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Research Article

Stability of Uncertain Impulsive Stochastic Genetic Regulatory Networks with Time-Varying Delay in the Leakage Term

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This paper is concerned with the stability problem for a class of uncertain impulsive stochastic genetic regulatory networks (UISGRNs) with time-varying delays both in the leakage term and in the regulator function. By constructing a suitable Lyapunov-Krasovskii functional which uses the information on the lower bound of the delay sufficiently, a delay-dependent stability criterion is derived for the proposed UISGRNs model by using the free-weighting matrices method and convex combination technique. The conditions obtained here are expressed in terms of LMIs whose feasibility can be checked easily by MATLAB LMI control toolbox. In addition, three numerical examples are given to justify the obtained stability results.

1. Introduction

Genetic regulatory networks (GRNs) which govern many essential functions of living cells have received much attention due to their extensive applications in many practical systems, especially in the biology, engineering, and other research fields [1–6]. That is why GRNs have become a hot topic of research recently. Several computational models have been applied to investigate the behaviours of GRNs: Petri net models [7–9], Bayesian network models [10–12], the Boolean models [13–15], the differential equation models [16–18], and so forth. In this paper, we will use differential equation models to encode genetic regulatory networks. The rate of change in concentration of a particular transcript is given by an influence function of other RNA concentrations.

Time delay is an interesting feature of signal transmission and becomes one of the main sources for causing divergence, instability, and poor performances for networks stability. So, it is important to consider the delay effects on the dynamical behavior of GRNs. Up to now, in almost all existing works on modeling GRNs [5, 19–21], time delay is included in the regulator function to describe the existing time delays peculiar to transcription, translation, and translocation processes in genetic networks. Chen and Aihara [5] firstly proposed a delay differential equation model for GRNs and studied its stability problem. In [19], Ren and Cao studied the asymptotic and robust stability of GRNs with time-varying delays. In [20], Zhang et al. investigated the stability analysis for GRNs with random discrete delays and distributed delays. Hu et al. [21] proposed a GRNs model with hybrid regulatory mechanism and studied its stability problem. Recently, Gopalsamy [22] put forward a neural network model with the incorporation of time delays in the leakage terms (i.e., negative feedback or decay terms which widely appeared in the models of neural networks, population dynamics, and GRNs). Along this line, a time delay will be taken into consideration in the decay terms of our GRNs model and we also call it "leakage delay."

When modeling the GRNs, stochastic disturbance should be taken into consideration since molecular noise plays important roles in biological functions of GRNs in practice. In [23, 24], the authors studied the model of GRNs with stochastic disturbances. Moreover, impulsive effects are also likely to exist in the genetic networks systems [25]. In [26], Li and Sun researched the stability of GRNs under impulsive control. On the other hand, it is well known that the stability of well-designed GRNs may often be destroyed by its unavoidable uncertainty in practice. In [27, 28], the authors investigated the stability for uncertain GRNs with interval time-varying delays. In [29, 30], the authors researched the stability problem of GRNs with stochastic disturbance and parameter uncertainties, simultaneously. In [31], Sakthivel et al. dealt with the asymptotic stability of delayed GRNs with both stochastic disturbance and impulsive effects. However, so far there has been very little published concerning the stability problem for GRNs with leakage delay, impulsive effects, stochastic disturbances, and parameter uncertainties, simultaneously.

Motivated by the above discussion, the stability analysis for UISGRNs with time-varying delays in the leakage term requires further consideration. By constructing a suitable Lyapunov-Krasovskii functional which uses the information on the lower bound of all the delays, the derived conditions are expressed in terms of LMIs whose feasibility can be easily checked by using numerically efficient MATLAB LMI control toolbox. It is believed that the result is meaningful and useful for the design and applications of UISGRNs. Finally, numerical examples are provided to show the usefulness of the derived LMI-based stability conditions.

Notations. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices. The superscript T denotes the transposition and the notation $X \geq Y$ (resp., $X > Y$), where X and Y are symmetric matrices, and it means that $X-Y$ is positive semidefinite (resp., positive definite). Diag$(\cdot)$ denotes the diagonal matrix, and col$\{\cdot\}$ means a column vector. In symmetric block matrices, we use an asterisk $(*)$ to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation and Preliminaries

In this paper, we consider the following model:

$$\begin{aligned}
dm_i(t) &= \big(-a_i m_i(t-d_1(t)) \\
&\quad + b_i\left(p_1(t-\sigma(t)), p_2(t-\sigma(t)), \ldots, p_n(t-\sigma(t))\right)\big)\,dt \\
&\quad + \eta_i(t, m_i(t-d_1(t)), p_i(t-\sigma(t)))\,d\omega(t), \\
&\qquad t \neq t_k, \\
\Delta m_i(t)\big|_{t=t_k} &= m_i(t_k) - m_i(t_k^-) = J_k(m_i(t_k^-)), \\
&\qquad k \in Z^+, \quad t = t_k, \\
dp_i(t) &= \left(-c_i p_i(t-d_2(t)) + l_i m_i(t-\tau(t))\right)dt \\
&\quad + \eta_i(t, m_i(t-\tau(t)), p_i(t-d_2(t)))\,d\omega(t), \\
&\qquad t \neq t_k, \\
\Delta p_i(t)\big|_{t=t_k} &= p_i(t_k) - p_i(t_k^-) = J_k(p_i(t_k^-)), \\
&\qquad k \in Z^+, \quad t = t_k \quad i = 1, 2, \ldots, n,
\end{aligned} \tag{1}$$

where $m_i(t)$, $p_i(t)$ are concentrations of mRNA and protein of the ith node at time t, respectively, a_i and c_i are positive real numbers that are the degradation rates of the mRNA and protein, l_i is a positive constant that represents the translation rate, and $b_i(\cdot)$ is the regulatory function of the ith gene. The first term in the first and third equations of the right side of (1) is called decay term and $d_i(t), i = 1, 2$, is called "leakage delay" as discussed in the Introduction. The regulatory function is of the form $b_i(p_1(t), p_2(t), \ldots, p_n(t)) = \sum_{j=1}^{n} b_{ij}(p_j(t))$, which is called SUM logic [32]. The stochastic disturbance $\omega(t)$ is one-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, P)$ with a natural filtration $\mathscr{F}_{t\geq 0}$ and $\eta_i(t, m_i(t-\tau(t)), p_i(t-d_2(t))) \in \mathbb{R}$ is the noise intensity.

The function $b_{ij}(p_j(t))$ is a monotonic function of the Hill form as follows:

$$b_{ij}(p_j(t)) = \begin{cases} \alpha_{ij} \dfrac{\left(p_j(t)/\beta_j\right)^{H_j}}{1+\left(p_j(t)/\beta_j\right)^{H_j}} & \text{if transcription factor } j \text{ is an activator of gene } i, \\ \alpha_{ij} \dfrac{1}{1+\left(p_j(t)/\beta_j\right)^{H_j}} & \text{if transcription factor } j \text{ is an repressor of gene } i, \end{cases} \tag{2}$$

where H_j is the Hill coefficient, β_j is a positive constant, and α_{ij} is the dimensionless transcriptional rate of transcription factor j to gene i, which is a bounded constant. Therefore, (1) can be rewritten into the following form:

$$\begin{aligned}
dm_i(t) &= \left(-a_i m_i(t-d_1(t)) + \sum_{j=1}^{n} v_{ij} h_j\left(p_j(t-\sigma(t))\right) + u_i\right)dt \\
&\quad + \eta_i(t, m_i(t-d_1(t)), p_i(t-\sigma(t)))\,d\omega(t), \\
&\qquad t \neq t_k, \\
\Delta m_i(t)\big|_{t=t_k} &= m_i(t_k) - m_i(t_k^-) = J_k(m_i(t_k^-)), \\
&\qquad k \in Z^+, \quad t = t_k, \\
dp_i(t) &= \left(-c_i p_i(t-d_2(t)) + l_i m_i(t-\tau(t))\right)dt \\
&\quad + \eta_i(t, m_i(t-\tau(t)), p_i(t-d_2(t)))\,d\omega(t), \\
&\qquad t \neq t_k,
\end{aligned}$$

$$\Delta p_i(t)\big|_{t=t_k} = p_i(t_k) - p_i(t_k^-) = J_k(p_i(t_k^-)),$$
$$k \in Z^+, \quad t = t_k, \quad i = 1, 2, \ldots, n, \tag{3}$$

where $h_j(x) = (x/\beta_j)^{H_j}/(1+(x/\beta_j)^{H_j})$, $u_i = \Sigma_{j\in I_i}\alpha_{ij}$ is defined as a basal rate, and I_i is the set of all the j which is a repressor of gene i. The matrix $W = (v_{ij}) \in \mathbb{R}^{n\times n}$ of the genetic network is defined as follows:

$$v_{ij} = \begin{cases} \alpha_{ij}, \\ \text{if transcription factor } j \text{ is an activator of gene } i, \\ 0, \\ \text{if there is no link from node } j \text{ to node } i, \\ -\alpha_{ij}, \\ \text{if transcription factor } j \text{ is a repressor of gene } i. \end{cases} \tag{4}$$

