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Research Article

Approximate Solutions of Fisher's Type Equations with Variable Coefficients

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The spectral collocation approximations based on Legendre polynomials are used to compute the numerical solution of time-dependent Fisher's type problems. The spatial derivatives are collocated at a Legendre-Gauss-Lobatto interpolation nodes. The proposed method has the advantage of reducing the problem to a system of ordinary differential equations in time. The four-stage A-stable implicit Runge-Kutta scheme is applied to solve the resulted system of first order in time. Numerical results show that the Legendre-Gauss-Lobatto collocation method is of high accuracy and is efficient for solving the Fisher's type equations. Also the results demonstrate that the proposed method is powerful algorithm for solving the nonlinear partial differential equations.

1. Introduction

Spectral methods (see, for instance, [1–5]) are powerful techniques that we use to numerically solve linear and nonlinear partial differential equations either in their strong or weak forms. What sets spectral methods apart from others like finite difference methods or finite element methods is that to get a spectral method we approximate the solutions by high order orthogonal polynomial expansions. The orthogonal polynomial approximations can have very high convergence rates, which allow us to use fewer degrees of freedom for a desired level of accuracy. The most common spectral method from the strong form of the equations is known as collocation. In collocation techniques, the partial differential equation must be satisfied at a set of grid, or more precisely, collocation points (see, for instance, [6–10]). Spectral methods also have become increasingly popular for solving fractional differential equations [11–21].

In this paper, we present an accurate numerical solution based on Legendre-Gauss-Lobatto collocation method for Fisher's type equations. The Fisher equation in the form

$$u_t = Du_{xx} + \nu u(1-u) \tag{1}$$

was firstly introduced by Fisher in [22] to describe the propagation of a mutant gene. Fisher equations have a wide application in a large number of the chemical kinetics [23], logistic population growth [24], flame propagation [25], population in one-dimensional habitual [26], neutron population in a nuclear reaction [27], neurophysiology [28], branching Brownian motion [23], autocatalytic chemical reactions [29], and nuclear reactor theory [30].

In recent years, many physicists and mathematicians have paid much attention to the Fisher equations due to their importance in mathematical physics. In [31], Öğün and Kart utilized truncated Painlevè expansions for presenting some exact solutions of Fisher and generalized Fisher equations. Tan et al. [28] proposed the homotopy analysis method to find analytical solution of Fisher equations. Gunzburger et al. [32] applied the discrete finite element approximation for obtaining a numerical solution of the forced Fisher equation. Dag et al. [33] discussed and applied the B-spline Galerkin method for Fisher's equation. Bastani and Salkuyeh [34] proposed the compact finite difference approach in combination with third-order Runge-Kutta scheme to solve Fisher's equation. More recently, Mittal and Jain [35] investigated the cubic B-spline scheme for solving Fisher's reaction-diffusion problem. However, the fisher equations have been studied in many other articles by numerous numerical methods such as pseudospectral method [36, 37], finite difference method [38–44], finite element method [45], B-spline algorithm [46], and Galerkin method [47, 48].

To increase the numerical solution accuracy, spectral collocation methods based on orthogonal polynomials are often chosen. Doha et al. [49] proposed and developed a new numerical algorithm for solving the initial-boundary system of nonlinear hyperbolic equations based on spectral collocation method; a Chebyshev-Gauss-Radau collocation method in combination with the implicit Runge-Kutta scheme are employed to obtain highly accurate approximations to this system of nonlinear hyperbolic equations. In [50], Bhrawy proposed an efficient Jacobi-Gauss-Lobatto collocation method for approximating the solution of the generalized Fitzhugh-Nagumo equation in which the Jacobi-Gauss-Lobatto points are used as collocation nodes for spatial derivatives. Moreover, the Jacobi spectral collocation methods are used to solve some problems in mathematical physics, (see, for instance, [51–53]).

Indeed, there are no results on Legendre-Gauss-Lobatto collocation method for solving nonlinear Fisher-type equations subject to initial-boundary conditions. Therefore, the objective of this work is to present a numerical algorithm for solving such equation based on Legendre-Gauss-Lobatto pseudospectral method. The spatial derivatives are approximated at these grid points by approximating the derivatives of Legendre polynomial that interpolates the solutions. Moreover, we set the boundary conditions in the collocation method. The problem is then reduced to system of first-order ordinary differential equations in time. The four-stage A-stable implicit Runge-Kutta scheme is proposed for treating the this system of equations. Finally, some illustrative examples are implemented to illustrate the efficiency and applicability of the proposed approach.

The rest of this paper is structured as follows. In the next section, some properties of Legendre polynomials, which are required for implementing our algorithm, are presented. Section 3 is devoted to the development of Gauss-Lobatto collocation technique for a general form of Fisher-type equations based on the Legendre polynomials, and in Section 4 the proposed method is implemented to obtain some numerical results for three problems of Fisher-type equations with known exact solutions. Finally, a brief conclusion is provided in Section 5.

