}
$$

We have $\langle D(x \otimes r), y \otimes s\rangle = 0$ for all $x, y \in H_1$ and for all $r, s \in H_2$. Consequently, if $z_1 = \sum_{i=1}^n x_i \otimes r_i$ and $z_2 = \sum_{i=1}^m y_i \otimes s_i$, then $\langle Dz_1, z_2\rangle = 0$. Since z_1, z_2 run through a dense subset of $H_1 \otimes H_2$ and since D is bounded, $D = 0$. Therefore, $\phi(a \otimes 1) = \widetilde{\pi}_1(a) \otimes 1$ for all $a \in A_1$. In the same way we can obtain $\phi(1 \otimes b) = 1 \otimes \widetilde{\pi}_2(b)$ for all $b \in A_2$. Since ϕ is a completely positive map on $A_1 \otimes A_2$ and $\phi(1 \otimes b) = 1 \otimes \widetilde{\pi}_2(b)$ for all $b \in A_2$, using a multiplicative domain argument (e.g., see [7, Lemma 2]), we have

$$
\phi(a \otimes b) = \phi(a \otimes 1)\big(1 \otimes \widetilde{\pi}_2(b)\big) = \big(1 \otimes \widetilde{\pi}_2(b)\big)\phi(a \otimes 1)
$$

for all $a \in A_1$, $b \in A_2$. Also $\phi(a \otimes 1) = \widetilde{\pi}_1(a) \otimes 1$ for all $a \in A_1$. Hence $\phi = \widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2$ on $A_1 \otimes_s A_2$. □

Corollary 2.2. *Let S_1 and S_2 be separable operator systems generating C^*-algebras A_1 and A_2, respectively. Assume that either A_1 or A_2 is a GCR algebra. Then S_1 and S_2 are hyperrigid in A_1 and A_2, respectively, if and only if $S_1 \otimes S_2$ is hyperrigid in $A_1 \otimes_s A_2$.*

Proof. Assume that $S_1 \otimes S_2$ is hyperrigid in the C^*-algebra $A_1 \otimes_s A_2$. By Theorem 1.3, every unital representation $\pi : A_1 \otimes_s A_2 \to B(H_1 \otimes H_2)$, $\pi_{|S_1 \otimes S_2}$ has unique extension property. We have that, if π is a unital representation of $A_1 \otimes_s A_2$, since one of the C^*-algebras is GCR, then by [5, Proposition 2] there are unique unital representations π_1 of A_1 and π_2 of A_2 such that $\pi = \pi_1 \otimes_s \pi_2$. Using Theorem 2.1, we can see that $\pi_{1|S_1}$ and $\pi_{2|S_2}$ have unique extension property. This implies that S_1 and S_2 are hyperrigid in A_1 and A_2, respectively, again by Theorem 1.3.

Conversely, assume that S_1 is hyperrigid in A_1 and that S_2 is hyperrigid in A_2. By Theorem 1.3, for every unital representations $\pi_1 : A_1 \to B(H_1)$ and $\pi_2 : A_2 \to B(H_2)$, $\pi_{1|S_1}$ and $\pi_{2|S_2}$ have unique extension property. We have, if π_1 and π_2 are unital representations of A_1 and A_2, respectively, that $\pi_1 \otimes_s \pi_2$ is an unital representation of $A_1 \otimes_s A_2$. Using Theorem 2.1, we can see that $\pi_1 \otimes_s \pi_{2|S_1 \otimes S_2}$ has unique extension property. Now, by Theorem 1.3, $S_1 \otimes S_2$ is hyperrigid in $A_1 \otimes_s A_2$. □

Let $A_1 \otimes_m A_2$ denote the closure of $A_1 \otimes A_2$ provided with maximal C^*-norm. There are C^*-algebras A_1 for which the minimal and the maximal norm on $A_1 \otimes A_2$ coincide for all C^*-algebras A_2, and consequently the C^*-norm on $A_1 \otimes A_2$ is unique. Such C^*-algebras are called *nuclear*. Clearly, the spatial norm assumption in the above result is redundant if the C^*-algebras are nuclear. But general C^*-algebras lacking the injectivity associated with other C^*-norms, including the maximal one, will require additional assumptions.

Let A_1 and A_2 be C^*-algebras, and let γ be any C^*-cross norm on $A_1 \otimes A_2$. If π_1 and π_2 are irreducible representations of A_1 and A_2, respectively, then $\pi_1 \otimes_\gamma \pi_2$ is an irreducible representation of $A_1 \otimes_\gamma A_2$. Conversely, every irreducible representation π on $A_1 \otimes_\gamma A_2$ need not factor as a product $\pi_1 \otimes_\gamma \pi_2$ of irreducible representations. If we assume one of the C^*-algebras is a GCR algebra, then by [5, Proposition 2] every irreducible representation factors. Since GCR algebras are nuclear, there is a unique C^*-cross norm on $A_1 \otimes A_2$, which we denote by $A_1 \otimes_\gamma A_2$.