Rewriting system (3) into compact matrix form, we obtain

$$\begin{aligned} dm(t) = {} & (-Am(t-d_1(t)) + Wh(p(t-\sigma(t))) + u)\,dt \\ & + \eta(t, m(t-d_1(t)), p(t-\sigma(t)))\,d\omega(t), \\ & t \neq t_k, \\ \Delta m(t)|_{t=t_k} = {} & m(t_k) - m(t_k^-) = J_k(m(t_k^-)), \\ & k \in Z^+, \quad t = t_k, \\ dp(t) = {} & (-Cp(t-d_2(t)) + Lm(t-\tau(t)))\,dt \\ & + \eta(t, m(t-\tau(t)), p(t-d_2(t)))\,d\omega(t), \\ & t \neq t_k, \\ \Delta p(t)\big|_{t=t_k} = {} & p(t_k) - p(t_k^-) = J_k(p(t_k^-)), \\ & k \in Z^+, \quad t = t_k, \end{aligned} \tag{5}$$

where $A = \text{diag}(a_1, a_2, \ldots, a_n)$, $u = \text{col}\{u_1, u_2, \ldots, u_n\}$, $C = \text{diag}(c_1, c_2, \ldots, c_n)$, $L = \text{diag}(l_1, l_2, \ldots, l_n)$, $m(t) = \text{col}\{m_1(t), m_2(t), \ldots, m_n(t)\}$, $p(t) = \text{col}\{p_1(t), p_2(t), \ldots, p_n(t)\}$, $h(p(t)) = \text{col}\{h_1(p_1(t)), h_2(p_2(t)), \ldots, h_n(p_n(t))\}$, and $\eta(t, x, y) = \text{col}\{\eta_1(t, x, y), \eta_2(t, x, y), \ldots, \eta_n(t, x, y)\}$.

Let (m^*, p^*) be a nonnegative equilibrium point of the system (5). In the following, we will always shift the equilibrium point (m^*, p^*) to the origin by letting $x(t) = m(t) - m^*$, $y(t) = p(t) - p^*$. Hence, system (5) can be transformed into the following form:

$$\begin{aligned} dx(t) = {} & (-Ax(t-d_1(t)) + Wf(y(t-\sigma(t))))\,dt \\ & + \eta(t, x(t-d_1(t)), y(t-\sigma(t)))\,d\omega(t), \\ & t \neq t_k, \\ \Delta x(t)\big|_{t=t_k} = {} & x(t_k) - x(t_k^-) = J_k(x(t_k^-)), \\ & k \in Z^+, \quad t = t_k, \\ dy(t) = {} & (-Cy(t-d_2(t)) + Lx(t-\tau(t)))\,dt \\ & + \eta(t, x(t-\tau(t)), y(t-d_2(t)))\,d\omega(t), \\ & t \neq t_k, \\ \Delta y(t)\big|_{t=t_k} = {} & y(t_k) - y(t_k^-) = J_k(y(t_k^-)), \\ & k \in Z^+, \quad t = t_k, \end{aligned} \tag{6}$$

where $x(t-d_1(t)) = \text{col}\{x_1(t-d_1(t)), x_2(t-d_1(t)), \ldots, x_n(t-d_1(t))\} \in \mathbb{R}^n$, $y(t-d_2(t)) = \text{col}\{y_1(t-d_2(t)), y_2(t-d_2(t)), \ldots, y_n(t-d_2(t))\} \in \mathbb{R}^n$, $f(y(t)) = \text{col}\{f_1(y_1(t)), f_2(y_2(t)), \ldots, f_n(y_n(t))\} \in \mathbb{R}^n$, the function $f_j(y_j(t)) = h_j(y_j(t) + p_j^*) - h_j(p_j^*)$, and *obviously* $\mathbf{f_j(0) = 0}$.

Due to the fact that h_j is a monotonically increasing function with saturation, from the relationship of $f(\cdot)$ and $h(\cdot)$, we know that, for any $y_i \in R$,

$$\gamma_i \leq \frac{f_i(y_i)}{y_i} \leq \alpha_i, \quad i = 1, 2, \ldots, n, \tag{7}$$

where γ_i and α_i are known constant scalars.

Taking parameter uncertainties into the GRNs model (6), we consider the following UISGRNs model:

$$\begin{aligned} dx(t) = {} & (-(A+\Delta A)x(t-d_1(t)) + (W+\Delta W) \\ & \times f(y(t-\sigma(t))))\,dt \\ & + \eta(t, x(t-d_1(t)), y(t-\sigma(t)))\,d\omega(t), \\ & t \neq t_k, \\ \Delta x(t)|_{t=t_k} = {} & x(t_k) - x(t_k^-) = J_k(x(t_k^-)), \\ & k \in Z^+, \quad t = t_k, \\ dy(t) = {} & (-(C+\Delta C)y(t-d_2(t)) \\ & + (L+\Delta L)x(t-\tau(t)))\,dt \\ & + \eta(t, x(t-\tau(t)), y(t-d_2(t)))\,d\omega(t), \\ & t \neq t_k, \\ \Delta y(t)\big|_{t=t_k} = {} & y(t_k) - y(t_k^-) = J_k(y(t_k^-)), \\ & k \in Z^+, \quad t = t_k, \\ x_0 = x(\theta) = \varphi(\theta), & \quad y_0 = y(\theta) = \psi(\theta), \quad \forall\theta \in [-\varepsilon, 0], \end{aligned} \tag{8}$$

where $\psi(\cdot)$ and $\varphi(\cdot)$ are the initial function which are continuously differentiable on $[-\varepsilon, 0]$ with $\varepsilon = \max\{h_2, h_4, h_6, h_8\}$. We extend $\varrho(\theta)$ on $\theta \in [-2\varepsilon, 0]$ to satisfy $\|\varrho\|_\varepsilon = \|\varrho\|_{2\varepsilon}$ with $\|\varrho\|_\varepsilon = \sup_{\theta\in[-\varepsilon,0]}\|\varrho(\theta)\|$, $\|\varrho\|_{2\varepsilon} = \sup_{\theta\in[-2\varepsilon,0]}\|\varrho(\theta)\|$, where $\varrho = \{\psi, \varphi\}$.

Moreover, the noise intensity η satisfies

$$\begin{aligned}
&\operatorname{tr}\left[\eta^T\left(t, x\left(t-\tau\left(t\right)\right), y\left(t-d_2\left(t\right)\right)\right)\right.\\
&\quad\left.\times\,\eta\left(t, x\left(t-\tau\left(t\right)\right), y\left(t-d_2\left(t\right)\right)\right)\right]\\
&\leq x^T\left(t-\tau\left(t\right)\right)\Sigma_1^T\Sigma_1 x\left(t-\tau\left(t\right)\right)\\
&\quad+ y^T\left(t-d_2\left(t\right)\right)\Sigma_2^T\Sigma_2 y_2\left(t-d\left(t\right)\right),
\end{aligned} \tag{9}$$

where Σ_1 and Σ_2 are constant matrices with appropriate dimensions.

In order to obtain our main theorem, the following assumptions and lemmas for the system (8) are always made throughout this paper.

Assumption 1. The parametric uncertainties $\Delta A(t)$, $\Delta W(t)$, $\Delta C(t)$, and $\Delta L(t)$ satisfy

$$\begin{aligned}
\Delta A\left(t\right) &= G_1F_1\left(t\right)H_a\\
\Delta W\left(t\right) &= G_1F_1\left(t\right)H_w\\
\Delta C\left(t\right) &= G_2F_2\left(t\right)H_c\\
\Delta L\left(t\right) &= G_2F_2\left(t\right)H_l,
\end{aligned} \tag{10}$$

where G_1, G_2, H_a, H_w, H_c, and H_l are some given constant matrices with appropriate dimensions and $F_i(t)$ satisfies $F_i^T(t)F_i(t) \leq I$, $i = 1, 2$, for any $t > 0$.

Assumption 2. $d_1(t), d_2(t), \tau(t)$, and $\sigma(t)$ are the time-varying delays satisfying

$$\begin{aligned}
&0 \leq h_1 \leq d_1\left(t\right) \leq h_2, && 0 \leq \dot{d}_1\left(t\right) \leq d_1 < \infty,\\
&0 \leq h_3 \leq \sigma\left(t\right) \leq h_4, && 0 \leq \dot{\sigma}\left(t\right) \leq \sigma < \infty,\\
&0 \leq h_5 \leq d_2\left(t\right) \leq h_6, && 0 \leq \dot{d}_2\left(t\right) \leq d_2 < \infty,\\
&0 \leq h_7 \leq \tau\left(t\right) \leq h_8, && 0 \leq \dot{\tau}\left(t\right) \leq \tau < \infty,\\
&h_{12} = h_2 - h_1, && h_{34} = h_4 - h_3,\\
&h_{56} = h_6 - h_5, && h_{78} = h_8 - h_7.
\end{aligned} \tag{11}$$

Lemma 3 (Schur complement, see [30]). *For a given matrix*

$$\begin{pmatrix} Q\left(x\right) & S\left(x\right)\\ S^T\left(x\right) & R\left(x\right)\end{pmatrix} > 0, \tag{12}$$

where

$$Q\left(x\right) = Q^T\left(x\right), \qquad R\left(x\right) = R^T\left(x\right), \tag{13}$$

and a vector function $w(x) : [0, r] \rightarrow R^n$ such that the intergrals concerned as well defined, then the following holds:

(i) $Q(x) > 0$, $R(x) - S^T(x)Q(x)^{-1}S(x) > 0$,

(ii) $R(x) > 0$, $Q(x) - S(x)R(x)^{-1}S^T(x) > 0$.

