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Research Article

A Note on Eulerian Polynomials

D. S. Kim,[1] T. Kim,[2] W. J. Kim,[3] and D. V. Dolgy[4]

[1] *Department of Mathematics, Sogang University, Seoul 121-742, Republic of Korea*
[2] *Department of Mathematics, Kwangwoon University, Seoul 139-701, Republic of Korea*
[3] *Division of General Education-Mathematics, Kwangwoon University, Seoul 139-701, Republic of Korea*
[4] *Hanrimwon, Kwangwoon University, Seoul 139-701, Republic of Korea*

Correspondence should be addressed to T. Kim, tkkim@kw.ac.kr

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We study Genocchi, Euler, and tangent numbers. From those numbers we derive some identities on Eulerian polynomials in connection with Genocchi and tangent numbers.

1. Introduction

As is well known, the Eulerian polynomials, $A_n(t)$, are defined by generating function as follows:

$$\frac{1-t}{\exp(x(t-1))-t} = e^{A(t)x} = \sum_{n=0}^{\infty} A_n(t)\frac{x^n}{n!}, \tag{1.1}$$

with the usual convention about replacing $A^n(t)$ by $A_n(t)$ (see [1–18]). From (1.1), we note that

$$(A(t)+(t-1))^n - tA_n(t) = (1-t)\delta_{0,n}, \tag{1.2}$$

where $\delta_{n,k}$ is the Kronecker symbol (see [3]).

Thus, by (1.2), we get

$$A_0(t) = 1, \qquad A_n(t) = \frac{1}{t-1}\sum_{l=0}^{n-1}\binom{n}{l}A_l(t)(t-1)^{n-l}, \quad (n \geq 1). \tag{1.3}$$

By (1.1), (1.2), and (1.3), we see that

$$\sum_{i=1}^{m} i^n t^i = \sum_{l=1}^{n} (-1)^{n+l} \binom{n}{l} \frac{t^{m+1} A_{n-l}(t)}{(t-1)^{n-l+1}} m^l + (-1)^n \frac{t(t^m-1)}{(t-1)^{n+1}} A_n(t), \tag{1.4}$$

where $m \geq 1$ and $n \geq 0$ (see [1]).

The Genocchi polynomials are defined by

$$\frac{2t}{e^t+1} e^{xt} = e^{G(x)t} = \sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!}, \tag{1.5}$$

(see [6–18]). In the special case, $x = 0$, $G_n(0) = G_n$ are called the nth Genocchi numbers (see [14, 17, 18]).

It is well known that the Euler polynomials are also defined by

$$\frac{2}{e^t+1} e^{xt} = e^{E(x)t} = \sum_{n=0}^{\infty} E_n(x) \frac{t^n}{n!}, \tag{1.6}$$

(see [1–5, 19–24]). Here $x = 0$, then $E_n(0) = E_n$ is called the nth Euler number. From (1.6), we have

$$E_0 = 1, \qquad (E+1)^n + E_n = 2\delta_{0,n}, \tag{1.7}$$

(see [3–5, 19–23]).

As is well known, the Bernoulli numbers are defined by

$$B_0 = 1, \qquad (B+1)^n - B_n = \delta_{0,n}, \tag{1.8}$$

(see [5, 18, 19]), with the usual convention about replacing B^n by B_n.

From (1.8), we note that the Bernoulli polynomials are also defined as

$$B_n(x) = \sum_{l=0}^{n} \binom{n}{l} B_l x^{n-l} = (B+x)^n, \tag{1.9}$$

(see [5, 18, 19]).

The tangent numbers T_{2n-1} $(n \geq 1)$ are defined as the coefficients of the Taylor expansion of $\tan x$:

$$\tan x = \sum_{n=1}^{\infty} \frac{T_{2n-1}}{(2n-1)!} x^{2n-1} = \frac{x}{1!} + \frac{x^3}{3!} 2 + \frac{x^5}{5!} 16 + \cdots, \tag{1.10}$$

(see [1–3, 5]).

In this paper, we give some identities on the Eulerian polynomials at $t = -1$ associated with Genocchi, Euler, and tangent numbers.

2. Witt's Formula for Eulerian Polynomials

In this section, we assume that $\mathbb{Z}_p$, $\mathbb{Q}_p$, and $\mathbb{C}_p$ will, respectively, denote the ring of p-adic integers, the field of p-adic numbers, and the completion of algebraic closure of $\mathbb{Q}_p$. The p-adic norm is normalized so that $|p|_p = 1/p$.

Let q be an indeterminate with $|1-q|_p < 1$. Then the q-number is defined by

$$[x]_q = \frac{1-q^x}{1-q}, \qquad [x]_{-q} = \frac{1-(-q)^x}{1+q}, \tag{2.1}$$

(see [6–18]).

Let $C(\mathbb{Z}_p)$ be the space of continuous functions on $\mathbb{Z}_p$. For $f \in C(\mathbb{Z}_p)$, the fermionic p-adic q-integral on $\mathbb{Z}_p$ is defined by

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)\,d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x)(-q)^x, \tag{2.2}$$

(see [7, 10–13]). From (2.2), we can derive the following:

$$q^{-1} I_{-q^{-1}}(f_1) + I_{-q^{-1}}(f) = [2]_{q^{-1}} f(0), \tag{2.3}$$

where $f_1(x) = f(x+1)$.

