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Research Article

The Study of the Solution to a Generalized KdV-mKdV Equation

Xiumei Lv,[1] Tengwei Shao,[1] and Jiacheng Chen[2]

[1] *School of Finance, Chongqing Technology and Business University, Chongqing 400067, China*
[2] *School of Finance, Southwestern University of Finance and Economics, Chengdu 611130, China*

Correspondence should be addressed to Xiumei Lv; lvxium@126.com

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A mathematical technique based on an auxiliary equation and the symbolic computation system Matlab is employed to investigate a generalized KdV-mKdV equation which possesses high-order nonlinear terms. Some new solutions including the Jacobi elliptic function solutions, the degenerated soliton-like solutions, and the triangle function solutions to the equation are obtained.

1. Introduction

Many powerful techniques have been established during the past decades for the study of the nonlinear dispersive partial differential equations [1–9]. The inverse scattering method, the Backlund transformation, the Darboux transformation, the Painleve' analysis, the pseudospectral method, the finite differences method, and the sine-cosine ansatz are used to acquire solitary wave solutions and compactons solutions for some nonlinear equations. Wadati [1] employed the potential function to handle the KdV equation, and the resulting equation was solved iteratively by making use of a formal series for the potential function. Wadati [2] developed the trace method to handle the KP equation. The tanh method, developed by Malfliet and Hereman [3], is heavily used in the literature to deal with nonlinear partial differential equations. Fan and Zhang [4] introduced a useful extended tanh method that combined the standard tanh method with the Riccati equation. The extensive method was effectively used by many researchers to investigate exact solutions for a lot of nonlinear models (see [10, 11]).

With the development of the symbolic computation system, the direct methods for constructing travelling wave solutions to differential equations become feasible. With the help of Mathematica, Sirendaoreji and Jiong [12, 13] used the auxiliary equation method to investigate KdV and mKdV equations, Boussinesq equations, sine-Gordon equations, and the nonlinear Klein-Gordon equations, respectively. The Jacobi elliptic function expansion method is confirmed as a powerful technique to solve some nonlinear differential equations (see [14]).

Zhang et al. [15] used a sub-ODE technique to investigate the exact solution of the following nonlinear dispersive KdV-mKdV equation:

$$u_t + \left(\alpha + \beta u^p + \lambda u^{2p}\right) u_x + u_{xxx} = 0, \tag{1}$$

where α, β, λ, and p are constants. The bell type solitary wave solution, the kink type solitary wave solution, the algebraic solitary wave solution, and the sinusoidal travelling wave solution of (1) with exponent $p > 0$ were expressed explicitly in [15]. In fact, (1) is turned into a KdV equation if $p = 1, \beta \neq 0, \lambda = 0$, and an mKdV equation if $p = 1, \beta = 0, \lambda \neq 0$.

In this paper, we further develop the work in [15] for the study of (1). By using a mathematical technique different from those in previous works [1–10], we obtain the exact travelling wave solutions including Jacobi elliptic function solutions, degenerated soliton solutions, and triangle function solutions for (1) with exponent p. Many of the solutions obtained are different from those presented in Zhang et al.'s work [15].

2. Brief Description of the Approach

To illustrate the basic concept of the auxiliary differential method, we consider that the nonlinear partial differential equation has the form

$$P\left(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt}, \ldots\right) = 0. \tag{2}$$

Using the transformation $\xi = \mu(x - ct)$ ($\mu \neq 0$ and $c \neq 0$), (2) turns into the following nonlinear ordinary different equation:

$$Q\left(u, u_{\xi}, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots\right) = 0. \quad (3)$$

We seek for the solutions of (3) in the form

$$u(\xi) = \sum_{i=0}^{N} g_i z^i(\xi), \quad (4)$$

where g_i ($i = 0, 1, 2, \ldots, N$) are constants which will be determined later. The parameter N is a positive integer and can be determined by balancing the highest-order derivative terms and the highest power nonlinear terms in (3). The highest degree of $\partial^p u/\partial \xi^p$ can be calculated by

$$\begin{aligned} O\left[\frac{\partial^p u}{\partial \xi^p}\right] &= N + p, \quad p = 0, 1, 2, \ldots, \\ O\left[u^q \frac{\partial^p u}{\partial \xi^p}\right] &= qN + p, \quad q, p = 0, 1, 2, \ldots. \end{aligned} \quad (5)$$

We assume that $z(\xi)$ represents the solutions of the following auxiliary differential equation

$$\left(\frac{dz}{d\xi}\right)^2 = c_1 + c_2 z^2 + \frac{c_3}{2} z^4, \quad (6)$$

where c_i ($i = 1, 2, 3$) are real constants.

Substituting (4) and (6) into (3) and equating the coefficients of all powers of $z(\xi)$ and $z^j(\xi)\sqrt{c_1 + c_2 z^2 + (c_3/2) z^4}$ ($j = 0, 1, 2, \ldots$) to be zero in the resulting equation, several algebraic equations will be obtained. Then, solving these algebraic equations by the symbolic computation system Matlab and combining (4) and the solutions of the auxiliary equation (6), we can get the exact solutions for (2).

3. Exact Travelling Wave Solutions to (1)

Firstly, setting $u^p = w$ yields

$$\begin{aligned} u_x &= \frac{1}{p} w^{1/p-1} w_x, \qquad u_t = \frac{1}{p} w^{1/p-1} w_t, \\ u_{xx} &= \frac{1}{p}\left(\frac{1}{p} - 1\right) w^{1/p-2} (w_x)^2 + \frac{1}{p} w^{1/p-1} w_{xx}, \\ u_{xxx} &= \frac{1}{p}\left(\frac{1}{p} - 1\right)\left(\frac{1}{p} - 2\right) w^{1/p-3} w_x^3 \\ &\quad + \frac{3}{p}\left(\frac{1}{p} - 1\right) w^{1/p-2} w_x w_{xx} \\ &\quad + \frac{1}{p} w^{1/p-1} w_{xxx}, \end{aligned} \quad (7)$$

which turns (1) into

$$\begin{aligned} & w^2 w_t + \left(\alpha + \beta w + \lambda w^2\right) w^2 w_x + \left(\frac{1}{p} - 1\right)\left(\frac{1}{p} - 2\right) w_x^3 \\ & \quad + 3\left(\frac{1}{p} - 1\right) w w_x w_{xx} + w^2 w_{xxx} = 0. \end{aligned} \quad (8)$$

To find the traveling solutions for (1), we use the wave variable $\xi = \mu(x - ct)$, where $c \neq 0$ and $\mu \neq 0$. The wave variable ξ transforms (8) into the following ordinary differential equation:

$$\begin{aligned} & \left(\alpha + \beta w + \lambda w^2\right) w^2 w' + \left(\frac{1}{p} - 1\right)\left(\frac{1}{p} - 2\right) \mu^2 w'^3 \\ & \quad + 3\mu^2\left(\frac{1}{p} - 1\right) w w' w'' - c w^2 w' + \mu^2 w^2 w''' = 0. \end{aligned} \quad (9)$$

From (9), we have $N = 1$. Therefore, we choose the ansatz

$$w(\xi) = g_0 + g_1 z, \quad (10)$$

where $z(\xi)$ may be determined by

$$\left(\frac{dz}{d\xi}\right)^2 = c_1 + c_2 z^2 + \frac{c_3}{2} z^4, \quad (11)$$

which possesses several types of solutions listed in Table 1 (see [16]).