Lemma 4 (see [33]). *For any constant symmetric matrix $M > 0$, scalar $\gamma > 0$,*

$$\left[\int_0^\gamma \varpi\left(s\right)ds\right]^T M\left[\int_0^\gamma \varpi\left(s\right)ds\right] \leq \gamma\int_0^\gamma \varpi^T\left(s\right)M\varpi\left(s\right)ds. \tag{14}$$

Lemma 5 (see [29]). *For any vectors $a, b \in \mathbb{R}^n$ and any positive matrix Y satisfying:*

$$\pm 2a^Tb \leq a^TYa + b^TY^{-1}b. \tag{15}$$

3. Main Result

In this section, mean square stability result for model (8) is summarized in the following theorem.

Theorem 6. *If* (7), (9), *and Assumptions 1 and 2 hold, there exist $\mu \geq 0$, $\lambda \geq 0$, $\rho_1 > 0$, $\rho_2 > 0$, $\boldsymbol{\chi_{im}} \in [\mathbf{0}, \mathbf{1}]$, $k = 0, 1, \ldots, r+2$, and $i = 1, \ldots, n, m \in Z^+$, such that the impulsive operator $J_m(\cdot)$ satisfies $J_{im}(x_i(t_m)) = -\chi_{im}x_i(t_m)$. The system* (8) *is stable in the mean square if there exist real matrices $P_1 > 0$, $P_2 > 0$, $Q_i > 0$ $(i = 1, 2, \ldots, 16)$, $Z_i > 0$ $(i = 1, 2, \ldots, 8)$, $V_1 > 0$, and $V_2 > 0$, diagonal matrices $Y_i > 0$ $(i = 1, 2, \ldots, 6)$, and any matrices N_{11}, N_{12}, N_{21}, N_{22}, M_{11}, M_{12}, M_{21}, M_{22}, M_{31}, M_{32}, S_{11}, S_{12}, S_{21}, S_{22}, S_{31}, S_{32}, E_{11}, E_{12}, E_{21}, E_{22}, E_{31}, and E_{32} to satisfy the following ten linear matrix inequalities:*

$$P_1 + V_1 < \rho_1 I, \tag{16}$$

$$P_2 + V_2 < \rho_2 I, \tag{17}$$

$$\phi_1 = \begin{bmatrix}\Xi & h_{12}N_2\\ * & -h_{12}Z_2\end{bmatrix} < 0, \tag{18}$$

$$\phi_2 = \begin{bmatrix}\Xi & h_{12}N_3\\ * & -h_{12}Z_2\end{bmatrix} < 0, \tag{19}$$

$$\phi_3 = \begin{bmatrix}\Xi & h_{34}M_2\\ * & -h_{34}Z_4\end{bmatrix} < 0, \tag{20}$$

$$\phi_4 = \begin{bmatrix}\Xi & h_{34}M_3\\ * & -h_{34}Z_4\end{bmatrix} < 0, \tag{21}$$

$$\phi_5 = \begin{bmatrix}\Xi & h_{56}S_2\\ * & -h_{56}Z_6\end{bmatrix} < 0, \tag{22}$$

$$\phi_6 = \begin{bmatrix}\Xi & h_{56}S_3\\ * & -h_{56}Z_6\end{bmatrix} < 0, \tag{23}$$

$$\phi_7 = \begin{bmatrix}\Xi & h_{78}E_2\\ * & -h_{78}Z_8\end{bmatrix} < 0, \tag{24}$$

$$\phi_8 = \begin{bmatrix}\Xi & h_{78}E_3\\ * & -h_{78}Z_8\end{bmatrix} < 0, \tag{25}$$

where

$$\Xi = \frac{1}{4}\begin{bmatrix} \phi & T_1X & T_2Y & h_1N_1 & h_3M_1 & h_5S_1 & h_7E_1 \\ * & -X & 0 & 0 & 0 & 0 & 0 \\ * & 0 & -Y & 0 & 0 & 0 & 0 \\ * & 0 & 0 & -h_1Z_1 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & -h_3Z_3 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & h_5Z_5 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & -h_7Z_7 \end{bmatrix},$$

$$\phi = \begin{bmatrix} \phi_9 & \phi_{10} \\ * & \phi_{11} \end{bmatrix},$$

$$\phi_9 = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & N_{21} & -N_{31} & \phi_{1,5} & E_{21} & E_{31} & S_{11}+M_{11} & \phi_{1,9} & M_{21} \\ * & \phi_{2,2} & N_{22} & -N_{32} & \phi_{2,5} & E_{22} & -E_{32} & M_{12}+S_{12} & \phi_{2,9} & M_{22} \\ * & * & \phi_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \phi_{5,5} & 0 & 0 & \phi_{5,8} & 0 & 0 \\ * & * & * & * & * & \phi_{6,6} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -Q_{16} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{8,8} & 0 & 0 \\ * & * & * & * & * & * & * & * & \phi_{9,9} & 0 \\ * & * & * & * & * & * & * & * & * & \phi_{10,10} \end{bmatrix},$$

$$\phi_{10} = \begin{bmatrix} -M_{31} & \phi_{1,12} & S_{12} & -S_{31} & \phi_{1,15} & 0 & \phi_{1,17} & 0 & 0 & 0 \\ -M_{32} & \phi_{2,12} & S_{22} & -S_{32} & \phi_{2,15} & 0 & 0 & \phi_{2,18} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_{5,16} & 0 & 0 & \phi_{5,19} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_{8,12} & 0 & 0 & 0 & \phi_{8,16} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_{9,17} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\phi_{11} = \begin{bmatrix} -Q_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \phi_{12,12} & 0 & 0 & 0 & \phi_{12,16} & 0 & 0 & 0 & \phi_{12,20} \\ * & * & \phi_{13,13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \phi_{15,15} & 0 & \phi_{15,17} & 0 & 0 & 0 \\ * & * & * & * & * & \phi_{16,16} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{17,17} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{18,18} & 0 & 0 \\ * & * & * & * & * & * & * & * & \phi_{19,19} & 0 \\ * & * & * & * & * & * & * & * & * & \phi_{20,20} \end{bmatrix},$$

$$J_{im}(x(t_m)) = (J_{1m}(x_1(t_m)), \dots, J_{nm}(x_n(t_m)))^T, \qquad \phi_{1,1} = Q_3 + Q_{15} + 2N_{11} + 2E_{11} - TY_6\Sigma,$$

$$\phi_{1,2} = -P_1A - P_1G_1F_1H_a - TV_2A - TV_2G_1F_1H_a - N_{11} + N_{12} + N_{13} - N_{21} + E_{12},$$

$$\phi_{1,5} = -E_{11} + E_{31} - E_{21}, \qquad \phi_{1,9} = -M_{11} + M_{31} - M_{21}, \qquad \phi_{1,12} = -S_{11} + S_{31} - S_{21},$$

$$\phi_{1,15} = \frac{1}{2}Y_6(T + \Sigma), \qquad \phi_{1,17} = -TV_2W - TV_2G_1F_1H_w + P_1W + P_1G_1F_1H_w,$$

$$\phi_{2,2} = -(1 - d_1)Q_1 - 2N_{12} - 2N_{22} + 2M_{32} + \rho_1\Sigma_1^T\Sigma_1 - TY_2\Sigma, \qquad \phi_{2,5} = -E_{12} + E_{32} - E_{22},$$

$$\phi_{2,9} = -M_{12} + M_{32} - M_{22}, \qquad \phi_{2,12} = -S_{12} + S_{32} - S_{22}, \qquad \phi_{2,15} = -V_2A - V_2G_1F_1H_a,$$

$$\phi_{2,18} = \frac{1}{2}Y_2(T + \Sigma), \qquad \phi_{3,3} = Q_1 + Q_4 - Q_3, \qquad \phi_{5,5} = -(1 - \tau)Q_{13} + \rho_2\Sigma_4\Sigma_4 - TY_5\Sigma,$$