2. Legendre Polynomials

The Legendre polynomials $L_k(x)$ $(k = 0, 1, \ldots,)$ satisfy the following Rodrigues' formula:

$$L_k(x) = \frac{(-1)^k}{2^k k!} D^k \left(\left(1 - x^2\right)^k\right); \tag{2}$$

we recall also that $L_k(x)$ is a polynomial of degree k, and therefore, the qth derivative of $L_k(x)$ is given by

$$L_k^{(q)}(x) = \sum_{i=0(k+i=\text{even})}^{k-q} C_q(k,i)\, L_i(x), \tag{3}$$

where

$$C_q(k,i) = \frac{2^{q-1}(2i+1)\,\Gamma\left[(q+k-i)/2\right]\Gamma\left[(q+k+i+1)/2\right]}{\Gamma[q]\,\Gamma\left[(2-q+k-i)/2\right]\Gamma\left[(3-q+k+i)/2\right]}. \tag{4}$$

The analytical form of Legendre polynomial is

$$L_n(x) = \sum_{i=0}^{[n/2]} c_k^{(n)} x^{n-2k}, \tag{5}$$

where $c_k^{(n)} = (-1)^k(2n-2k)!/2^n(n-k)!(n-2k)!k!$, and

$$\left[\frac{n}{2}\right] = \begin{cases} \dfrac{n}{2}, & \text{even}, \\ \dfrac{n-1}{2}, & \text{odd}. \end{cases} \tag{6}$$

It is also generating from the following relation:

$$L_{k+2}(x) = \frac{2k+3}{k+2} x L_{k+1}(x) - \frac{k+1}{k+2} L_k(x), \tag{7}$$

with $L_0(x) = 1$, $L_1(x) = x$, and satisfies the orthogonality condition

$$\left(L_k(x), L_l(x)\right)_w = \int_{-1}^{1} L_k(x)\, L_l(x)\, w(x) = h_k \delta_{lk}. \tag{8}$$

where $w(x) = 1$, $h_k = 2/(2k+1)$. Let S_N be the space of all polynomials of degree $\leq N$, then for any $\phi \in S_{2N-1}(0, L)$,

$$\int_{-1}^{1} w(x)\,\phi(x)\,dx = \sum_{j=0}^{N} \varpi_{N,j}\phi\left(x_{N,j}\right). \tag{9}$$

Let us define the following discrete inner product and norm:

$$(u, v)_w = \sum_{j=0}^{N} u\left(x_{N,j}\right) v\left(x_{N,j}\right) \varpi_{N,j}, \tag{10}$$

where $x_{N,j}$ and $\varpi_{N,j}$ are the nodes and the corresponding weights of the Legendre-Gauss-Lobatto quadrature formula on the interval $(-1, 1)$, respectively.

3. Legendre Spectral Collocation Method

Because of the pseudospectral method is an efficient and accurate numerical scheme for solving various problems in physical space, including variable coefficient and singularity (see, [54, 55]), we propose this method based on Legendre polynomials for approximating the solution of the nonlinear generalized Burger-Fisher model equation and Fisher model with variable coefficient.

3.1. (1+1)-Dimensional Generalized Burger-Fisher Equation. In this subsection, we derive a Legendre pseudospectral algorithm to solve numerically the generalized Burger-Fisher problem:

$$u_t + \nu u^{\delta} u_x - u_{xx} - \gamma u\left(1 - u^{\delta}\right) = 0, \quad (x,t) \in D \times [0,T], \tag{11}$$

where $D = \{x : -1 \le x \le 1\}$. Subject to

$$u(x,t) = g(t), \quad x = -1, 1, \tag{12}$$

$$u(x,0) = f(x), \quad x \in D. \tag{13}$$

In the following, we shall derive an efficient algorithm for the numerical solution of (11)–(13). Let the approximation of $u(x,t)$ be given in terms of the Legendre polynomials expansion:

$$u(x,t) = \sum_{j=0}^{N} a_j(t) L_j(x), \quad \mathbf{a} = \left(a_0, a_1, \ldots, a_N\right)^T. \tag{14}$$

Making use of relations (8) and (10) gives

$$u(x,t) = \sum_{j=0}^{N} \left(\frac{1}{h_j} \sum_{i=0}^{N} L_j(x_i) L_j(x) \varpi_{N,i} u(x_i,t) \right) \tag{15}$$

or equivalently

$$u(x,t) = \sum_{i=0}^{N} \left(\sum_{j=0}^{N} \frac{1}{h_j} L_j(x_i) L_j(x) \varpi_{N,i} \right) u(x_i,t). \tag{16}$$