In Table 1, functions $\operatorname{sn}(\xi) = \operatorname{sn}(\xi, r)$, $\operatorname{cn}(\xi) = \operatorname{cn}(\xi, r)$, and $\operatorname{dn}(\xi) = \operatorname{dn}(\xi, r)$, are Jacobian elliptic functions with modulus r ($0 < r < 1$), which have the properties $\operatorname{sn}(-\xi) = -\operatorname{sn}(\xi)$, $\operatorname{cn}(-\xi) = \operatorname{cn}(\xi)$, $\operatorname{dn}(-\xi) = \operatorname{dn}(\xi)$, $\operatorname{sn}^2(\xi) + \operatorname{cn}^2(\xi) = 1$, $\operatorname{dn}^2(\xi) + r^2 \operatorname{sn}^2(\xi) = 1$, $(\operatorname{sn}(\xi))' = \operatorname{cn}(\xi)\operatorname{dn}(\xi)$, $(\operatorname{cn}(\xi))' = -\operatorname{sn}(\xi)\operatorname{dn}(\xi)$, and $(\operatorname{dn}(\xi))' = -r^2 \operatorname{sn}(\xi)\operatorname{cn}(\xi)$. Setting $r \to 0$ yields $\operatorname{sn}(\xi) \to \sin(\xi)$, $\operatorname{cn}(\xi) \to \cos(\xi)$, and $\operatorname{dn}(\xi) \to 1$. When $r \to 1$, it derives that $\operatorname{sn}(\xi) \to \tanh(\xi)$, $\operatorname{cn}(\xi) \to \operatorname{sech}(\xi)$, and $\operatorname{dn}(\xi) \to \operatorname{sech}(\xi)$.

Substituting (10) and (11) into (9) and letting each coefficient of $z^j \sqrt{c_1 + c_2 z^2 + (c_3/2) z^4}$ ($0 \leq j \leq 4$) be zero, we obtain

$$\begin{aligned} & \lambda g_1^5 + \frac{\mu^2}{2}\left(\frac{1}{p} - 1\right)\left(\frac{1}{p} - 2\right) g_1^3 c_3 \\ & \quad + 3\left(\frac{1}{p} - 1\right) \mu^2 g_1^3 c_3 + 3\mu^2 g_1^3 c_3 = 0, \\ & \left(\beta g_1 + 2\lambda g_0 g_1\right) g_1^3 + 2\lambda g_0 g_1^4 \\ & \quad + 3\left(\frac{1}{p} - 1\right) \mu^2 g_0 g_1^2 c_3 + 6\mu^2 g_0 g_1^2 c_3 = 0, \\ & -c g_1^3 + \left(\alpha + \beta g_0 + \lambda g_0^2\right) g_1^3 + 2\left(\beta g_1 + 2\lambda g_0 g_1\right) g_0 g_1^2 \\ & \quad + \lambda g_0^2 g_1^3 + \mu^2\left(\frac{1}{p} - 1\right)\left(\frac{1}{p} - 2\right) g_1^3 c_2 \end{aligned}$$

Table 1

No.	$z(\xi)$	c_3	c_2	c_1
1	$\mathrm{sn}(\xi), \mathrm{cd}(\xi) = \dfrac{\mathrm{cn}(\xi)}{\mathrm{dn}(\xi)}$	$2r^2$	$-(r^2+1)$	1
2	$\mathrm{cn}(\xi)$	$-2r^2$	$2r^2-1$	$1-r^2$
3	$\mathrm{dn}(\xi)$	-2	$2-r^2$	r^2-1
4	$\mathrm{nc}(\xi) = \dfrac{1}{\mathrm{cn}(\xi)}$	$2(1-r^2)$	$2r^2-1$	$-r^2$
5	$\mathrm{ns}(\xi) = \dfrac{1}{\mathrm{sn}(\xi)}, \mathrm{dc}(\xi) = \dfrac{\mathrm{dn}(\xi)}{\mathrm{cn}(\xi)}$	2	$-(r^2+1)$	r^2
6	$\mathrm{nd}(\xi) = \dfrac{1}{\mathrm{dn}(\xi)}$	$2(r^2-1)$	$2-r^2$	-1
7	$\mathrm{cs}(\xi) = \dfrac{\mathrm{cn}(\xi)}{\mathrm{sn}(\xi)}$	2	$2-r^2$	$1-r^2$
8	$\mathrm{sc}(\xi) = \dfrac{\mathrm{sn}(\xi)}{\mathrm{cn}(\xi)}$	$2(1-r^2)$	$2-r^2$	1
9	$\mathrm{sd}(\xi) = \dfrac{\mathrm{sn}(\xi)}{\mathrm{dn}(\xi)}$	$2r^2(r^2-1)$	$2r^2-1$	1
10	$\mathrm{ds}(\xi) = \dfrac{\mathrm{dn}(\xi)}{\mathrm{sn}(\xi)}$	2	$2r^2-1$	r^4-r^2
11	$r\mathrm{cn}(\xi) \pm \mathrm{dn}(\xi)$	$-\dfrac{1}{2}$	$\dfrac{r^2+1}{2}$	$-\dfrac{(1-r^2)^2}{4}$
12	$\dfrac{1}{\mathrm{sn}(\xi)} \pm \dfrac{\mathrm{cn}(\xi)}{\mathrm{sn}(\xi)}$	$\dfrac{1}{2}$	$\dfrac{-2r^2+1}{2}$	$\dfrac{1}{4}$
13	$\dfrac{1}{\mathrm{cn}(\xi)} \pm \dfrac{\mathrm{sn}(\xi)}{\mathrm{cn}(\xi)}$	$\dfrac{1-r^2}{2}$	$\dfrac{r^2+1}{2}$	$\dfrac{1-r^2}{4}$
14	$\dfrac{1}{\mathrm{sn}(\xi)} \pm \dfrac{\mathrm{dn}(\xi)}{\mathrm{sn}(\xi)}$	$\dfrac{1}{2}$	$\dfrac{r^2-2}{2}$	$\dfrac{r^4}{4}$
15	$\mathrm{sn}(\xi) \pm i\mathrm{cn}(\xi), \dfrac{\mathrm{dn}(\xi)}{\sqrt{1-r^2}\mathrm{sn}(\xi) \pm \mathrm{cn}(\xi)}$	$\dfrac{r^2}{2}$	$\dfrac{r^2-2}{2}$	$\dfrac{r^2}{4}$
16	$r\mathrm{sn}(\xi) \pm i\mathrm{dn}(\xi), \dfrac{\mathrm{sn}(\xi)}{1 \pm \mathrm{cn}(\xi)}$	$\dfrac{1}{2}$	$\dfrac{1-2r^2}{2}$	$\dfrac{1}{4}$
17	$\dfrac{\mathrm{sn}(\xi)}{1 \pm \mathrm{dn}(\xi)}$	$\dfrac{r^2}{2}$	$\dfrac{r^2-2}{2}$	$\dfrac{1}{4}$
18	$\dfrac{\mathrm{dn}(\xi)}{1 \pm r\mathrm{sn}(\xi)}$	$\dfrac{r^2-1}{2}$	$\dfrac{r^2+1}{2}$	$\dfrac{r^2-1}{4}$
19	$\dfrac{\mathrm{cn}(\xi)}{1 \pm \mathrm{sn}(\xi)}$	$\dfrac{1-r^2}{2}$	$\dfrac{r^2+1}{2}$	$\dfrac{-r^2+1}{4}$
20	$\dfrac{\mathrm{sn}(\xi)}{\mathrm{dn}(\xi) \pm \mathrm{cn}(\xi)}$	$\dfrac{(1-r^2)^2}{2}$	$\dfrac{r^2+1}{2}$	$\dfrac{1}{4}$
21	$\dfrac{\mathrm{cn}(\xi)}{\sqrt{1-r^2} \pm \mathrm{dn}(\xi)}$	$\dfrac{r^4}{2}$	$\dfrac{r^2-2}{2}$	$\dfrac{1}{4}$