$$
\begin{gathered}
\phi_{5,8} = -TV_1L - TV_1G_2F_2H_l + P_2L + P_2G_2F_2H_l, \qquad \phi_{5,16} = V_1L + V_1G_2F_2H_l, \\
\phi_{5,19} = \frac{1}{2}Y_5(T+\Sigma), \qquad \phi_{6,6} = Q_{13} + Q_{16} - Q_{15}, \qquad \phi_{8,8} = Q_7 + Q_{11} - TY_1\Sigma, \\
\phi_{8,12} = -P_2C - P_2G_2F_2H_c + TV_1C + TV_1G_2F_2H_c, \qquad \phi_{8,16} = \frac{1}{2}Y_1(T+\Sigma), \\
\phi_{9,9} = -(1-\sigma)Q_5 + \rho_1\Sigma_2\Sigma_2 - TY_3\Sigma, \qquad \phi_{9,17} = \frac{1}{2}Y_3(T+\Sigma), \qquad \phi_{10,10} = Q_5 - Q_7 + Q_8, \\
\phi_{12,12} = -\left(1-d_2\right)Q_9 + \rho_2\Sigma_3\Sigma_3 - TY_4\Sigma, \qquad \phi_{12,16} = -V_1C - V_1G_2F_2H_c, \\
\phi_{12,20} = \frac{1}{2}Y_4(T+\Sigma), \qquad \phi_{13,13} = Q_9 - Q_{11} + Q_{12}, \qquad \phi_{15,15} = Q_{14} - Y_6 + Q_2, \\
\phi_{15,17} = V_2W + V_2G_1F_1H_w, \qquad \phi_{16,16} = Q_6 - Y_1 + Q_{10}, \qquad \phi_{17,17} = -(1-\sigma)Q_6 - Y_3, \\
\phi_{18,18} = -\left(1-d_1\right)Q_2 - Y_2, \qquad \phi_{19,19} = -(1-\tau)Q_{15} - Y_5, \qquad \phi_{20,20} = -\left(1-d_2\right)Q_{10} - Y_4, \\
X = h_1Z_1 + h_{12}Z_2 + h_7Z_7 + h_{78}Z_8, \qquad Y = h_2Z_3 + h_{34}Z_4 + h_5Z_5 + h_{56}Z_6, \\
T_{11} = \begin{bmatrix} 0 & -A - G_1F_1H_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
T_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & W + G_1F_1H_w & 0 & 0 & 0 \end{bmatrix}, \\
T_{21} = \begin{bmatrix} 0 & 0 & 0 & L + G_2F_2H_l & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
T_{22} = \begin{bmatrix} 0 & -C - G_2H_2H_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
T_1 = \begin{bmatrix} T_{11} & T_{12} \end{bmatrix}^T, \qquad T_2 = \begin{bmatrix} T_{21} & T_{22} \end{bmatrix}^T, \\
N_1 = \begin{bmatrix} N_{11}^T & N_{12}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
N_2 = \begin{bmatrix} N_{21}^T & N_{22}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
N_3 = \begin{bmatrix} N_{31}^T & N_{32}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
S_1 = \begin{bmatrix} S_{11}^T & S_{12}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
S_2 = \begin{bmatrix} S_{21}^T & S_{22}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
S_3 = \begin{bmatrix} S_{31}^T & S_{32}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
M_1 = \begin{bmatrix} M_{11}^T & M_{12}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
M_2 = \begin{bmatrix} M_{21}^T & M_{22}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
M_3 = \begin{bmatrix} M_{31}^T & M_{32}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
E_1 = \begin{bmatrix} E_{11}^T & E_{12}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
E_2 = \begin{bmatrix} E_{21}^T & E_{22}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
E_3 = \begin{bmatrix} E_{31}^T & E_{32}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
T = \operatorname{diag}\left(\gamma_1, \gamma_2, \ldots, \gamma_n\right), \qquad \Sigma = \operatorname{diag}\left(\alpha_1, \alpha_2, \ldots, \alpha_n\right).
\end{gathered}
\tag{26}
$$

Proof. We consider the following Lyapunov functional candidate for system (8):

$$V(t,x_t,y_t)=V_1(t,x_t,y_t)+V_2(t,x_t,y_t)+V_3(t,x_t,y_t), \tag{27}$$

where

$$\begin{aligned}V_1(t,x_t,y_t)&=2\sum_{i=1}^{n}V_{1i}\int_0^{y_i}(f_i(s)-\gamma_i s)\,ds\\&\quad+2\sum_{i=1}^{n}V_{2i}\int_0^{x_i}(f_i(s)-\gamma_i s)\,ds,\end{aligned} \tag{28}$$

$$\begin{aligned}V_2(t,x_t,y_t)&=x^T(t)P_1x(t)+y^T(t)P_2y(t)\\&\quad+\int_{t-d_1(t)}^{t-h_1}x^T(s)Q_1x(s)\,ds\\&\quad+\int_{t-d_1(t)}^{t}f^T(x(s))Q_2f(x(s))\,ds\\&\quad+\int_{t-h_1}^{t}x^T(s)Q_3x(s)\,ds\\&\quad+\int_{t-h_2}^{t-h_1}x^T(s)Q_4x(s)\,ds\\&\quad+\int_{t-\sigma(t)}^{t-h_3}y^T(s)Q_5y(s)\,ds\\&\quad+\int_{t-\sigma(t)}^{t}f^T(y(s))Q_6f(y(s))\,ds\\&\quad+\int_{t-h_3}^{t}y^T(s)Q_7y(s)\,ds\\&\quad+\int_{t-h_4}^{t-h_3}y^T(s)Q_8y(s)\,ds\\&\quad+\int_{t-d_2(t)}^{t-h_5}y^T(s)Q_9y(s)\,ds\\&\quad+\int_{t-d_2(t)}^{t}f^T(y(s))Q_{10}f(y(s))\,ds\\&\quad+\int_{t-h_5}^{t}y^T(s)Q_{11}y(s)\,ds\\&\quad+\int_{t-h_6}^{t-h_5}y^T(s)Q_{12}y(s)\,ds\\&\quad+\int_{t-\tau(t)}^{t-h_7}x^T(s)Q_{13}x(s)\,ds\\&\quad+\int_{t-\tau(t)}^{t}f^T(x(s))Q_{14}f(x(s))\,ds\\&\quad+\int_{t-h_7}^{t}x^T(s)Q_{15}x(s)\,ds\\&\quad+\int_{t-h_8}^{t-h_7}x^T(s)Q_{16}x(s)\,ds,\end{aligned} \tag{29}$$

$$\begin{aligned}V_3(t,x_t,y_t)&=\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\,ds\,d\theta\\&\quad+\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}^T(s)Z_2\dot{x}(s)\,ds\,d\theta\\&\quad+\int_{-h_3}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)Z_3\dot{y}(s)\,ds\,d\theta\\&\quad+\int_{-h_4}^{-h_3}\int_{t+\theta}^{t}\dot{y}^T(s)Z_4\dot{y}(s)\,ds\,d\theta\\&\quad+\int_{-h_5}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)Z_5\dot{y}(s)\,ds\,d\theta\\&\quad+\int_{-h_6}^{-h_5}\int_{t+\theta}^{t}\dot{y}^T(s)Z_6\dot{y}(s)\,ds\,d\theta\\&\quad+\int_{-h_7}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_7\dot{x}(s)\,ds\,d\theta\\&\quad+\int_{-h_8}^{-h_7}\int_{t+\theta}^{t}\dot{x}^T(s)Z_8\dot{x}(s)\,ds\,d\theta.\end{aligned} \tag{30}$$

Then, by Itô's differential formula, taking the derivative of $V(t)$ along the trajectories of the system (8), we can obtain the following stochastic differential [29]:

$$\begin{aligned}dV(t)&=\mathcal{F}V(t)\,dt\\&\quad+2x^T(t)P_1\eta(t,x(t-d_1(t)),y(t-\sigma(t)))\\&\quad+2y^TP_2\eta(t,x(t-\tau(t)),y(t-d_2(t))),\end{aligned} \tag{31}$$

where $\mathcal{F}$ is the diffusion operator and

$$\begin{aligned}\mathcal{F}V(t,x_t,y_t)&=\mathcal{F}V_1(t,x_t,y_t)\\&\quad+\mathcal{F}V_2(t,x_t,y_t)+\mathcal{F}V_3(t,x_t,y_t),\end{aligned} \tag{32}$$

with

$$\begin{aligned}\mathcal{F}V_1(t,x_t,y_t)&=2\left[f^T(y(t))-y^T(t)\Gamma\right]\\&\quad\times V_1\left[-Cy(t-d_2(t))+Lx(t-\tau(t))\right.\\&\quad-G_2F_2H_cy(t-d_2(t))\\&\quad\left.+G_2F_2H_lx(t-\tau(t))\right]\end{aligned}$$

$$+\operatorname{tr}\left[\eta^T(t,x(t-\tau(t)),y(t-d_2(t)))\times V_1\eta(t,x(t-\tau(t)),y(t-d_2(t)))\right]$$
$$+2\left[f^T(x(t))-x^T(t)\Gamma\right]\times V_2\left[-Ax(t-d_1(t))+Wf(y(t-\sigma(t)))-G_1F_1H_ax(t-d_1(t))+G_1F_1H_wf(y(t-\sigma(t)))\right]$$
$$+\operatorname{tr}\left[\eta^T(t,y(t-\sigma(t)),y(t-d_1(t)))\times V_2\eta(t,y(t-\sigma(t)),y(t-d_1(t)))\right],$$