The Gauss-Lobatto points were introduced by way of (9). We then saw that the polynomial approximation $u(x,t)$ can be characterized by $(N+1)$ nodal values $u(x_i,t)$. The approximation of the spatial partial derivatives of first-order for $u(x,t)$ can be computed at the Legendre Gauss-Lobatto interpolation nodes as

$$\begin{aligned} u_x(x_n,t) &= \sum_{i=0}^{N} \left(\sum_{j=0}^{N} \frac{1}{h_j} L_j(x_i) \left(L_j(x_n)\right)' \varpi_{N,i} \right) u(x_i,t) \\ &= \sum_{i=0}^{N} A_{ni} u(x_i,t) \\ &= \sum_{i=0}^{N} A_{ni} u_i(t), \quad n = 0, 1, \ldots, N, \end{aligned} \tag{17}$$

where

$$\begin{gathered} A_{ni} = \sum_{j=0}^{N} \frac{1}{h_j} L_j(x_i) \left(L_j(x_n)\right)' \varpi_{N,i}, \\ u_i(t) = u(x_i,t). \end{gathered} \tag{18}$$

Subsequently, the second-order spatial partial derivatives of $u(x,t)$ may be written at the same collocation nodes as

$$\begin{aligned} u_{xx}(x_n,t) &= \sum_{i=0}^{N} \left(\sum_{j=0}^{N} \frac{1}{h_j} L_j(x_i) \left(L_j(x_n)\right)'' \varpi_{N,i} \right) u(x_i,t) \\ &= \sum_{i=0}^{N} B_{ni} u(x_i,t) \\ &= \sum_{i=0}^{N} B_{ni} u_i, \end{aligned} \tag{19}$$

where

$$B_{ni} = \sum_{j=0}^{N} \frac{1}{h_j} L_j(x_i) \left(L_j(x_n)\right)'' \varpi_{N,i}. \tag{20}$$

In collocation methods, one specifically seeks the approximate solution such that the problem (11) is satisfied exactly at the Legendre Gauss-Lobatto set of interpolation points x_n; $n = 1, \ldots, N-1$. The approximation is exact at the $N-1$ collocation points. Therefore, (11) after using relations (17)–(20), can be written as

$$\begin{gathered} \dot{u_n}(t) + \nu u_n^{\delta}(t) \sum_{i=0}^{N} A_{ni} u_i(t) \\ - \sum_{i=0}^{N} B_{ni} u_i(t) - \gamma u_n(t) \left(1 - u_n^{\delta}(t)\right) = 0, \\ n = 1, \ldots, N-1, \end{gathered} \tag{21}$$

where $u_n(t) = u(x_n,t)$ and $\dot{u_n}(t) = \partial \dot{u_n}(t)/\partial t$.

Now the two values $u_0(t)$ and $u_N(t)$ can be determined from the boundary conditions (12), then (21) can be reformulated as

$$\begin{gathered} \dot{u_n}(t) + \nu u_n^{\delta}(t) \sum_{i=1}^{N-1} A_{ni} u_i(t) \\ - \sum_{i=1}^{N-1} B_{ni} u_i(t) + \nu u_n^{\delta}(t) d_n(t) - \widetilde{d_n}(t) \\ - \gamma u_n(t) \left(1 - u_n^{\delta}(t)\right) = 0, \quad n = 1, \ldots, N-1, \end{gathered} \tag{22}$$

where

$$\begin{aligned} d_n(t) &= A_{n0} u_0(t) + A_{nN} u_N(t), \\ \widetilde{d_n}(t) &= B_{n0} u_0(t) + B_{nN} u_N(t). \end{aligned} \tag{23}$$

Approximation (22) automatically satisfies the boundary conditions (12), but we need an initial condition for each of the $u_n(t)$ to integrate (22) in time. The initial condition is usually taken to be the interpolant of the initial function $f(x)$; that is $u_n(0) = f(x_n)$. Therefore, the approximation of (11)–(13) is reduced to the solution of system of ordinary differential equations in time. Consider

$$\dot{u_n}(t) + \nu u_n^{\delta}(t) \sum_{i=1}^{N-1} A_{ni} u_i(t) - \sum_{i=1}^{N-1} B_{ni} u_i(t) + \nu u_n^{\delta}(t) d_n(t) - \widetilde{d_n}(t) - \gamma u_n(t)\left(1 - u_n^{\delta}(t)\right) = 0, \quad n = 1,\dots,N-1, \quad u_n(0) = f(x_n). \tag{24}$$