$$
\begin{aligned}
&+3\left(\frac{1}{p}-1\right)\mu^2 g_1^3 c_2 + 3c_3\mu^2 g_0^2 + \mu^2 c_2 g_1^3 = 0,\\
&-2cg_0g_1^2 + 2\left(\alpha+\beta g_0+\lambda g_0^2\right)g_1^2 g_0 + \left(\beta g_1 + 2\lambda g_0 g_1\right)g_0^2 g_1\\
&\quad+3\left(\frac{1}{p}-1\right)\mu^2 g_1^2 g_0 c_2 + 2\mu^2 g_1^2 g_0 c_2 = 0,\\
&-cg_0^2 g_1 + \left(\alpha+\beta g_0+\lambda g_0^2\right)g_0^2 g_1\\
&\quad+\left(\frac{1}{p}-1\right)\left(\frac{1}{p}-2\right)\mu^2 g_1^3 c_1 + \mu^2 g_0^2 g_1 c_2 = 0.
\end{aligned}
\tag{12}
$$

Solving (12) with the help of Matlab, we acquire the following solutions:

$$
\begin{aligned}
&p=1, \quad g_0 = -\frac{\beta}{2\lambda}, \quad g_1 = \pm\frac{\beta}{2\lambda}\sqrt{-\frac{c_3}{c_2}},\\
&\mu = \pm\frac{\beta}{2}\sqrt{\frac{1}{3\lambda c_2}}, \quad c = \alpha - \frac{\beta^2}{6\lambda},
\end{aligned}
\tag{13}
$$

$$
p=\frac{1}{2}, \quad g_0 = -\frac{2\beta}{5\lambda}, \quad g_1 = \pm\frac{2\beta}{5\lambda}\sqrt{-\frac{c_3}{c_2}},
$$

$$\mu = \pm\frac{\beta}{5}\sqrt{\frac{2}{3\lambda c_2}}, \quad c = \alpha - \frac{16\beta^2}{75\lambda}, \tag{14}$$

$$p = 2, \quad \mu = \pm\left(\frac{5c_2\beta^2 + 5\epsilon\beta^2\sqrt{c_2^2 - 2c_1c_3}}{48c_1c_3\lambda}\right)^{1/2},$$
$$g_0 = -\frac{5\beta}{8\lambda}, \quad g_1 = \pm\frac{\mu}{4}\sqrt{-\frac{30c_3}{\lambda}}, \quad c = \alpha - \frac{5\beta^2}{32\lambda} + \frac{1}{4}\mu^2 c_2, \tag{15}$$

$$g_0 = -\frac{\beta(2p+1)}{2\lambda(p+2)}, \quad g_1 = \pm\frac{\beta(2p+1)}{2\lambda(p+2)}\sqrt{-\frac{c_3}{c_2}}, \quad 2c_1c_3 = c_2^2,$$
$$\mu = \pm\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{2\lambda(p+1)c_2}}, \quad c = \alpha - \frac{\beta^2(2p+1)}{\lambda(p+2)^2(p+1)}, \tag{16}$$

where the exponent $p \neq -2, -1$, and $\epsilon = \pm 1$.

3.1. The Jacobi Elliptic Function Solutions to (1) *in the Case of $p = 1$.* In order to make that wave number μ a real-valued number, we have to choose constants λ and modulus r (c_2 depends on r) to satisfy some restrictions. However, in this section, we allow μ to take values in complex number domain. From the expression of g_1, g_2, μ, and c in (13) and the solutions listed in Table 1, we derive the following Jacobi elliptic function solutions for (1) in the case $p = 1$:

$$u_{1.1}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta r}{2\lambda}\sqrt{\frac{2}{r^2+1}} \times \operatorname{sn}\left[\frac{\beta}{2}\sqrt{-\frac{1}{3\lambda(r^2+1)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.2}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta r}{2\lambda}\sqrt{\frac{2}{r^2+1}} \times \operatorname{cd}\left[\frac{\beta}{2}\sqrt{-\frac{1}{3\lambda(r^2+1)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.3}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta r}{2\lambda}\sqrt{\frac{2}{2r^2-1}} \times \operatorname{cn}\left[\frac{\beta}{2}\sqrt{\frac{1}{3\lambda(2r^2-1)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.4}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{2}{2-r^2}} \times \operatorname{dn}\left[\frac{\beta}{2}\sqrt{\frac{1}{3\lambda(2-r^2)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.5}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{2(r^2-1)}{2r^2-1}} \times \operatorname{nc}\left[\frac{\beta}{2}\sqrt{\frac{1}{3\lambda(2r^2-1)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.6}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{2}{r^2+1}} \times \operatorname{ns}\left[\frac{\beta}{2}\sqrt{-\frac{1}{3\lambda(r^2+1)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.7}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{2}{r^2+1}} \times \operatorname{dc}\left[\frac{\beta}{2}\sqrt{-\frac{1}{3\lambda(r^2+1)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.8}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{2(r^2-1)}{r^2-2}} \times \operatorname{nd}\left[\frac{\beta}{2}\sqrt{\frac{1}{3\lambda(2-r^2)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.9}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{2}{r^2-2}} \times \operatorname{cs}\left[\frac{\beta}{2}\sqrt{\frac{1}{3\lambda(2-r^2)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.10}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{2(1-r^2)}{r^2-2}} \times \operatorname{sc}\left[\frac{\beta}{2}\sqrt{\frac{1}{3\lambda(2-r^2)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.11}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta r}{2\lambda}\sqrt{\frac{2(r^2-1)}{1-2r^2}} \times \operatorname{sd}\left[\frac{\beta}{2}\sqrt{-\frac{1}{3\lambda(1-2r^2)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.12}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{2}{1-2r^2}} \times \operatorname{ds}\left[\frac{\beta}{2}\sqrt{-\frac{1}{3\lambda(1-2r^2)}}\left(x - \alpha t + \frac{\beta^2}{6\lambda}t\right)\right],$$

$$u_{1.13}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{1}{1+r^2}} \times \left\{r\operatorname{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}} \times \left(x - \alpha t + \frac{\beta^2 t}{6\lambda}\right)\right]\right.$$