$$\begin{aligned}
&\mathscr{F}V_2(t,x_t,y_t)\\
&\le 2x^T(t)P_1\left[-Ax(t-d_1(t))+Wf(y(t-\sigma(t)))-G_1F_1H_ax(t-d_1(t))+G_1F_1H_wf(y(t-\sigma(t)))\right]\\
&\quad+\operatorname{tr}\left[\eta^T(t,y(t-\sigma(t)),y(t-d_1(t)))\times P_1\eta(t,y(t-\sigma(t)),y(t-d_1(t)))\right]\\
&\quad+2y^T(t)P_2\left[-Cy(t-d_2(t))+Lx(t-\tau(t))-G_2F_2H_cy(t-d_2(t))+G_2F_2H_lx(t-\tau(t))\right]\\
&\quad+\operatorname{tr}\left[\eta^T(t,y(t-\tau(t)),y(t-d_2(t)))\times P_2\eta(t,y(t-\tau(t)),y(t-d_2(t)))\right]\\
&\quad+x^T(t-h_1)Q_1x(t-h_1)-(1-d_1)x^T(t-d_1(t))Q_1x(t-d_1(t))\\
&\quad+f^T(x(s))Q_2f(x(s))-(1-d_1)f^T(y(t-d_1(t)))\times Q_2f(y(t-d_1(t)))\\
&\quad+x^T(t)Q_3x(t)-x^T(t-h_1)Q_3x(t-h_1)\\
&\quad+x^T(t-h_1)Q_4x(t-h_1)-x^T(t-h_2)Q_4x(t-h_2)\\
&\quad+y^T(t-h_5)Q_5x(t-h_5)-(1-\sigma)y^T(t-\sigma(t))Q_5y(t-\sigma(t))\\
&\quad+f^T(y(t))Q_6f(y(t))-(1-\sigma)f(y^T(t-\sigma(t)))\times Q_5f(y(t-\sigma(t)))\\
&\quad-y^T(t-h_3)Q_7y(t-h_3)+y^T(t)Q_7y(t)+y^T(t-h_3)Q_8y(t-h_3)\\
&\quad-y^T(t-h_4)Q_8y(t-h_4)+y^T(t-h_5)Q_9y(t-h_5)\\
&\quad-(1-d_2)y^T(t-d_2(t))Q_9y(t-d_2(t))\\
&\quad+f^T(y(t))Q_{10}f(y(t))-(1-d_2)f^T(y(t-d_2(t)))\times Q_{10}f(y(t-d_2(t)))\\
&\quad+y^TQ_{11}y(t)-y^T(t-h_5)Q_{11}y(t-h_5)\\
&\quad+y^T(t-h_5)Q_{12}y(t-h_5)-y^T(t-h_6)Q_{12}y(t-h_6)\\
&\quad+x^T(t-h_7)Q_{13}x(t-h_7)-(1-\tau)x^T(t-\tau(t))Q_{13}x(t-\tau(t))\\
&\quad+f^T(x(t))Q_{14}f(x(t))-(1-\tau)f^T(x(t-\tau(t)))\times Q_{14}f(x(t-\tau(t)))+x^T(t)Q_{15}x(t)\\
&\quad-x^T(t-h_7)Q_{15}x(t-h_7)+x^T(t-h_7)Q_{16}x(t-h_7)\\
&\quad-x^T(t-h_8)Q_{16}x(t-h_8),
\end{aligned}$$

$$\begin{aligned}
&\mathscr{F}V_3(t,x_t,y_t)\\
&=h_1\dot{x}^T(t)Z_1\dot{x}(t)-\int_{t-h_1}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\,ds\\
&\quad+h_{12}\dot{x}^T(t)Z_2\dot{x}(t)-\int_{t-h_2}^{t-h_1}\dot{x}^T(s)Z_2\dot{x}(s)\,ds\\
&\quad+h_2\dot{y}^T(t)Z_3\dot{y}(t)-\int_{t-h_3}^{t}\dot{y}^T(s)Z_3\dot{y}(s)\,ds
\end{aligned}$$

$$\begin{aligned}
&+ h_{34}\dot{y}^T(t) Z_4 \dot{y}(t) \\
&- \int_{t-h_4}^{t-h_3} \dot{y}^T(s) Z_4 \dot{y}(s)\, ds \\
&+ h_5 \dot{y}^T(t) Z_5 \dot{y}(t) \\
&- \int_{t-h_5}^{t} \dot{y}^T(s) Z_5 \dot{y}(s)\, ds \\
&+ h_{56}\dot{y}^T(t) Z_6 \dot{y}(t) \\
&- \int_{t-h_6}^{t-h_5} \dot{y}^T(s) Z_6 \dot{y}(s)\, ds \\
&+ h_7 \dot{x}^T(t) Z_7 \dot{y}(t) \\
&- \int_{t-h_7}^{t} \dot{y}^T(s) Z_7 \dot{y}(s)\, ds \\
&+ h_{78}\dot{y}^T(s) Z_8 \dot{y}(s) \\
&- \int_{t-h_8}^{t-h_7} \dot{y}^T(t) Z_8 \dot{y}(t)\, ds.
\end{aligned} \tag{33}$$

By Newton-Leibnitz formula, we have that

$$\begin{gathered}
2\varepsilon^T(t) N_1 \left[x(t) - x(t-h_1) - \int_{t-h_1}^{t} \dot{x}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) N_2 \left[x(t-h_1) - x(t-d_1(t)) - \int_{t-d_1(t)}^{t-h_1} \dot{x}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) N_3 \left[x(t-d_1(t)) - x(t-h_2) - \int_{t-h_2}^{t-d_1(t)} \dot{x}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) M_1 \left[y(t) - y(t-\sigma(t)) - \int_{t-\sigma(t)}^{t} \dot{y}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) M_2 \left[y(t-h_3) - y(t-\sigma(t)) - \int_{t-\sigma(t)}^{t-h_3} \dot{y}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) M_3 \left[y(t-\sigma(t)) - y(t-h_4) - \int_{t-h_4}^{t-\sigma(t)} \dot{y}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) S_1 \left[y(t) - y(t-d_2(t)) - \int_{t-d_2(t)}^{t} \dot{y}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) S_2 \left[y(t-h_5) - y(t-d_2(t)) - \int_{t-d_2(t)}^{t-h_5} \dot{y}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) S_3 \left[y(t-d_2(t)) - y(t-h_6) - \int_{t-h_6}^{t-d_2(t)} \dot{y}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) E_1 \left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) E_2 \left[x(t-h_7) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-h_7} \dot{x}(s)\, ds \right] = 0, \\
2\varepsilon^T(t) E_3 \left[x(t-h_8) - x(t-h_7) - \int_{t-h_8}^{t-h_7} \dot{x}(s)\, ds \right] = 0,
\end{gathered} \tag{34}$$

where

$$\begin{aligned}
\varepsilon(t) &= \left[\varepsilon_1(t), \varepsilon_2(t), \varepsilon_3(t), \varepsilon_4(t)\right]^T, \\
\varepsilon_1(t) &= \left[x^T(t), x^T(t-d_1(t)), x^T(t-h_1), \right. \\
&\qquad \left. x^T(t-h_2), x^T(t-\tau(t))\right], \\
\varepsilon_2(t) &= \left[x^T(t-h_7), x^T(t-h_8), y^T(t), \right. \\
&\qquad \left. y^T(t-\sigma(t)), y^T(t-h_3)\right], \\
\varepsilon_3(t) &= \left[y^T(t-h_4), y^T(t-d_2(t)),\ y^T(t-h_5), \right. \\
&\qquad \left. y^T(t-h_6), f^T(x)\right], \\
\varepsilon_4(t) &= \left[f^T(y), f^T(y-\sigma(t)), f^T(x-d_1(t)), \right. \\
&\qquad \left. f^T(x-\tau(t)), f^T(y-d_2(t))\right].
\end{aligned} \tag{35}$$

By using Lemmas 4 and 5, we have

$$\begin{aligned}
&-2\varepsilon^T(t) N_1 \int_{t-h_1}^{t} \dot{x}(s)\, ds \\
&\quad \le h_1 \varepsilon^T(t) N_1 Z_1^{-1} N_1^T \varepsilon(t) \\
&\qquad + \int_{t-h_1}^{t} \dot{x}^T(s) Z_1 \dot{x}(s)\, ds, \\
&\quad -2\varepsilon^T(t) N_2 \int_{t-d_1(t)}^{t-h_1} \dot{x}(s)\, ds \\
&\quad \le (d_1(t) - h_1) \varepsilon^T(t) N_2 Z_2^{-1} N_2^T \varepsilon(t) \\
&\qquad + \int_{t-d_1(t)}^{t-h_1} \dot{x}^T(s) Z_2 \dot{x}(s)\, ds, \\
&-2\varepsilon^T(t) N_3 \int_{t-h_2}^{t-d_1(t)} \dot{x}(s)\, ds \\
&\quad \le (h_2 - d_1(t)) \varepsilon^T(t) N_3 Z_2^{-1} N_3^T \varepsilon(t) \\
&\qquad + \int_{t-h_2}^{t-d_1(t)} \dot{x}^T(s) Z_2 \dot{x}(s)\, ds, \\
&-2\varepsilon^T(t) M_1 \int_{t-h_3}^{t} \dot{y}(s)\, ds
\end{aligned}$$