Let us denote

$$\begin{gathered} U^{\cdot}(t) = \left[\dot{u_1}(t), \dot{u_2}(t), \dots, \dot{u_{N-1}}(t)\right]^T, \\ U(0) = \left[u_1(0), u_2(0), \dots, u_{N-1}(0)\right]^T, \\ f = \left[f(x_1), f(x_2), \dots, f(x_{N-1})\right]^T, \\ F(t,u(t)) = \left[F_1(t,u(t)), F_2(t,u(t)), \dots, F_{N-1}(t,u(t))\right]^T, \\ F_n(t,u(t)) = -\nu u_n^{\delta}(t) \sum_{i=1}^{N-1} A_{ni} u_i(t) + \sum_{i=1}^{N-1} B_{ni} u_i(t) - \nu u_n^{\delta}(t) d_n(t) + \widetilde{d_n}(t) + \gamma u_n(t)\left(1 - u_n^{\delta}(t)\right), \\ n = 1,\dots,N-1. \end{gathered} \tag{25}$$

Then (24) can be written in the matrix form

$$U^{\cdot}(t) = F(t,u(t)) \qquad U(0) = f. \tag{26}$$

This system of ordinary differential equations can be solved by using four-stage A-stable implicit Runge-Kutta scheme.

3.2. (1+1)-Dimensional Fisher Equation with Variable Coefficient. In this subsection, we extend the application of the Legendre pseudospectral method to solve numerically the Fisher equation with variable coefficient,

$$u_t - b(t) u_{xx} - cu(1-u) = 0, \quad (x,t) \in D \times [0,T], \tag{27}$$

subject to the initial-boundary conditions

$$u(x,0) = f(x), \quad x \in D, \qquad u(x,t) = g(t). \tag{28}$$

Proceeding as in the previous subsection we can obtain u_{xx} in the same form as (19), and then (27) can be collocated in the Legendre Gauss-Lobatto points as:

$$\dot{u_n}(t) - b(t) \sum_{i=1}^{N-1} B_{ni} u_i(t) - b(t) \widetilde{d_n}(t) - cu_n(t)\left(1 - u_n(t)\right) = 0, \quad n = 1,\dots,N-1, \tag{29}$$

Table 1: Absolute errors for Example 1.

x	t	$E(x,t)$	x	t	$E(x,t)$
−1		5.56×10^{-11}	−1		8.93×10^{-11}
−0.5		1.46×10^{-8}	−0.5		8.23×10^{-9}
0	0.1	1.95×10^{-8}	0	0.2	1.41×10^{-8}
0.5		1.55×10^{-8}	0.5		1.17×10^{-8}
1		5.56×10^{-11}	1		8.93×10^{-11}

which can be written in the matrix form

$$U^{\cdot}(t) = F(t,u(t)), \qquad U(0) = f, \tag{30}$$

where

$$F_n(t,u(t)) = b(t) \sum_{i=1}^{N-1} B_{ni} u_i(t) + b(t) \widetilde{d_n}(t) + cu_n(t)\left(1 - u_n(t)\right), \quad n = 1,\dots,N-1. \tag{31}$$

4. Numerical Examples

In this section, three nonlinear time-dependent Fisher-type equations on finite interval are implemented to demonstrate the accuracy and capability of the proposed algorithm, and all of them were performed on the computer using a program written in Mathematica 8.0. The absolute errors in the given tables are $E(x,t) = |u(x,t) - \tilde{u}(x,t)$ where $u(x,t)$ and $\tilde{u}(x,t)$ are the exact and numerical solution at selected points (x,t).

Example 1. Consider the nonlinear time-dependent one-dimensional Fisher-type equations

$$u_t = u_{xx} + u(1-u)(u-\gamma), \quad (x,t) \in D \times [0,T], \tag{32}$$

where $D = \{x : -1 < x < 1\}$. Subject to

$$\begin{aligned} u(1,t) &= \frac{1+\gamma}{2} - \frac{\gamma-1}{2} \tanh\left[\frac{\gamma-1}{2\sqrt{2}}\left(1 - \frac{1+\gamma}{\sqrt{2}}t\right)\right], \\ u(-1,t) &= \frac{1+\gamma}{2} + \frac{\gamma-1}{2} \tanh\left[\frac{\gamma-1}{2\sqrt{2}}\left(1 + \frac{1+\gamma}{\sqrt{2}}t\right)\right], \\ u(x,0) &= \frac{1+\gamma}{2} - \frac{\gamma-1}{2} \tanh\left[\frac{\gamma-1}{2\sqrt{2}}(x)\right], \quad x \in D. \end{aligned} \tag{33}$$

The exact solution is

$$u(x,t) = \frac{1+\gamma}{2} - \frac{\gamma-1}{2} \tanh\left[\frac{\gamma-1}{2\sqrt{2}}\left(x - \frac{1+\gamma}{\sqrt{2}}t\right)\right]. \tag{34}$$

In Table 1, we introduce the absolute errors between the approximate and exact solutions for problem (32) using the proposed method for different values of x and t, with $\gamma = 10^{-2}$ and $N = 20$.