$$\pm \operatorname{nd}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\Bigg\},$$

$$\begin{aligned}u_{1.14}(x,t) &= -\frac{\beta}{2\lambda}+\frac{\epsilon\beta}{2\lambda}\sqrt{\frac{1}{2r^2-1}} \\ &\quad\times\left\{1\times\left(\operatorname{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(1-2r^2)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right. \\ &\quad\pm\left(\operatorname{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(1-2r^2)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \\ &\quad\left.\times\left(\operatorname{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(1-2r^2)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right\},\end{aligned}$$

$$\begin{aligned}u_{1.15}(x,t) &= -\frac{\beta}{2\lambda}+\frac{\epsilon\beta}{2\lambda}\sqrt{\frac{r^2-1}{1+r^2}} \\ &\quad\times\left\{1\times\left(\operatorname{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right. \\ &\quad\pm\left(\operatorname{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \\ &\quad\left.\times\left(\operatorname{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right\},\end{aligned}$$

$$\begin{aligned}u_{1.16}(x,t) &= -\frac{\beta}{2\lambda}+\frac{\epsilon\beta}{2\lambda}\sqrt{\frac{1}{2-r^2}} \\ &\quad\times\left\{1\times\left(\operatorname{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right. \\ &\quad\pm\left(\operatorname{dn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \\ &\quad\left.\times\left(\operatorname{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right\},\end{aligned}$$

$$\begin{aligned}u_{1.17}(x,t) &= -\frac{\beta}{2\lambda}+\frac{\epsilon\beta r}{2\lambda}\sqrt{\frac{1}{2-r^2}} \\ &\quad\times\left\{\operatorname{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right. \\ &\quad\left.\pm i\operatorname{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right\},\end{aligned}$$

$$\begin{aligned}u_{1.18}(x,t) &= -\frac{\beta}{2\lambda}+\frac{\epsilon\beta r}{2\lambda}\sqrt{\frac{1}{2-r^2}} \\ &\quad\times\left(\left(\operatorname{dn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)\right. \\ &\quad\times\left(\sqrt{1-r^2}\operatorname{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right. \\ &\quad\left.\left.\pm\operatorname{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\times\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right),\end{aligned}$$

$$u_{1.19}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{1}{2r^2-1}} \times \left\{ r\,\mathrm{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(1-2r^2)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right] \pm i\,\mathrm{dn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(1-2r^2)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right\},$$

$$u_{1.20}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{1}{2r^2-1}} \times \left(\left(\mathrm{sn}\left[\frac{\beta}{2}\sqrt{\frac{1}{3\lambda(1-2r^2)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \times \left(1 \pm \mathrm{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(1-2r^2)}} \times \left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right),$$

$$u_{1.21}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{r^2}{2-r^2}} \times \left(\left(\mathrm{sn}\left[\frac{\beta}{2}\sqrt{\frac{1}{3\lambda(r^2-2)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \times \left(1 \pm \mathrm{dn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}} \times \left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right),$$

$$u_{1.22}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{1-r^2}{r^2+1}} \times \left(\left(\mathrm{dn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \times \left(1 \pm r\,\mathrm{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}} \times \left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right),$$

$$u_{1.23}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta}{2\lambda}\sqrt{\frac{r^2-1}{1+r^2}} \times \left(\left(\mathrm{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \times \left(1 \pm \mathrm{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}} \times \left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right),$$

$$u_{1.24}(x,t) = -\frac{\beta}{2\lambda} + \frac{\epsilon\beta(1-r^2)}{2\lambda}\sqrt{-\frac{1}{r^2+1}} \times \left(\left(\mathrm{sn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \times \left(\mathrm{dn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}} \times \left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right] \pm \mathrm{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2+1)}} \times \left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right),$$

$$u_{1.25}(x,t) = \frac{-\beta}{2\lambda} + \frac{\epsilon\beta r^2}{2\lambda\sqrt{2-r^2}} \times \left(\left(\mathrm{cn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}}\left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right) \times \left(\sqrt{1-r^2} \pm \mathrm{dn}\left[\frac{\beta}{2}\sqrt{\frac{2}{3\lambda(r^2-2)}} \times \left(x-\alpha t+\frac{\beta^2 t}{6\lambda}\right)\right]\right)^{-1}\right). \quad (17)$$

Remark 1. From the properties of Jacobi elliptic functions, we know that when the module $r \to 0$ or $r \to 1$, sn, cn, and dn degenerate into triangular or hyperbolic functions, from which we can obtain soliton and triangular function solutions of (1) in the case $p = 1$. In other word, when $p = 1$, we can deduce Jacobi elliptic functions solutions, soliton, and triangular function solutions for (1).

Remark 2. When $\alpha = 0$, $k = (\beta/2)\sqrt{2/3\lambda(1-2r^2)}$, $a_0 = -\beta/2\lambda$, and $a_1 = \pm k\sqrt{-3/2\lambda}$, $u_{1.14}(x,t)$ turns into

$$\begin{aligned} u_{1.14}(x,t) &= a_0 \\ &\quad + a_1 \left\{ \operatorname{ns} k \left[x - \left(\frac{1-2r^2}{2} k^2 + \lambda a_0^2 + \beta a_0 \right) t \right] \right. \\ &\quad \left. + \operatorname{cs} k \left[x - \left(\frac{1-2r^2}{2} k^2 + \lambda a_0^2 + \beta a_0 \right) t \right] \right\}, \end{aligned} \tag{18}$$

which was obtained by Yomba [17].

Remark 3. Supposing $k = (\beta/6)\sqrt{-1/q\nu\lambda}$, $p = (2 + \sqrt{3}/3)\sqrt{-6q\nu\lambda}$ (q and ν are arbitrary constants). When $r \to 1$ in $u_{1.17}(x,t)$, we obtain

$$\begin{aligned} u_{1.17.1}(x,t) &= a_0 - \frac{\epsilon\beta}{12\lambda q}\sqrt{-\frac{6}{q\nu\lambda}} \\ &\quad \times \left\{ \pm\sqrt{-2q\nu\lambda} + \sqrt{-6q\nu\lambda} \left(\tanh\sqrt{-6q\nu\lambda}\xi \right) \right. \\ &\quad \left. \pm i \operatorname{sech}\sqrt{-6q\nu\lambda}\xi \right\}, \end{aligned} \tag{19}$$

where $a_0 = -(\beta p/2\lambda)\sqrt{-6q\nu\lambda} + \beta/2\lambda, \xi = k(x - (\lambda a_0^2 + \beta a_0)t)$. Solution (19) is in full agreement with that presented in [17] by Yomba.