$$
\begin{aligned}
&\leq h_3 \varepsilon^T(t) M_1 Z_3^{-1} M_1^T \varepsilon(t) \\
&\quad + \int_{t-h_3}^{t} \dot{y}^T(s) Z_3 \dot{y}(s)\, ds, \\
&- 2\varepsilon^T(t) M_2 \int_{t-\sigma(t)}^{t-h_3} \dot{y}(s)\, ds \\
&\leq (\sigma(t) - h_3) \varepsilon^T(t) M_2 Z_4^{-1} M_2^T \varepsilon(t) \\
&\quad + \int_{t-\sigma(t)}^{t-h_3} \dot{y}^T(s) Z_4 \dot{y}(s)\, ds, \\
&- 2\varepsilon^T(t) M_3 \int_{t-h_4}^{t-\sigma(t)} \dot{y}(s)\, ds \\
&\leq (h_4 - \sigma(t)) \varepsilon^T(t) M_3 Z_4^{-1} M_3^T \varepsilon(t) \\
&\quad + \int_{t-h_4}^{t-\sigma(t)} \dot{y}^T(s) Z_4 \dot{y}(s)\, ds, \\
&- 2\varepsilon^T(t) S_1 \int_{t-h_5}^{t} \dot{y}(s)\, ds \\
&\leq h_5 \varepsilon^T(t) S_1 Z_5^{-1} S_1^T \varepsilon(t) \\
&\quad + \int_{t-h_5}^{t} \dot{y}^T(s) Z_5 \dot{y}(s)\, ds, \\
&- 2\varepsilon^T(t) S_2 \int_{t-d_2(t)}^{t-h_5} \dot{y}(s)\, ds \\
&\leq (d_2(t) - h_5) \varepsilon^T(t) S_2 Z_6^{-1} S_2^T \varepsilon(t) \\
&\quad + \int_{t-d_2(t)}^{t-h_5} \dot{y}^T(s) Z_6 \dot{y}(s)\, ds, \\
&- 2\varepsilon^T(t) S_3 \int_{t-h_6}^{t-d_2(t)} \dot{y}(s)\, ds \\
&\leq (h_6 - d_2(t)) \varepsilon^T(t) S_3 Z_6^{-1} S_3^T \varepsilon(t) \\
&\quad + \int_{t-h_6}^{t-d_2(t)} \dot{y}^T(s) Z_6 \dot{y}(s)\, ds, \\
&- 2\varepsilon^T(t) E_1 \int_{t-h_7}^{t} \dot{x}(s)\, ds \\
&\leq h_7 \varepsilon^T(t) E_1 Z_7^{-1} E_1^T \varepsilon(t) \\
&\quad + \int_{t-h_7}^{t} \dot{x}^T(s) Z_7 \dot{x}(s)\, ds, \\
&- 2\varepsilon^T(t) E_2 \int_{t-\tau(t)}^{t-h_7} \dot{x}(s)\, ds \\
&\leq (\tau(t) - h_7) \varepsilon^T(t) E_2 Z_8^{-1} E_2^T \varepsilon(t) \\
&\quad + \int_{t-\tau(t)}^{t-h_7} \dot{x}^T(s) Z_8 \dot{x}(s)\, ds, \\
&- 2\varepsilon^T(t) E_3 \int_{t-h_8}^{t-\tau(t)} \dot{x}(s)\, ds \\
&\leq (h_8 - \tau(t)) \varepsilon^T(t) E_3 Z_8^{-1} E_3^T \varepsilon(t) \\
&\quad + \int_{t-h_8}^{t-\tau(t)} \dot{x}^T(s) Z_8 \dot{x}(s)\, ds.
\end{aligned}
\tag{36}
$$

It follows from (7) that

$$
\begin{aligned}
0 &= f^T(y(t)) Y_1 f(y(t)) - f^T(y(t)) Y_1 f(y(t)) \\
&\leq -y^T(t) T Y_1 \Sigma y(t) + y^T(t) Y_1 (T + \Sigma) f(y(t)) \\
&\quad - f^T(y(t)) Y_1 f(y(t)), \\
0 &= f^T(y(t - d_1(t))) Y_2 f(y(t - d_1(t))) \\
&\quad - f^T(y(t - d_1(t))) Y_2 f(y(t - d_1(t))) \\
&\leq -y^T(t - d_1(t)) T Y_2 \Sigma y(t - d_1(t)) \\
&\quad + y^T(t - d_1(t)) Y_2 (T + \Sigma) f(y(t - d_1(t))) \\
&\quad - f^T(y(t - d_1(t))) Y_2 f(y(t - d_1(t))), \\
0 &= f^T(y(t - \sigma(t))) Y_3 f(y(t - \sigma(t))) \\
&\quad - f^T(y(t - \sigma(t))) Y_3 f(y(t - \sigma(t))) \\
&\leq -y^T(t - \sigma(t)) T Y_3 \Sigma y(t - \sigma(t)) \\
&\quad + y^T(t - \sigma(t)) Y_3 (T + \Sigma) f(y(t - \sigma(t))) \\
&\quad - f^T(y(t - \sigma(t))) Y_3 f(y(t - \sigma(t))), \\
0 &= f^T(y(t - d_2(t))) Y_4 f(y(t - d_2(t))) \\
&\quad - f^T(y(t - d_2(t))) Y_4 f(y(t - d_2(t))) \\
&\leq -y^T(t - d_2(t)) T Y_2 \Sigma y(t - d_2(t)) \\
&\quad + y^T(t - d_2(t)) Y_2 (T + \Sigma) f(y(t - d_2(t))) \\
&\quad - f^T(y(t - d_2(t))) Y_2 f(y(t - d_2(t))), \\
0 &= f^T(y(t - \tau(t))) Y_5 f(y(t - \tau(t))) \\
&\quad - f^T(y(t - \tau(t))) Y_5 f(y(t - \tau(t))) \\
&\leq -y^T(t - \tau(t)) T Y_5 \Sigma y(t - \tau(t)) \\
&\quad + y^T(t - \tau(t)) Y_5 (T + \Sigma) f(y(t - \tau(t))) \\
&\quad - f^T(y(t - \tau(t))) Y_5 f(y(t - \tau(t))), \\
0 &= f^T(x(t)) Y_6 f(x(t)) - f^T(x(t)) Y_6 f(x(t)) \\
&\leq -x^T(t) T Y_6 \Sigma x(t) + x^T(t) Y_6 (T + \Sigma) f(x(t)) \\
&\quad - f^T(x(t)) Y_6 f(x(t)).
\end{aligned}
\tag{37}
$$

Next, it follows from (9) and (27) that

$$\begin{aligned}
&\operatorname{tr}\left[\eta^{T}\left(t, y\left(t-\sigma\left(t\right)\right), x\left(t-d_{1}\left(t\right)\right)\right)\right.\\
&\qquad\left.\times\left(P_{1}+V_{1}\right)\eta\left(t, y\left(t-\sigma\left(t\right)\right), x\left(t-d_{1}\left(t\right)\right)\right)\right]\\
&\leq \lambda_{\max}\left(P_{1}+V_{1}\right)\left[\eta^{T}\left(t, y\left(t-\sigma\left(t\right)\right), x\left(t-d_{1}\left(t\right)\right)\right)\right.\\
&\qquad\left.\times\eta\left(t, y\left(t-\sigma\left(t\right)\right), x\left(t-d_{1}\left(t\right)\right)\right)\right]\\
&\leq y^{T}\left(t-\sigma\left(t\right)\right)\rho_{1}\Sigma_{1}^{T}\Sigma_{1}y\left(t-\sigma\left(t\right)\right)\\
&\quad+x^{T}\left(t-d_{1}\left(t\right)\right)\rho_{1}\Sigma_{2}^{T}\Sigma_{2}x\left(t-d_{1}\left(t\right)\right),\\
&\operatorname{tr}\left[\eta^{T}\left(t, y\left(t-d_{2}\left(t\right)\right), x\left(t-\tau\left(t\right)\right)\right)\right.\\
&\qquad\left.\times\left(P_{2}+V_{2}\right)\eta\left(t, y\left(t-d_{2}\left(t\right)\right), x\left(t-\tau\left(t\right)\right)\right)\right]\\
&\leq \lambda_{\max}\left(P_{2}+V_{2}\right)\left[\eta^{T}\left(t, y\left(t-d_{2}\left(t\right)\right), x\left(t-\tau\left(t\right)\right)\right)\right.\\
&\qquad\left.\times\eta\left(t, y\left(t-d_{2}\left(t\right)\right), x\left(t-\tau\left(t\right)\right)\right)\right]\\
&\leq x^{T}\left(t-\tau\left(t\right)\right)\rho_{2}\Sigma_{4}^{T}\Sigma_{4}x\left(t-\tau\left(t\right)\right)\\
&\quad+y^{T}\left(t-d_{2}\left(t\right)\right)\rho_{2}\Sigma_{3}^{T}\Sigma_{3}y\left(t-d_{2}\left(t\right)\right).
\end{aligned}\tag{38}$$

Add both sides of (34) and (38) to both sides of (32) and apply (36)–(37); one can obtain that

$$\mathscr{F}V\left(t, x_{t}, y_{t}\right)\leq\varepsilon^{T}\left(t\right)\Upsilon\varepsilon\left(t\right),\tag{39}$$

where $\Upsilon = \phi+T_1XT_1+T_2YT_2+h_1N_1Z_1^{-1}N_1^T+h_3M_1Z_3^{-1}M_1^T+h_5S_1Z_5^{-1}S_1^T+h_7E_1Z_7^{-1}E_1^T+\Theta$, with $\Theta = (d_1(t)-h_1)N_2Z_2^{-1}N_2^T+(h_2-d_1(t))N_3Z_2^{-1}N_3^T+(d_2(t)-h_5)S_2Z_6^{-1}S_2^T+(h_6-d_2(t))S_3Z_6^{-1}S_3^T+(\sigma(t)-h_3)M_2Z_4^{-1}M_2^T+(h_4-\sigma(t))M_3Z_4^{-1}M_3^T+(\tau(t)-h_7)E_2Z_8^{-1}E_2^T+(h_8-\tau(t))E_3Z_8^{-1}E_3^T$.