Using the above facts, the result by Hopenwasser [7] relating boundary representations of tensor products of C^*-algebras will become a corollary to our Theorem 2.1.

Corollary 2.3. *Let S_1 and S_2 be unital operator subspaces of generating C^*-algebras A_1 and A_2, respectively. Assume that either A_1 or A_2 is a GCR algebra. Then the representation $\pi_1 \otimes_\gamma \pi_2$ of $A_1 \otimes_\gamma A_2$ is a boundary representation for $S_1 \otimes S_2$ if and only if the representations π_1 of A_1 and π_2 of A_2 are boundary representations for S_1 and S_2, respectively.*

Now, we will provide some examples which illustrate the results above.

Example 2.4. Let $G = \text{linear span}(I, S, S^*)$, where S is the unilateral right shift in $B(H)$ and I is the identity operator. Let $A = C^*(G)$ be the C^*-algebra generated by G. We have that $K(H) \subseteq A$, $A/K(H) \cong C(\mathbb{T})$ is commutative, where $\mathbb{T}$ denotes the unit circle in $\mathbb{C}$. Let Id denote the identity representation of the C^*-algebra A. Let $S^*\mathrm{Id}(\cdot)S$ be a completely positive map on the C^*-algebra A such that $S^*\mathrm{Id}S_{|_G} = \mathrm{Id}_{|_G}$; it is easy to see that $S^*\mathrm{Id}S_{|_A} \neq \mathrm{Id}_{|_A}$. Therefore, the unital representation $\mathrm{Id}_{|_G}$ does not have unique extension property. Using Theorem 1.3, we conclude that G is not a hyperrigid operator system in a C^*-algebra A.

Let $G_1 = G$, let $A_1 = A$, and let Id_1 denote the identity representation of A_1. Let $G_2 = A_2 = M_n(\mathbb{C})$, and let Id_2 denote the identity representation of the C^*-algebra A_2. The completely positive map $S^*\mathrm{Id}_1 S \otimes \mathrm{Id}_2$ on the C^*-algebra $A_1 \otimes A_2$ is such that $S^*\mathrm{Id}_1 S \otimes \mathrm{Id}_2 = \mathrm{Id}_1 \otimes \mathrm{Id}_2$ on operator system $G_1 \otimes G_2$. By the above conclusion, we see that $S^*\mathrm{Id}_1 S \otimes \mathrm{Id}_2 \neq \mathrm{Id}_1 \otimes \mathrm{Id}_2$ on the C^*-algebra $A_1 \otimes A_2$. Therefore, the unital representation $\mathrm{Id}_1 \otimes \mathrm{Id}_2$ does not have unique extension property for $G_1 \otimes G_2$. Hence by Theorem 1.3, $G_1 \otimes G_2$ is not a hyperrigid operator system in a C^*-algebra $A_1 \otimes A_2$.

Example 2.5. Let the Volterra integration operator V acting on the Hilbert space $H = L^2[0,1]$ be given by

$$Vf(x) = \int_0^x f(t)\,dt, \quad f \in L^2[0,1].$$

Note that V generates the C^*-algebra $K = K(H)$ of all compact operators. Let $S = \text{linear span}(V, V^*, V^2, V^{2*})$ and let S be hyperrigid (see [2, Theorem 1.7]). Then $\tilde{S} = S + \mathbb{C} \cdot \mathbf{1}$ is a hyperrigid operator system generating the C^*-algebra $\tilde{A} = K + \mathbb{C} \cdot \mathbf{1}$. Let $S_1 = S_2 = \tilde{S}$ and let $A_1 = A_2 = \tilde{A}$. We know that S_1 and S_2 are hyperrigid operator systems in the C^*-algebra A_1 and A_2, respectively. By Corollary 2.2, we conclude that $S_1 \otimes S_2$ is hyperrigid operator system in the C^*-algebra $A_1 \otimes A_2$.

Example 2.6. Let $G = \text{linear span}(I, S, S^*, SS^*)$, where S is the unilateral right shift in $B(H)$ and I is the identity operator. Let $A = C^*(G)$ be the C^*-algebra generated by the operator system G. We have that $K(H) \subseteq A$; also, $A/K(H) \cong C(\mathbb{T})$ is commutative, where $\mathbb{T}$ denotes the unit circle in $\mathbb{C}$. Since S is an isometry, G is a hyperrigid operator system (see [2, Theorem 3.3]) in the C^*-algebra A. Let $G_1 = G$, let $A_1 = A$, and let $G_2 = A_2 = M_n(\mathbb{C})$. It is clear that G_2 is a hyperrigid operator system in the C^*-algebra $A_2 = C^*(G_2)$. By Corollary 2.2, $G \otimes M_n(\mathbb{C})$ is a hyperrigid operator system in $A \otimes M_n(\mathbb{C})$.

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