3.2. The Jacobi Elliptic Function Solutions to (1) *in the Case of* $p = 1/2$. Similarly as in Section 3.1, we allow μ to take values in complex number field. Making use of (14) and the solutions listed in Table 1, we obtain the following Jacobi elliptic function solutions for (1) in the case $p = 1/2$:

$$\begin{aligned} u_{2.1}(x,t) &= \left\{ -\frac{2\beta}{5\lambda} + \frac{2\epsilon\beta r}{5\lambda}\sqrt{\frac{2}{r^2+1}} \right. \\ &\quad \left. \times \operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{-2}{3\lambda(r^2+1)}} \left(x - \alpha t + \frac{16\beta^2}{75\lambda}t \right) \right] \right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2.2}(x,t) &= \left\{ -\frac{2\beta}{5\lambda} + \frac{2\epsilon\beta r}{5\lambda}\sqrt{\frac{2}{r^2+1}} \right. \\ &\quad \left. \times \operatorname{cd}\left[\frac{\beta}{5}\sqrt{\frac{-2}{3\lambda(r^2+1)}} \left(x - \alpha t + \frac{16\beta^2}{75\lambda}t \right) \right] \right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2.3}(x,t) &= \left\{ \frac{-2\beta}{5\lambda} + \frac{2\epsilon\beta r}{5\lambda}\sqrt{\frac{2}{2r^2-1}} \right. \\ &\quad \left. \times \operatorname{cn}\left[\sqrt{\frac{2\beta^2}{75\lambda(2r^2-1)}} \left(x - \alpha t + \frac{16\beta^2}{75\lambda}t \right) \right] \right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2.4}(x,t) &= \left\{ -\frac{2\beta}{5\lambda} + \frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{2}{2-r^2}} \right. \\ &\quad \left. \times \operatorname{dn}\left[\frac{\beta}{5}\sqrt{\frac{2}{3\lambda(2-r^2)}} \left(x - \alpha t + \frac{16\beta^2}{75\lambda}t \right) \right] \right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2.5}(x,t) &= \left\{ \frac{-2\beta}{5\lambda} + \frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{2r^2-2}{2r^2-1}} \right. \\ &\quad \left. \times \operatorname{nc}\left[\sqrt{\frac{2\beta^2}{75\lambda(2r^2-1)}} \left(x - \alpha t + \frac{16\beta^2}{75\lambda}t \right) \right] \right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2.6}(x,t) &= \left\{ -\frac{2\beta}{5\lambda} + \frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{2}{r^2+1}} \right. \\ &\quad \left. \times \operatorname{ns}\left[\frac{\beta}{5}\sqrt{-\frac{2}{3\lambda(r^2+1)}} \left(x - \alpha t + \frac{16\beta^2}{75\lambda}t \right) \right] \right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2.7}(x,t) &= \left\{ -\frac{2\beta}{5\lambda} + \frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{2}{r^2+1}} \right. \\ &\quad \left. \times \operatorname{dc}\left[\frac{\beta}{5}\sqrt{-\frac{2}{3\lambda(r^2+1)}} \left(x - \alpha t + \frac{16\beta^2}{75\lambda}t \right) \right] \right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2.8}(x,t) &= \left\{ \frac{-2\beta}{5\lambda} + \frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{2-2r^2}{2-r^2}} \right. \\ &\quad \left. \times \operatorname{nd}\left[\frac{\beta}{5}\sqrt{\frac{2}{3\lambda(2-r^2)}} \left(x - \alpha t + \frac{16\beta^2}{75\lambda}t \right) \right] \right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2.9}(x,t) &= \left\{ -\frac{2\beta}{5\lambda} + \frac{2\epsilon\beta}{5\lambda}\sqrt{-\frac{2}{2-r^2}} \right. \end{aligned}$$

$$\times \operatorname{cs}\left[\frac{\beta}{5}\sqrt{\frac{2}{3\lambda(2-r^2)}}\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\Bigg\}^2,$$

$$u_{2.10}(x,t) = \left\{\frac{-2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{2r^2-2}{2-r^2}} \times \operatorname{sc}\left[\frac{\beta}{5}\sqrt{\frac{2}{3\lambda(2-r^2)}}\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right\}^2,$$

$$u_{2.11}(x,t) = \left\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{2r^2(1-r^2)}{2r^2-1}} \times \operatorname{sd}\left[\frac{\beta}{5}\sqrt{\frac{2}{3\lambda(2r^2-1)}}\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right\}^2,$$

$$u_{2.12}(x,t) = \left\{\frac{-2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{2}{1-2r^2}} \times \operatorname{ds}\left[\sqrt{\frac{2\beta^2}{75\lambda(2r^2-1)}}\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right\}^2,$$

$$u_{2.13}(x,t) = \left\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{1}{r^2+1}} \times \left(r\operatorname{cn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right] \pm \operatorname{dn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)\right\}^2,$$

$$u_{2.14}(x,t) = \left\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{-\frac{1}{1-2r^2}} \times \left(1\times\left(\operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(-2r^2+1)}} \times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1} \pm\left(\operatorname{cn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(-2r^2+1)}} \times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right) \times\left(\operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(-2r^2+1)}} \times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1}\right)\right\}^2,$$

$$u_{2.15}(x,t) = \left\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{-\frac{1-r^2}{r^2+1}} \times \left(1\times\left(\operatorname{cn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}} \times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1} \pm\left(\operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}} \times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right) \times\left(\operatorname{cn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}} \times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1}\right)\right\}^2,$$

$$u_{2.16}(x,t) = \left\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{-\frac{1}{r^2-2}} \times \left(1\times\left(\operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2-2)}} \times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1} \pm\left(\operatorname{dn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2-2)}} \times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right) \times\left(\operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2-2)}}\right.\right.\right.$$

$$\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)^{-1}\Bigg)\Bigg\}^2,$$

$u_{2.17}(x,t)$

$$\begin{aligned}
&=\Bigg\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta r}{5\lambda}\sqrt{-\frac{1}{r^2-2}}\\
&\quad\times\Bigg(\operatorname{sn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2-2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\\
&\quad\pm i\operatorname{cn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2-2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)\Bigg\}^2,
\end{aligned}$$

$u_{2.18}(x,t)$

$$\begin{aligned}
&=\Bigg\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta r}{5\lambda}\sqrt{-\frac{1}{r^2-2}}\\
&\quad\times\Bigg(\Bigg(\operatorname{dn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2-2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)\\
&\quad\times\Bigg(\sqrt{1-r^2}\operatorname{sn}\Bigg[\sqrt{\frac{4\beta^2}{75\lambda(r^2-2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\\
&\quad\pm\operatorname{cn}\Bigg[\sqrt{\frac{4\beta^2}{75\lambda(r^2-2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)^{-1}\Bigg)\Bigg\}^2,
\end{aligned}$$

$u_{2.19}(x,t)$

$$\begin{aligned}
&=\Bigg\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda\sqrt{2r^2-1}}\\
&\quad\times\Bigg(\Bigg(\operatorname{sn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(1-2r^2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)\\
&\quad\times\Bigg(1\pm\operatorname{cn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(1-2r^2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)^{-1}\Bigg)\Bigg\}^2,
\end{aligned}$$

$u_{2.20}(x,t)$

$$\begin{aligned}
&=\Bigg\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{1}{2r^2-1}}\\
&\quad\times\Bigg(r\operatorname{sn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(1-2r^2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\\
&\quad\pm i\operatorname{dn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(1-2r^2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)\Bigg\}^2,
\end{aligned}$$