Noting Assumption 2, Θ can be seen as the convex combination of $N_2Z_2^{-1}N_2^T$ and $N_3Z_2^{-1}N_3^T$ on $d_1(t)$, $M_2Z_4^{-1}M_2^T$ and $M_3Z_4^{-1}M_3^T$ on $\sigma(t)$, $S_2Z_6^{-1}S_2^T$ and $S_3Z_6^{-1}S_3^T$ on $d_2(t)$, and $E_2Z_8^{-1}E_2^T$ and $E_3Z_8^{-1}E_3^T$ on $\tau(t)$. Therefore, $\Upsilon<0$ holds if

$$\begin{gathered}
\Lambda+h_{12}N_2Z_2^{-1}N_2^T<0,\\
\Lambda+h_{12}N_3Z_2^{-1}N_3^T<0,\\
\Lambda+h_{34}M_2Z_4^{-1}M_2^T<0,\\
\Lambda+h_{34}M_3Z_4^{-1}M_3^T<0,\\
\Lambda+h_{56}S_2Z_6^{-1}S_2^T<0,\\
\Lambda+h_{56}S_3Z_6^{-1}S_3^T<0,\\
\Lambda+h_{78}E_2Z_8^{-1}E_2^T<0,\\
\Lambda+h_{78}E_3Z_8^{-1}E_3^T<0,
\end{gathered}\tag{40}$$

where

$$\begin{aligned}
\Lambda=\frac{1}{4}&\left\{\phi+T_1XT_1+T_2YT_2+h_1N_1Z_1^{-1}N_1^T\right.\\
&\left.+h_3M_1Z_3^{-1}M_1^T+h_5S_1Z_5^{-1}S_1^T+h_7E_1Z_7^{-1}E_1^T\right\}.
\end{aligned}\tag{41}$$

By Schur complements, (40) is equivalent to $\phi_i<0$, $i=1,2,3,\ldots,8$, respectively. Then

$$\dot{V}\left(t, x_t, y_t\right)<0.\tag{42}$$

On the other hand, from (27) and Theorem 6 conditions, we note that

$$\begin{aligned}
\mathbf{V_1}\left(\mathbf{t_k}, \mathbf{x_t}, \mathbf{y_t}\right)&=2\sum_{i=1}^{n}\lambda_i\int_0^{y_i(t_k)}\left(f_i\left(s\right)-\gamma_i s\right)ds\\
&\quad+2\sum_{i=1}^{n}\lambda_i\int_0^{x_i(t_k)}\left(f_i\left(s\right)-\gamma_i s\right)ds\\
&=2\sum_{i=1}^{n}\lambda_i\int_0^{\{1-\chi_{ik}\}y_i(t_k^-)}\left(f_i\left(s\right)-\gamma_i s\right)ds\\
&\quad+2\sum_{i=1}^{n}\lambda_i\int_0^{\{1-\chi_{ik}\}x_i(t_k^-)}\left(f_i\left(s\right)-\gamma_i s\right)ds\\
&\leq 2\sum_{i=1}^{n}\lambda_i\int_0^{y_i(t_k^-)}\left(f_i\left(s\right)-\gamma_i s\right)ds\\
&\quad+2\sum_{i=1}^{n}\lambda_i\int_0^{x_i(t_k^-)}\left(f_i\left(s\right)-\gamma_i s\right)ds\\
&=V_1\left(t_k^-, x_t, y_t\right).
\end{aligned}\tag{43}$$

Moreover, it is obvious that $V_2(t_k,x_t,y_t)=V_2(t_k^-,x_t,y_t)$, $V_3(t_k,x_t,y_t)=V_3(t_k^-,x_t,y_t)$. Hence, we get $V(t_k,x_t,y_t)\leq V(t_k^-,x_t,y_t)$. By Lyapunov-Krasovskii stability theorem, the equilibrium point of (8) is stable in the mean square. The proof is completed. □

Remark 7. For the UISGRNs (8) without stochastic disturbances, leakage delay, and impulsive effects, it reduces to the model of [27]. And when the model of (8) without impulsive, it reduces to the model of [30]. In addition, it is easy to see that the main theorem obtained above covers the sparse results available in the literature in the concern of only one or two of the complex dynamics generally being involved with GRNs, leakage delays, parameter uncertainties, impulsive effects, and stochastic disturbances.

We give a couple of corollaries below in order to show further that our main result is general enough to cover two cases that have not been investigated in the literature. Hence, they are new and significant. Firstly, for model (6) or the UISGRNs (8) without parameter uncertainties (i.e., $\Delta A(t)=\Delta W(t)=\Delta C(t)=\Delta L(t)=0$), we have the following corollary.

Corollary 8. *If* (7), (9), *and Assumption 1 hold, there exist* $\mu \geq 0$, $\lambda \geq 0$, $\rho_1 > 0$, $\rho_2 > 0$, $\boldsymbol{\chi}_{\mathbf{im}} \in [\mathbf{0}, \mathbf{1}]$, $k = 0, 1, \ldots, r+2$, $i = 1, \ldots, n, m \in Z^+$, *such that the impulsive operator* $J_m(\cdot)$ *satisfies* $J_{im}(x_i(t_m)) = -\chi_{im} x_i(t_m)$. *The system* (6) *is stable in the mean square if there exist real matrices* $P_1 > 0$, $P_2 > 0$, $Q_i > 0$ *(*$i = 1, 2, \ldots, 16$*)*, $Z_i > 0$ *(*$i = 1, 2, \ldots, 8$*)*, $V_1 > 0$, *and* $V_2 > 0$, *diagonal matrices* $Y_i > 0$ *(*$i = 1, 2, \ldots, 6$*), and any matrices* N_{11}, N_{12}, N_{21}, N_{22}, M_{11}, M_{12}, M_{21}, M_{22}, M_{31}, M_{32}, S_{11}, S_{12}, S_{21}, S_{22}, S_{31}, S_{32}, E_{11}, E_{12}, E_{21}, E_{22}, E_{31}, *and* E_{32}, *to satisfy conditions* (16)–(25) *replaced accordingly by the following:*

$$\begin{gathered}
\phi_{1,2} = -P_1 A - T V_2 A - N_{11} + N_{12} + N_{13} - N_{21} + E_{12}, \\
\phi_{1,17} = -T V_2 W + P_1 W, \qquad \phi_{2,15} = -V_2 A, \\
\phi_{5,8} = -T V_1 L + P_2 L, \qquad \phi_{5,16} = V_1 L, \\
\phi_{8,12} = -P_2 C + T V_1 C, \qquad \phi_{12,16} = -V_1 C, \\
\phi_{15,17} = V_2 W, \\
T_1 = [0 \;\; -A \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; W \;\; 0 \;\; 0 \;\; 0]^T, \\
T_2 = [0 \;\; 0 \;\; 0 \;\; L \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -C \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T.
\end{gathered} \tag{44}$$

In the second case, we suppose that there are no stochastic disturbances in the UISGRNs model (8). Hence, model (8) can be reduced to

$$\begin{aligned}
\dot{x}(t) &= -(A + \Delta A)\, x(t - d_1(t)) \\
&\quad + (W + \Delta W)\, f(y(t - \sigma(t))) \\
\Delta x(t)|_{t=t_k} &= x(t_k) - x(t_k^-) = J_k(x(t_k^-)) \\
&\qquad k \in Z^+, \quad t = t_k, \\
\dot{y}(t) &= -(C + \Delta C)\, y(t - d_2(t)) + (L + \Delta L)\, x(t - \tau(t)) \\
\Delta y(t)|_{t=t_k} &= y(t_k) - y(t_k^-) = J_k(y(t_k^-)) \\
&\qquad t = t_k, \quad k \in Z^+, \\
x_0 &= x(\theta) = \varphi(\theta), \quad y_0 = y(\theta) = \psi(\theta), \\
&\qquad \forall \theta \in [-\omega, 0].
\end{aligned} \tag{45}$$

Then we have the following new result.

Corollary 9. *If* (7) *and Assumptions 1 and 2 hold, there exist* $\mu \geq 0$, $\lambda \geq 0$, $\boldsymbol{\chi}_{\mathbf{im}} \in [\mathbf{0}, \mathbf{1}]$, $k = 0, 1, \ldots, r+2$, $i = 1, \ldots, n, m \in Z^+$, *such that the impulsive operator* $J_m(\cdot)$ *satisfies* $J_{im}(x_i(t_m)) = -\chi_{im} x_i(t_m)$. *The system* (45) *is stable if there exist real matrices* $P_1 > 0$, $P_2 > 0$, $Q_i > 0$ *(*$i = 1, 2, \ldots, 16$*)*, $Z_i > 0$ *(*$i = 1, 2, \ldots, 8$*)*, $V_1 > 0$, *and* $V_2 > 0$, *diagonal matrices* $Y_i > 0$ *(*$i = 1, 2, \ldots, 6$*), and any matrices* N_{11}, N_{12}, N_{21}, N_{22}, M_{11}, M_{12}, M_{21}, M_{22}, M_{31}, M_{32}, S_{11}, S_{12}, S_{21}, S_{22}, S_{31}, S_{32}, E_{11}, E_{12}, E_{21}, E_{22}, E_{31}, *and* E_{32}, *to satisfy conditions* (18)–(25) *replaced accordingly by the following:*

$$\begin{gathered}
\phi_{2,2} = -(1 - d_1)\, Q_1 - 2N_{12} - 2N_{22} + 2M_{32}, \\
\phi_{5,5} = -(1 - \tau)\, Q_{13} - T Y_5 \Sigma, \\
\phi_{9,9} = -(1 - \sigma)\, Q_5 - T Y_3 \Sigma, \\
\phi_{12,12} = -(1 - d_2)\, Q_9 - T Y_4 \Sigma.
\end{gathered} \tag{46}$$

4. Numerical Examples

In this section, we present three numerical examples so as to illustrate the usefulness of our results derived in this paper.