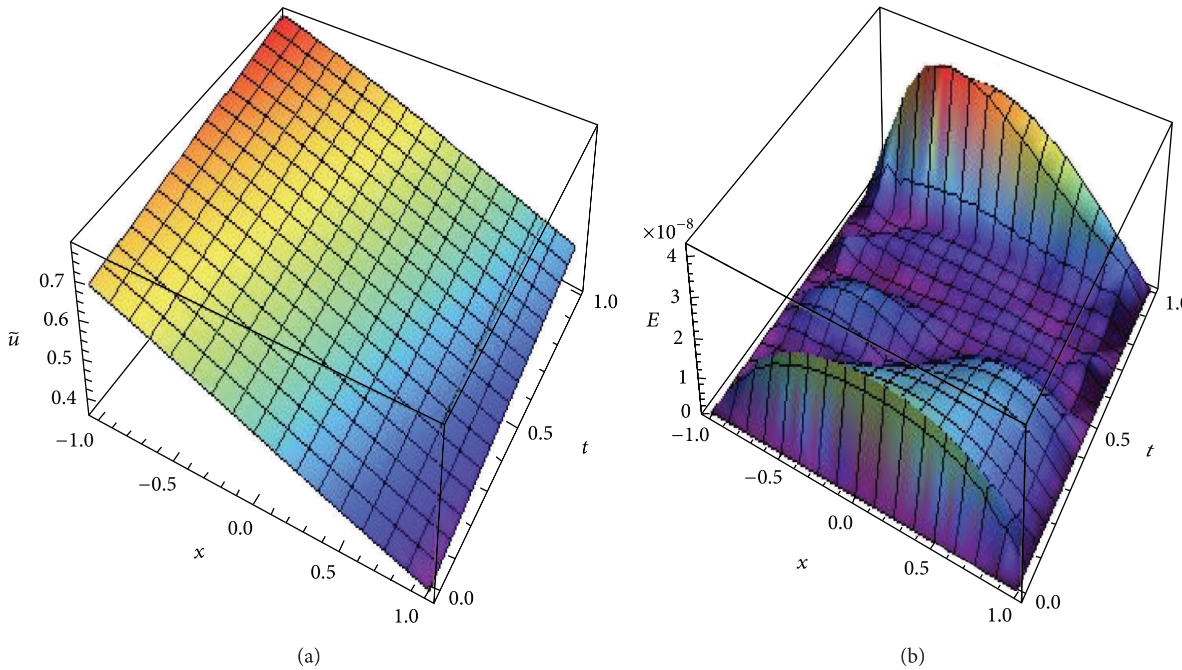


FIGURE 1: The result of the L-GL-C method at $\gamma = 10^{-2}$ and $N = 20$. (a) The approximate solution. (b) The absolute error.

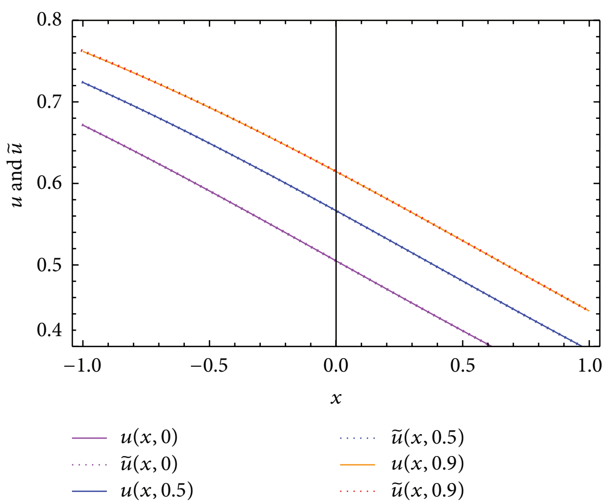


FIGURE 2: The curves of approximate solutions and the exact solutions of problem (32) at $t = 0.0$, $t = 0.5$, and $t = 0.9$ with $\gamma = 10^{-2}$ and $N = 20$.