$u_{2.21}(x,t)$

$$\begin{aligned}
&=\Bigg\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta r}{5\lambda}\sqrt{\frac{1}{2-r^2}}\\
&\quad\times\Bigg(\Bigg(\operatorname{sn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(2-r^2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)\\
&\quad\times\Bigg(1\pm\operatorname{dn}\Bigg[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(2-r^2)}}\\
&\qquad\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\Bigg]\Bigg)^{-1}\Bigg)\Bigg\}^2,
\end{aligned}$$

$u_{2.22}(x,t)$

$$=\Bigg\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{1-r^2}{r^2+1}}$$

$$\times\left(\left(\operatorname{dn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)\times\left(1\pm r\operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1}\right)\Bigg\}^2,$$

$u_{2.23}(x,t)$

$$=\left\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\sqrt{\frac{r^2-1}{r^2+1}}\times\left(\left(\operatorname{cn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)\times\left(1\pm\operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1}\right)\right\}^2,$$

$u_{2.24}(x,t)$

$$=\left\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta\left(1-r^2\right)}{5\lambda}\sqrt{-\frac{1}{r^2+1}}\times\left(\left(\operatorname{sn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)\times\left(\operatorname{dn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\pm\operatorname{cn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2+1)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1}\right)\right\}^2,$$

$u_{2.25}(x,t)$

$$=\left\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta r^2}{5\lambda}\sqrt{\frac{1}{2-r^2}}\times\left(\left(\operatorname{cn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2-2)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)\times\left(\sqrt{1-r^2}\pm\operatorname{dn}\left[\frac{\beta}{5}\sqrt{\frac{4}{3\lambda(r^2-2)}}\times\left(x-\alpha t+\frac{16\beta^2}{75\lambda}t\right)\right]\right)^{-1}\right)\right\}^2. \tag{20}$$

When $p=1/2$, we can also obtain Jacobi elliptic functions solutions, soliton, and triangular function solutions for (1).

3.3. The Jacobi Elliptic Function Solutions to (1) *in the Case of* $p=2$. From the expressions of (15), we get the following jacobi elliptic function solutions to (1):

$u_{3.1}(x,t)$

$$=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu r}{2}\sqrt{-\frac{15}{\lambda}}\times\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t+\frac{t}{4}\left(r^2+1\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{4r}\left(\frac{-5\left(r^2+1\right)+5\epsilon\left(r^2-1\right)}{6\lambda}\right)^{1/2},$$

$u_{3.2}(x,t)$

$$=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu r}{2}\sqrt{-\frac{15}{\lambda}}\times\operatorname{cd}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t+\frac{t}{4}\left(r^2+1\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{4r}\left(\frac{-5\left(r^2+1\right)+5\epsilon\left(r^2-1\right)}{6\lambda}\right)^{1/2},$$

$u_{3.3}(x,t)$

$$=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu r}{2}\sqrt{\frac{15}{\lambda}}\times\operatorname{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{4}\left(2r^2-1\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4r}\left(\frac{5\left(2r^2-1\right)+5\epsilon}{6\left(r^2-1\right)\lambda}\right)^{1/2},$$

$u_{3.4}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{2}\sqrt{\frac{15}{\lambda}} \times \mathrm{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{4}\left(2-r^2\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4}\left(\frac{5\left(2-r^2\right)+5\epsilon r^2}{6\left(1-r^2\right)\lambda}\right)^{1/2},$$

$u_{3.5}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{2}\sqrt{-\frac{15\left(1-r^2\right)}{\lambda}} \times \mathrm{nc}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{4}\left(2r^2-1\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4r}\left(\frac{5\left(2r^2-1\right)+5\epsilon}{6\left(r^2-1\right)\lambda}\right)^{1/2},$$

$u_{3.6}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{2}\sqrt{-\frac{15}{\lambda}} \times \mathrm{ns}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t+\frac{t}{4}\left(r^2+1\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4r}\left(\frac{-5\left(r^2+1\right)+5\epsilon\left(r^2-1\right)}{6\lambda}\right)^{1/2},$$

$u_{3.7}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{2}\sqrt{-\frac{15}{\lambda}} \times \mathrm{dc}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t+\frac{t}{4}\left(r^2+1\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4r}\left(\frac{-5\left(r^2+1\right)+5\epsilon\left(r^2-1\right)}{6\lambda}\right)^{1/2},$$

$u_{3.8}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{2}\sqrt{-\frac{15\left(r^2-1\right)}{\lambda}} \times \mathrm{nd}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{4}\left(2-r^2\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4}\left(\frac{5\left(2-r^2\right)+5\epsilon r^2}{6\left(1-r^2\right)\lambda}\right)^{1/2},$$

$u_{3.9}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{2}\sqrt{-\frac{15}{\lambda}} \times \mathrm{cs}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{4}\left(2-r^2\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4}\left(\frac{5\left(2-r^2\right)+5\epsilon r^2}{6\lambda\left(1-r^2\right)}\right)^{1/2},$$

$u_{3.10}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{2}\sqrt{-\frac{15\left(1-r^2\right)}{\lambda}} \times \mathrm{sc}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{4}\left(2-r^2\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4}\left(\frac{5\left(2-r^2\right)+5\epsilon r^2}{6\lambda\left(1-r^2\right)}\right)^{1/2},$$

$u_{3.11}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu r}{2}\sqrt{\frac{15\left(1-r^2\right)}{\lambda}} \times \mathrm{sd}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{4}\left(2r^2-1\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4r}\left(\frac{5\left(2r^2-1\right)+5\epsilon}{6\left(r^2-1\right)\lambda}\right)^{1/2},$$

$u_{3.12}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{2}\sqrt{-\frac{15}{\lambda}} \times \mathrm{ds}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{4}\left(2r^2-1\right)\mu^2\right)\right]\right\}^{1/2},$$

$$\mu = \pm\frac{\beta}{4r}\left(\frac{5\left(2r^2-1\right)+5\epsilon}{6\left(r^2-1\right)\lambda}\right)^{1/2},$$

$u_{3.13}(x,t)$

$$= \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{4}\sqrt{\frac{15}{\lambda}}\right.$$

$$\times\left(r\,\mathrm{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right.$$
$$\left.\left.\pm\,\mathrm{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right)\right\}^{1/2},$$
$$\mu=\pm\frac{\beta}{2\left(1-r^2\right)}\left(\frac{5\left(r^2+1\right)+10\epsilon r}{3\lambda}\right)^{1/2},$$

$u_{3.14}(x,t)$

$$=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{4}\sqrt{-\frac{15}{\lambda}}\right.$$
$$\times\left(1\times\left(\mathrm{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(1-2r^2\right)\right)\right]\right)^{-1}\right.$$
$$\pm\left(\mathrm{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(1-2r^2\right)\mu^2\right)\right]\right)$$
$$\left.\left.\times\left(\mathrm{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(1-2r^2\right)\mu^2\right)\right]\right)^{-1}\right)\right\}^{1/2},$$
$$\mu=\pm\frac{\beta}{2}\left(\frac{5\left(1-2r^2\right)+10\epsilon r\sqrt{r^2-1}}{3\lambda}\right)^{1/2},$$