Example 1. Let us firstly consider the system (8) with parameters as follows:

$$\begin{gathered}
C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} 1.1 & 0 \\ 0 & 1.1 \end{bmatrix}, \\
\mathbf{L} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, \qquad \mathbf{W} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, \\
\Sigma_1 = \Sigma_2 = \begin{bmatrix} 0.1667 & 0 \\ 0 & 0.1067 \end{bmatrix}, \\
I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{G_1} = 0.2 * I, \qquad \mathbf{G_2} = 0.2 * I, \\
\mathbf{H_a} = \mathbf{H_w} = \mathbf{H_c} = \mathbf{H_l} = 0.02 * I, \\
F_1 = 0.5 * I, \qquad F_2 = 0.6 * I.
\end{gathered} \tag{47}$$

For convenience, we assume that $\chi_{im} = 1/3$, $d_1(t) = 0.1 + 0.05\sin(t)$, $\sigma(t) = 0.2 + 0.03\cos(t)$, $d_2(t) = 0.3 + 0.01\cos(t)$, $\tau(t) = 0.4 + 0.05\cos(t)$, and $\mathbf{f_1(x)} = \mathbf{f_2(x)} = \mathbf{1/(1 + x^2)}$; then we can obtain

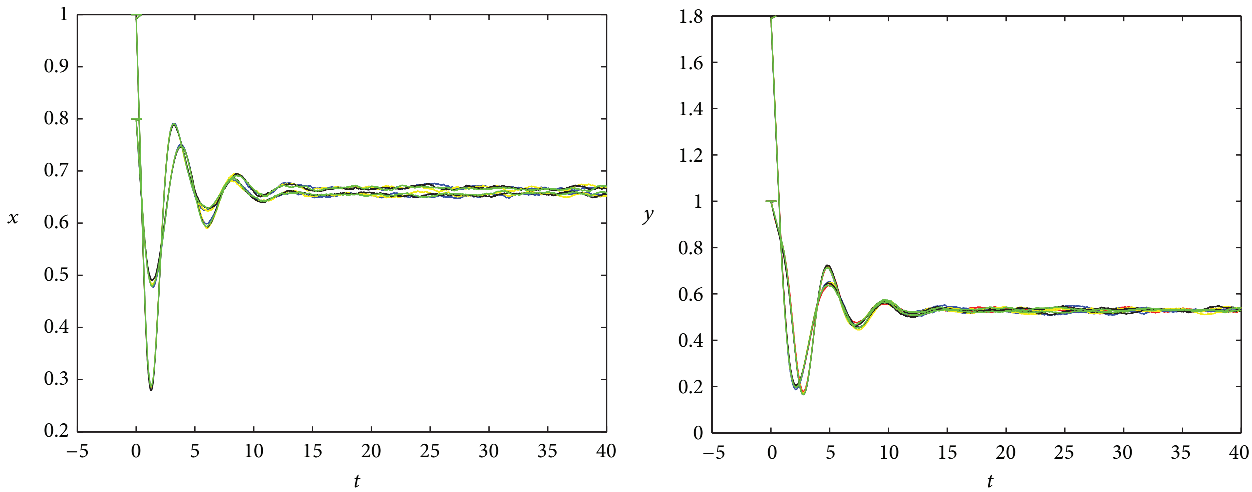
$$\begin{gathered}
h_1 = 0.05, \qquad h_2 = 0.15, \qquad h_3 = 0.17, \\
h_4 = 0.23, \qquad h_5 = 0.29, \qquad h_6 = 0.31, \\
h_7 = 0.35, \qquad h_8 = 0.45, \\
d_1 = 0.5, \qquad d_2 = 0.6, \qquad \tau = 0.7, \\
\sigma = 0.8, \qquad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad T = 0.
\end{gathered} \tag{48}$$

Using MATLAB LMI control toolbox and by solving the LMIs (16) in Theorem 6, we can obtain the feasible solutions. Due to space limitations, we do not list them here. We find that the delayed UISGRNs (8) are stable in the mean square which is shown in Figure 1.

Example 2. Consider the system (6) with the parameters that are the same as in Example 1 other than

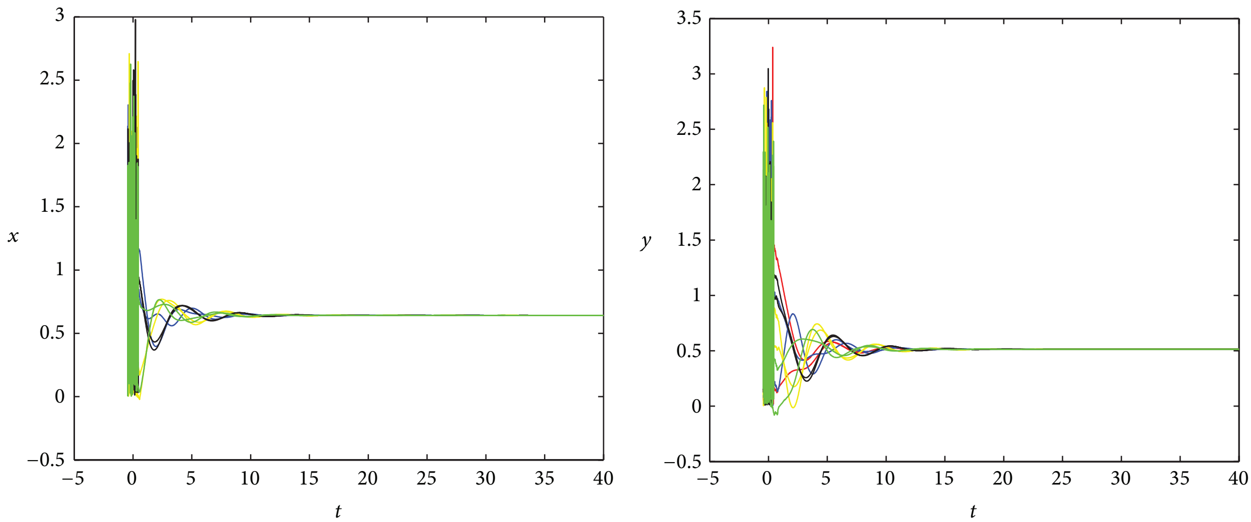
$$H_a = H_c = H_w = H_l = 0. \tag{49}$$

We can find feasible solutions for the LMIs in Corollary 8, so the impulsive stochastic GRNs (6) are stable in the mean square which can be shown in Figure 2.



(a) (b)

Figure 1: Trajectories of $x(t)$ and $y(t)$ of the genetic network (8) with randomly chosen initial values.



(a) (b)

Figure 2: Trajectories of $x(t)$ and $y(t)$ of the genetic network (6) with randomly chosen initial values.

Example 3. When the parameters in (45) are the same as in Example 1 in addition to

$$\eta = 0. \tag{50}$$

It is easy to see that the uncertain impulsive GRNs (45) are stable in the mean square by checking Corollary 9 conditions. Figure 3 shows that the result is valid.

5. Conclusions

In this paper, we have investigated the stability problem for a new UISGRNs model with the introduction of leakage delay. By employing the Lyapunov stability theory, free-weighting matrix, and convex combination technique combining with stochastic stability approach and the LMI framework, we have obtained a sufficient condition to justify the stability of the proposed UISGRNs model. The obtained stability condition is expressed in terms of LMIs which can be easily solved by the efficient MATALB LMI toolbox. Finally, numerical examples have been provided to illustrate the usefulness of the derived stability results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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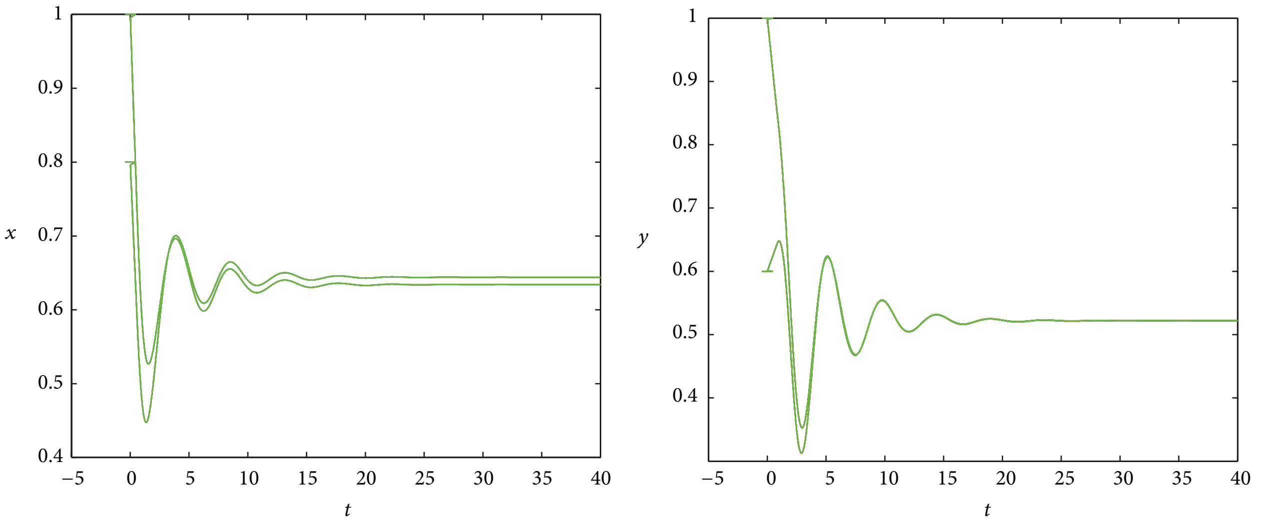

(a) (b)

Figure 3: Trajectories of $x(t)$ and $y(t)$ of the genetic network (45) with randomly chosen initial values.

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