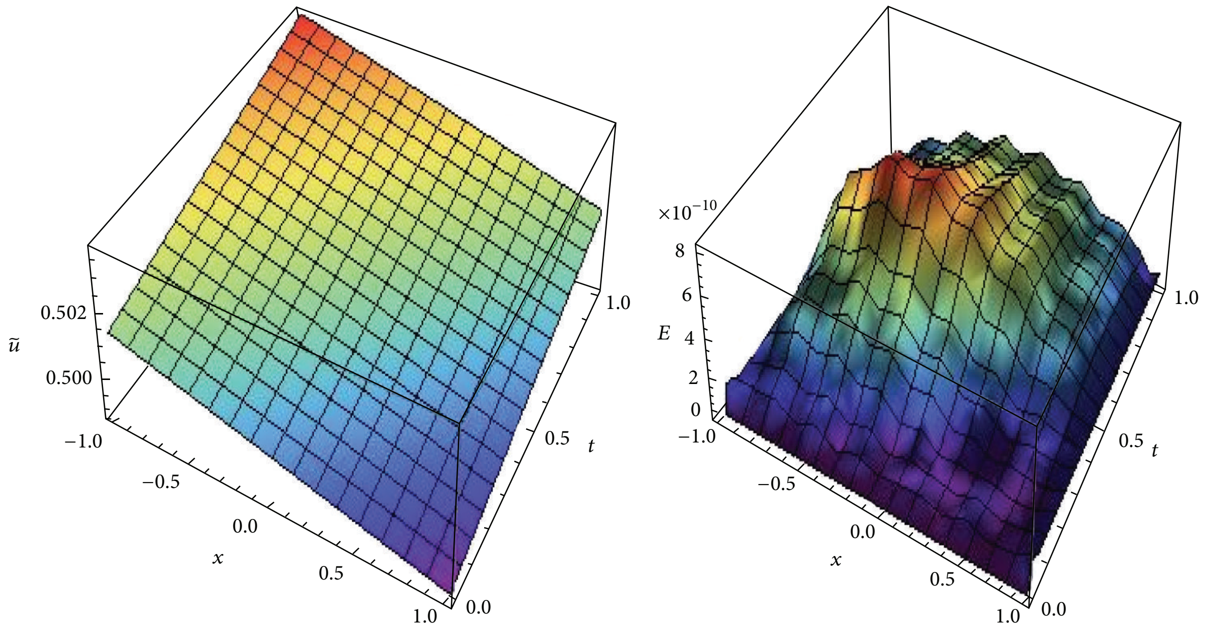
In case of $\gamma = 10^{-2}$ and $N = 20$, the approximate solution and absolute errors of problem (32) are displayed in Figures 1(a) and 1(b), respectively. In Figure 2, we plotted the curves of approximate solutions and exact solutions of problem (32) for different values of t (0.0,0.5 and 0.9) with $\gamma = 10^{-2}$ and $N = 20$. It is clear from this figure that approximate solutions and exact solutions completely coincide for the chosen values of t.

Example 2. Consider the nonlinear time-dependent one-dimensional generalized Burger-Fisher-type equations

$$u_t = u_{xx} - \nu u^{\delta} u_x + \gamma u \left(1 - u^{\delta}\right), \quad (x,t) \in D \times [0,T], \tag{35}$$

where $D = \{x : -1 < x < 1\}$. Subject to

$$\begin{aligned} u(1,t) &= \left(\frac{1}{2} - \frac{1}{2}\tanh\left[\frac{\nu\delta}{2(\delta+1)} \times \left(1 - \left(\frac{\nu}{\delta+1} + \frac{\gamma(\delta+1)}{\nu}\right)t\right)\right]\right)^{1/\delta}, \\ u(-1,t) &= \left(\frac{1}{2} + \frac{1}{2}\tanh\left[\frac{\nu\delta}{2(\delta+1)} \times \left(1 + \left(\frac{\nu}{\delta+1} + \frac{\gamma(\delta+1)}{\nu}\right)t\right)\right]\right)^{1/\delta}, \\ u(x,0) &= \left(\frac{1}{2} - \frac{1}{2}\tanh\left[\frac{\nu\delta}{2(\delta+1)}x\right]\right)^{1/\delta}, \quad x \in D. \end{aligned} \tag{36}$$



(a) (b)

FIGURE 3: The result of the L-GL-C method at $\nu = \gamma = 10^{-2}$, $\delta = 1$, and $N = 20$. (a) The approximate solution. (b)The absolute error.

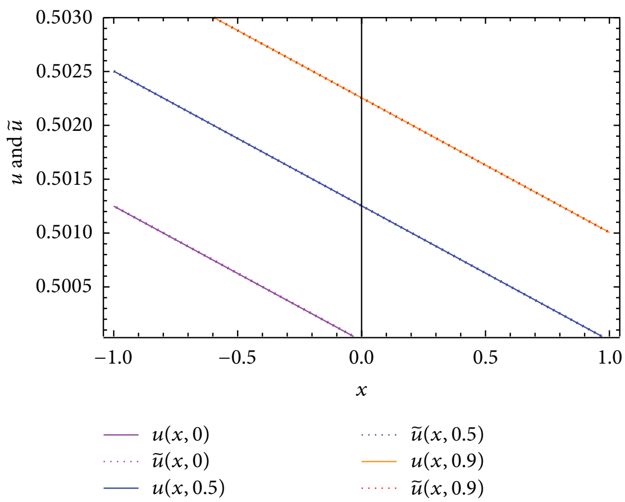


FIGURE 4: The curves of approximate solutions and the exact solutions of problem (35) at $t = 0.0$, $t = 0.5$, and $t = 0.9$ with $\nu = \gamma = 10^{-2}$, $\delta = 1$, and $N = 20$.