$u_{3.15}(x,t)$

$$=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{4}\sqrt{-\frac{15\left(1-r^2\right)}{\lambda}}\right.$$
$$\times\left(1\times\left(\mathrm{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right)^{-1}\right.$$
$$\pm\left(\mathrm{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right)$$
$$\left.\left.\times\left(\mathrm{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right)^{-1}\right)\right\}^{1/2},$$
$$\mu=\pm\frac{\beta}{2\left(1-r^2\right)}\left(\frac{5\left(r^2+1\right)+10\epsilon r}{3\lambda}\right)^{1/2},$$

$u_{3.16}(x,t)$

$$=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{4}\sqrt{-\frac{15}{\lambda}}\right.$$
$$\times\left(1\times\left(\mathrm{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2-2\right)\mu^2\right)\right]\right)^{-1}\right.$$
$$\pm\left(\mathrm{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2-2\right)\mu^2\right)\right]\right)$$
$$\left.\left.\times\left(\mathrm{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2-2\right)\mu^2\right)\right]\right)^{-1}\right)\right\}^{1/2},$$
$$\mu=\pm\frac{\beta}{2r^2}\left(\frac{5\left(r^2-2\right)+10\epsilon\sqrt{1-r^2}}{3\lambda}\right)^{1/2},$$

$u_{3.17}(x,t)$

$$=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu r}{4}\sqrt{-\frac{15}{\lambda}}\right.$$
$$\times\left(\mathrm{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2-2\right)\mu^2\right)\right]\right.$$
$$\left.\left.\pm\,i\,\mathrm{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2-2\right)\mu^2\right)\right]\right)\right\}^{1/2},$$
$$\mu=\pm\frac{\beta}{2r^2}\left(\frac{5\left(r^2-2\right)+10\epsilon\sqrt{1-r^2}}{3\lambda}\right)^{1/2},$$

$$u_{3.18}(x,t) = \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu r}{4}\sqrt{-\frac{15}{\lambda}} \times\left(\left(\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t - \frac{t}{8}\left(r^2-2\right)\mu^2\right)\right]\right) \times\left(\sqrt{1-r^2}\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t - \frac{t\mu^2\left(r^2-2\right)}{8}\right)\right] \pm \operatorname{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t - \frac{t\mu^2\left(r^2-2\right)}{8}\right)\right]\right)^{-1}\right)\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{2r^2}\left(\frac{5\left(r^2-2\right)+10\epsilon\sqrt{1-r^2}}{3\lambda}\right)^{1/2},$$

$$u_{3.19}(x,t) = \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{4}\sqrt{-\frac{15}{\lambda}} \times\left(r\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t - \frac{t}{8}\left(1-2r^2\right)\mu^2\right)\right] \pm i\operatorname{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t - \frac{t}{8}\left(1-2r^2\right)\mu^2\right)\right]\right)\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{2}\left(\frac{5\left(1-2r^2\right)+10\epsilon r\sqrt{r^2-1}}{3\lambda}\right)^{1/2},$$

$$u_{3.20}(x,t) = \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{4}\sqrt{-\frac{15}{\lambda}} \times\left(\left(\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(1-2r^2\right)\mu^2\right)\right]\right) \times\left(1\pm\operatorname{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t - \frac{t}{8}\left(1-2r^2\right)\mu^2\right)\right]\right)^{-1}\right)\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{2}\left(\frac{5\left(1-2r^2\right)+10\epsilon r\sqrt{r^2-1}}{3\lambda}\right)^{1/2},$$

$$u_{3.21}(x,t) = \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu r}{4}\sqrt{-\frac{15}{\lambda}} \times\left(\left(\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2-2\right)\mu^2\right)\right]\right) \times\left(1\pm\operatorname{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t - \frac{t}{8}\left(r^2-2\right)\mu^2\right)\right]\right)^{-1}\right)\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{2r}\left(\frac{5\left(r^2-2\right)+5\epsilon\sqrt{r^4-5r^2+4}}{3\lambda}\right)^{1/2},$$

$$u_{3.22}(x,t) = \left\{\frac{-5\beta}{8\lambda}+\frac{\epsilon\mu}{4}\sqrt{\frac{15\left(1-r^2\right)}{\lambda}} \times\left(\left(\operatorname{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right) \times\left(1\pm r\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t - \frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right)^{-1}\right)\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{2\left(1-r^2\right)}\left(\frac{5\left(r^2+1\right)+10\epsilon r}{3\lambda}\right)^{1/2},$$

$$u_{3.23}(x,t) = \left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu}{4}\sqrt{\frac{15\left(r^2-1\right)}{\lambda}} \times\left(\left(\operatorname{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right) \times\left(1\pm\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t\right.\right.\right.\right.\right.$$

$$-\frac{t}{8}\left(r^2+1\right)\mu^2\Big)\Big]\Big)^{-1}\Big)\Big\}^{1/2},$$

$$\mu=\pm\frac{\beta}{2\left(1-r^2\right)}\left(\frac{5\left(r^2+1\right)+10\epsilon r}{3\lambda}\right)^{1/2},$$

$$u_{3.24}(x,t)=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu\left(r^2-1\right)}{4}\sqrt{-\frac{15}{\lambda}}\times\left(\left(\operatorname{sn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right)\times\left(\operatorname{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\pm\operatorname{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t}{8}\left(r^2+1\right)\mu^2\right)\right]\right)^{-1}\right)\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{2\left(1-r^2\right)}\left(\frac{5\left(r^2+1\right)+10\epsilon r}{3\lambda}\right)^{1/2},$$

$$u_{3.25}(x,t)=\left\{-\frac{5\beta}{8\lambda}+\frac{\epsilon\mu r^2}{4}\sqrt{-\frac{15}{\lambda}}\times\left(\left(\operatorname{cn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t\mu^2\left(r^2-2\right)}{8}\right)\right]\right)\times\left(\sqrt{1-r^2}\pm\operatorname{dn}\left[\mu\left(x-\alpha t+\frac{5\beta^2}{32\lambda}t-\frac{t\mu^2\left(r^2-2\right)}{8}\right)\right]\right)^{-1}\right)\right\}^{1/2},$$

$$\mu=\pm\frac{\beta}{2r^2}\left(\frac{5\left(r^2-2\right)+10\epsilon\sqrt{1-r^2}}{3\lambda}\right)^{1/2}. \tag{21}$$

3.4. Soliton and Triangular Function Solutions of (1). From the expression of $2c_1c_3=c_2^2$, we know that $r\to 0$ or $r\to 1$. According to the properties of Jacobi elliptic functions sn, cn, and dn, the following degenerated soliton and triangular function solutions for (1) can be obtained:

$$u_{4.1}(x,t)=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\tanh\left[\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{4\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right]\right\}^{1/p},$$

$$u_{4.2}(x,t)=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+\epsilon\frac{\sqrt{2}\beta(2p+1)}{2\lambda(p+2)}\times\operatorname{sech}\left[\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{2\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right]\right\}^{1/p},$$

$$u_{4.3}(x,t)=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\coth\left[\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{4\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right]\right\}^{1/p},$$

$$u_{4.4}(x,t)=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+i\epsilon\frac{\sqrt{2}\beta(2p+1)}{2\lambda(p+2)}\times\operatorname{csch}\left[\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{2\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right]\right\}^{1/p},$$

$$u_{4.5}(x,t)=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\left(\left(\cosh\left(\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right)\pm 1\right)\right.$$

$$\times\left(\sinh\left(\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right)\right)^{-1}\Bigg)\Bigg\}^{1/p},$$