The exact solution of (35) is

$$u(x,t) = \left(\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{\nu\delta}{2(\delta+1)} \times \left(x - \left(\frac{\nu}{\delta+1} + \frac{\gamma(\delta+1)}{\nu}\right)t\right)\right)\right)^{1/\delta}. \tag{37}$$

The absolute errors for problem (35) are listed in Table 2 using the L-GL-C method with $\nu = \gamma = 10^{-2}$, $N = 20$, and various choices of δ.

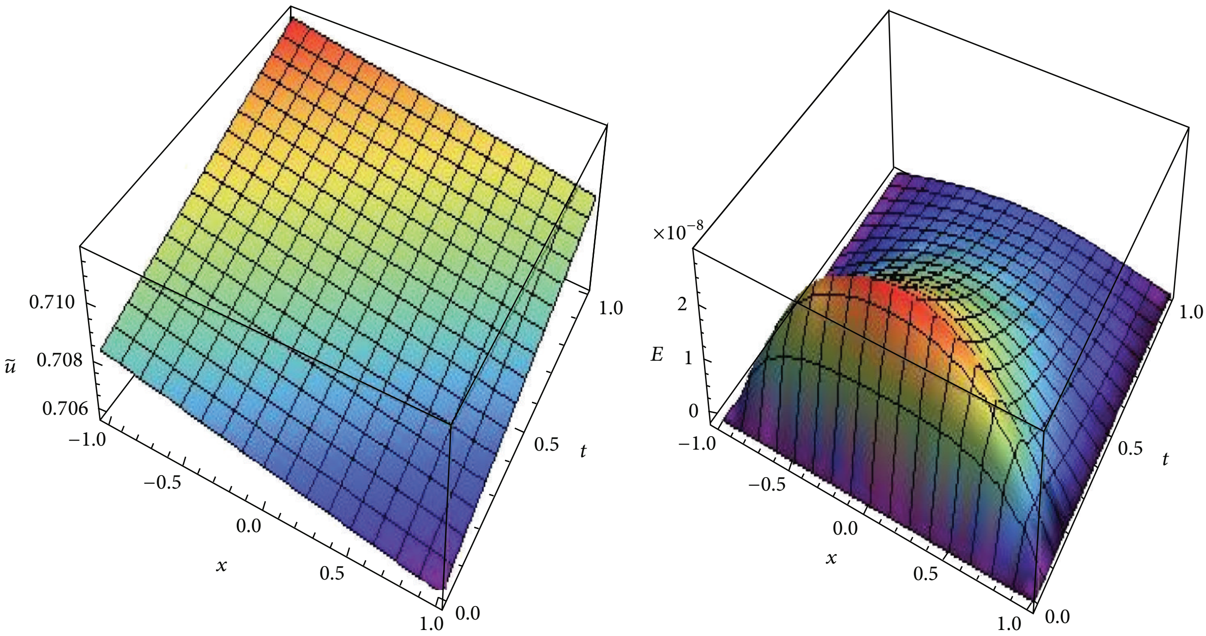
To illustrate the effectiveness of the Legendre pseudospectral method for problem (35), we displayed in Figures 3(a) and 3(b) the approximate solution and the absolute error with $\nu = \gamma = 10^{-2}$, $\delta = 1$, and $N = 20$. The graph of curves of exact and approximate solutions with different values of t (0.0, 0.5, and 0.9) is given in Figure 4. Moreover, the approximate solution and the absolute error with $\nu = \gamma = 10^{-2}$, $\delta = 2$, and $N = 20$ are displayed in Figures 5(a) and 5(b), respectively. The curves of exact and approximate solutions of problem (35) with $\delta = 2$ are displayed in Figure 6 with values of parameters listed in its caption.

Example 3. Consider the nonlinear time-dependent one-dimensional Fisher-type equations with variable coefficient

$$u_t = -\frac{a}{6\mu^2}\coth\left(\frac{a}{6}t + c\right)u_{xx} + au(1-u), \quad (x,t) \in D \times [0,T], \tag{38}$$

where $D = \{x : -1 < x < 1\}$. Subject to

$$u(1,t) = \frac{1}{4}\coth\left(\frac{a}{6}t + c\right)\operatorname{sech}^2\left(\frac{\mu}{2} + \frac{5a}{12}t\right) + \frac{1}{2}\tanh\left(\frac{\mu}{2} + \frac{5a}{12}t\right) + \frac{1}{2},$$



(a) (b)

Figure 5: The result of the L-GL-C method at $\nu = \gamma = 10^{-2}$, $\delta = 2$, and $N = 20$. (a) The approximate solution. (b)The absolute error.

Table 2: Absolute errors for Example 2.