$u_{4.6}(x,t)$

$$=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\left(\left(\sinh\left(\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{\lambda(p+1)}}\times\left[x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right]\right)\pm i\right)\times\left(\cosh\left(\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{\lambda(p+1)}}\times\left[x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right]\right)\right)^{-1}\right)\right\}^{1/p},$$

$u_{4.7}(x,t)$

$$=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\left(\left(\sinh\left(\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right)\right)\times\left(1\pm\cosh\left(\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{\lambda(p+1)}}\times\left[x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right]\right)\right)^{-1}\right)\right\}^{1/p},$$

$u_{4.8}(x,t)$

$$=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+\epsilon\frac{\sqrt{2}\beta(2p+1)}{2\lambda(p+2)}\times\sec\left[\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{2\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right]\right\}^{1/p},$$

$u_{4.9}(x,t)$

$$=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+\epsilon\frac{\sqrt{2}\beta(2p+1)}{2\lambda(p+2)}\times\csc\left[\frac{\beta p}{p+2}\sqrt{-\frac{2p+1}{2\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right]\right\}^{1/p},$$

$u_{4.10}(x,t)$

$$=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+i\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\cot\left[\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{4\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right]\right\}^{1/p},$$

$u_{4.11}(x,t)$

$$=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+i\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\tan\left[\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{4\lambda(p+1)}}\times\left(x-\alpha t+\frac{\beta^2(2p+1)t}{\lambda(p+2)^2(p+1)}\right)\right]\right\}^{1/p},$$

$u_{4.12}(x,t)$

$$=\left\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+i\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\right.$$

$$\times\Bigg(\Bigg(\Bigg(1\pm\cos\Bigg(\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{\lambda(p+1)}}\times\Bigg(x-\alpha t+\frac{\beta^{2}(2p+1)t}{\lambda(p+2)^{2}(p+1)}\Bigg)\Bigg)\Bigg)\times\Bigg(\sin\Bigg(\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{\lambda(p+1)}}\times\Bigg(x-\alpha t+\frac{\beta^{2}(2p+1)t}{\lambda(p+2)^{2}(p+1)}\Bigg)\Bigg)\Bigg)^{-1}\Bigg)\Bigg\}^{1/p},$$

$$u_{4.13}(x,t)=\Bigg\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+i\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\Bigg(\Bigg(1\pm\sin\Bigg(\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{\lambda(p+1)}}\times\Bigg[x-\alpha t+\frac{\beta^{2}(2p+1)t}{\lambda(p+2)^{2}(p+1)}\Bigg]\Bigg)\Bigg)\times\Bigg(\cos\Bigg(\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{\lambda(p+1)}}\times\Bigg[x-\alpha t+\frac{\beta^{2}(2p+1)t}{\lambda(p+2)^{2}(p+1)}\Bigg]\Bigg)\Bigg)^{-1}\Bigg)\Bigg\}^{1/p},$$

$$u_{4.14}(x,t)=\Bigg\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+i\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\Bigg(\Bigg(\sin\Bigg(\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{\lambda(p+1)}}\times\Bigg(x-\alpha t+\frac{\beta^{2}(2p+1)t}{\lambda(p+2)^{2}(p+1)}\Bigg)\Bigg)\Bigg)\times\Bigg(1\pm\cos\Bigg(\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{\lambda(p+1)}}\times\Bigg[x-\alpha t+\frac{\beta^{2}(2p+1)t}{\lambda(p+2)^{2}(p+1)}\Bigg]\Bigg)\Bigg)^{-1}\Bigg)\Bigg\}^{1/p},$$

$$u_{4.15}(x,t)=\Bigg\{-\frac{\beta(2p+1)}{2\lambda(p+2)}+i\epsilon\frac{\beta(2p+1)}{2\lambda(p+2)}\times\Bigg(\Bigg(\cos\Bigg(\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{\lambda(p+1)}}\times\Bigg[x-\alpha t+\frac{\beta^{2}(2p+1)t}{\lambda(p+2)^{2}(p+1)}\Bigg]\Bigg)\Bigg)\times\Bigg(1\pm\sin\Bigg(\frac{\beta p}{p+2}\sqrt{\frac{2p+1}{2\lambda(p+1)}}\times\Bigg(x-\alpha t+\frac{\beta^{2}(2p+1)t}{\lambda(p+2)^{2}(p+1)}\Bigg)\Bigg)\Bigg)^{-1}\Bigg)\Bigg\}^{1/p}. \tag{22}$$

Remark 4. When $r\to 0$ and $r\to 1$ in the solutions $u_{1.1}(x,t)$ to $u_{3.25}(x,t)$, the soliton and triangular function solutions we obtain can be found in $u_{4.1}(x,t)$ to $u_{4.15}(x,t)$ while $p=1,2,1/2$, respectively. For example, when $r\to 1$, $u_{2.17}$ and $u_{3.17}$ can be expressed by

$$u_{2.17.1}(x,t)=\Bigg\{-\frac{2\beta}{5\lambda}+\frac{2\epsilon\beta}{5\lambda}\times\Bigg(\tanh\Bigg[\frac{\beta}{5}\sqrt{-\frac{4}{3\lambda}}\Bigg(x-\alpha t+\frac{16\beta^{2}}{75\lambda}t\Bigg)\Bigg]\pm i\,\mathrm{sech}\Bigg[\frac{\beta}{5}\sqrt{-\frac{4}{3\lambda}}\Bigg(x-\alpha t+\frac{16\beta^{2}}{75\lambda}t\Bigg)\Bigg]\Bigg)\Bigg\}^{2},$$

$$u_{3.17.1}(x,t) = \left\{-\frac{5\beta}{8\lambda} + \frac{5\epsilon\beta}{8\lambda} \times \left(\tanh\left[\frac{\beta}{2}\sqrt{-\frac{5}{3\lambda}}\left(x-\alpha t+\frac{5\beta^2}{48\lambda}t\right)\right] \pm i\,\mathrm{sech}\left[\frac{\beta}{2}\sqrt{-\frac{5}{3\lambda}}\left(x-\alpha t+\frac{5\beta^2}{48\lambda}t\right)\right]\right)\right\}^{1/2}, \quad (23)$$

which are the same as $u_{4.6}$ in the case of $p = 1/2, 2$, respectively.

Remark 5. In these expressions of solutions, $u_{4.1}(x,t)$ and $u_{4.2}(x,t)$ are in full agreement with the result which was obtained by Zhang et al. [15].

4. Conclusion

In this paper, by using the auxiliary differential equation method, we obtain the Jacobi function solutions, the degenerated solitons, and the triangular function solutions to the generalized dispersive KdV-mKdV equation (1) in the case of $p = 1$, $p = 2$, and $p = 1/2$ with the help of symbolic computation system Matlab. However, for arbitrary exponent p, we only obtain the degenerated solitons and triangle function solutions to (1). It may need other technique to deal with the Jacobi elliptic function solutions for the generalized dispersive KdV-mKdV equation with arbitrary exponent p.

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