$\delta = 1$			$\delta = 2$			$\delta = 3$		
x	t	$E(x,t)$	x	t	$E(x,t)$	x	t	$E(x,t)$
−1		8.51×10^{-11}	−1		1.26×10^{-10}	−1		6.91×10^{-11}
−0.5		1.16×10^{-10}	−0.5		2.34×10^{-8}	−0.5		1.09×10^{-8}
0	0.1	5.36×10^{-12}	0	0.1	2.61×10^{-8}	0	0.1	1.30×10^{-8}
0.5		1.53×10^{-11}	0.5		2.33×10^{-8}	0.5		1.09×10^{-8}
1		8.51×10^{-11}	1		1.26×10^{-10}	1		6.91×10^{-11}
−1		8.71×10^{-11}	−1		1.22×10^{-10}	−1		1.41×10^{-10}
−0.5		7.77×10^{-10}	−0.5		8.24×10^{-9}	−0.5		4.27×10^{-9}
0	0.5	6.49×10^{-10}	0	0.5	1.16×10^{-8}	0	0.5	5.89×10^{-9}
0.5		3.81×10^{-10}	0.5		8.26×10^{-9}	0.5		4.28×10^{-9}
1		8.71×10^{-11}	1		1.22×10^{-10}	1		1.41×10^{-10}

$$u(-1,t) = \frac{1}{4}\coth\left(\frac{a}{6}t + c\right)\operatorname{sech}^2\left(-\frac{\mu}{2} + \frac{5a}{12}t\right) + \frac{1}{2}\tanh\left(-\frac{\mu}{2} + \frac{5a}{12}t\right) + \frac{1}{2}, \tag{39}$$

$$u(x,0) = \frac{1}{4}\coth(c)\operatorname{sech}^2\left(\frac{\mu x}{2}\right) + \frac{1}{2}\tanh\left(\frac{\mu x}{2}\right) + \frac{1}{2}, \quad x \in D. \tag{40}$$

The exact solution of (38) is

$$u(x,t) = \frac{1}{4}\coth\left(\frac{a}{6}t + c\right)\operatorname{sech}^2\left(\frac{\mu x}{2} + \frac{5a}{12}t\right) + \frac{1}{2}\tanh\left(\frac{\mu x}{2} + \frac{5a}{12}t\right) + \frac{1}{2}. \tag{41}$$

Table 3 lists the absolute errors for problem (38) using the L-GL-C method. From numerical results of this table, it can be concluded that the numerical solutions are in excellent agreement with the exact solutions.

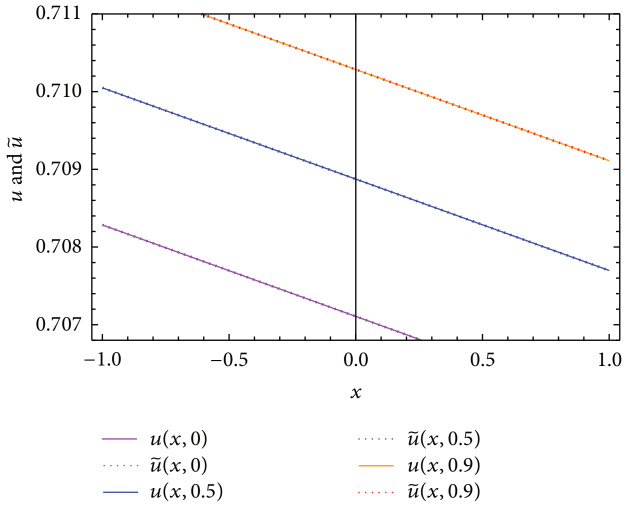


FIGURE 6: The curves of approximate solutions and the exact solutions of problem (35) at $t = 0.0$, $t = 0.5$, and $t = 0.9$ with $\nu = \gamma = 10^{-2}$, $\delta = 2$, and $N = 20$.

TABLE 3: Absolute errors for Example 3.

x	t	$E(x,t)$	x	t	$E(x,t)$
−1		9.81×10^{-10}	−1		7.05×10^{-9}
−0.5		1.00×10^{-7}	−0.5		1.93×10^{-6}
0	0.1	1.45×10^{-7}	0	0.5	2.84×10^{-6}
0.5		1.19×10^{-7}	0.5		2.12×10^{-6}
1		9.81×10^{-10}	1		7.06×10^{-9}

5. Conclusion

In this paper, based on the Legendre-Gauss-Lobatto pseudospectral approximation we proposed an efficient numerical algorithm to solve nonlinear time-dependent Fisher-type equations with constant and variable coefficients. The method is based upon reducing the nonlinear partial differential equation into a system of first-order ordinary differential equations in the expansion coefficient of the spectral solution. Numerical examples were also provided to illustrate the effectiveness of the derived algorithms. The numerical experiments show that the Legendre pseudospectral approximation is simple and accurate with a limited number of collocation nodes.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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