Let us take $f(x) = e^{-x(1+q)t}$. Then, by (2.3), we get

$$\left(\frac{q+e^{-(1+q)t}}{q}\right) \int_{\mathbb{Z}_p} e^{-x(1+q)t}\,d\mu_{-q^{-1}}(x) = [2]_{q^{-1}}. \tag{2.4}$$

Thus, from (2.4), we have

$$\int_{\mathbb{Z}_p} e^{-x(1+q)t}\,d\mu_{-q^{-1}}(x) = \frac{1+q}{e^{-(1+q)t}+q} = \sum_{n=0}^{\infty} A_n(-q)\frac{t^n}{n!}. \tag{2.5}$$

By Taylor expansion on the left-hand side of (2.5), we get

$$\sum_{n=0}^{\infty} (-1)^n \int_{\mathbb{Z}_p} x^n\,d\mu_{-q^{-1}}(x)(1+q)^n \frac{t^n}{n!} = \sum_{n=0}^{\infty} A_n(-q)\frac{t^n}{n!}. \tag{2.6}$$

Comparing coefficients on the both sides of (2.6), we have

$$\int_{\mathbb{Z}_p} x^n\,d\mu_{-q^{-1}}(x) = \frac{(-1)^n}{(1+q)^n} A_n(-q). \tag{2.7}$$

Therefore, by (2.7), we obtain the following theorem.

Theorem 2.1. *For $n \in \mathbb{Z}_+$, one has*

$$\int_{\mathbb{Z}_p} x^n d\mu_{-q^{-1}}(x) = \frac{(-1)^n}{(1+q)^n} A_n(-q), \tag{2.8}$$

where $A_n(-q)$ is an Eulerian polynomials.

It seems interesting to study Theorem 2.1 at $q = 1$. By (2.3), we get

$$I_{-1}(f_1) + I_{-1}(f) = 2f(0), \tag{2.9}$$

where $f_1(x) = f(x+1)$. From (2.9), we can derive the following equation:

$$\int_{\mathbb{Z}_p} f(x+n) d\mu_{-1}(x) + (-1)^{n-1} \int_{\mathbb{Z}_p} f(x) d\mu_{-1}(x) = 2\sum_{l=0}^{n-1} (-1)^{n-l+1} f(l), \tag{2.10}$$

where $n \in \mathbb{Z}_+$ (see [5–13]).

From (2.9), we can derive the following:

$$\begin{aligned} 0 &= \int_{\mathbb{Z}_p} \sin\, a(x+1) d\mu_{-1}(x) + \int_{\mathbb{Z}_p} \sin\, ax d\mu_{-1}(x) \\ &= (\cos\, a + 1) \int_{\mathbb{Z}_p} \sin\, ax d\mu_{-1}(x) + \sin\, a \int_{\mathbb{Z}_p} \cos ax d\mu_{-1}(x), \\ 2 &= \int_{\mathbb{Z}_p} \cos\, a(x+1) d\mu_{-1}(x) + \int_{\mathbb{Z}_p} \cos\, ax d\mu_{-1}(x) \\ &= (\cos\, a + 1) \int_{\mathbb{Z}_p} \cos\, ax d\mu_{-1}(x) - \sin\, a \int_{\mathbb{Z}_p} \sin\, ax d\mu_{-1}(x). \end{aligned} \tag{2.11}$$

By (2.11), we get

$$\int_{\mathbb{Z}_p} \sin\, ax d\mu_{-1}(x) = -\frac{\sin\, a}{\cos\, a + 1} = -\tan \frac{a}{2}. \tag{2.12}$$

From (1.10) and (2.12), we have

$$\sum_{n=1}^{\infty} \frac{T_{2n-1}}{(2n-1)!} \left(\frac{a}{2}\right)^{2n-1} = -\int_{\mathbb{Z}_p} \sin ax d\mu_{-1}(x) = \sum_{n=1}^{\infty} \frac{(-1)^n a^{2n-1}}{(2n-1)!} \int_{\mathbb{Z}_p} x^{2n-1} d\mu_{-1}(x). \tag{2.13}$$

By comparing coefficients on the both sides of (2.13), we get

$$\int_{\mathbb{Z}_p} x^{2n-1} d\mu_{-1}(x) = (-1)^n \frac{T_{2n-1}}{2^{2n-1}}, \quad \text{for } n \in \mathbb{N}, \tag{2.14}$$

where T_{2n-1} is the $(2n-1)$th tangent number.

Therefore, by (2.14), we obtain the following theorem.

Theorem 2.2. *For $n \in \mathbb{N}$, one has*

$$\int_{\mathbb{Z}_p} x^{2n-1} d\mu_{-1}(x) = (-1)^n \frac{T_{2n-1}}{2^{2n-1}}, \tag{2.15}$$

where T_{2n-1} is the $(2n-1)$th tangent numbers.

From Theorem 2.1, one has

$$\int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) = \frac{(-1)^n}{2^n} A_n(-1). \tag{2.16}$$

Therefore, by Theorem 2.2 and (2.16), we obtain the following corollary.

Corollary 2.3. *For $n \in \mathbb{N}$, one has*

$$A_{2n-1}(-1) = (-1)^{n-1} T_{2n-1}. \tag{2.17}$$

From (1.6) and (2.9), we have

$$\int_{\mathbb{Z}_p} e^{xt} d\mu_{-1}(x) = \frac{2}{e^t+1} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}, \tag{2.18}$$

(see [5]). Thus, by (2.16) and (2.18), we get

$$\int_{\mathbb{Z}_p} x^{2n-1} d\mu_{-1}(x) = E_{2n-1} = (-1)^n \frac{T_{2n-1}}{2^{2n-1}}. \tag{2.19}$$

Therefore, by Corollary 2.3 and (2.19), we obtain the following corollary.

Corollary 2.4. *For $n \in \mathbb{N}$, one has*

$$E_{2n-1} = (-1)^n \frac{T_{2n-1}}{2^{2n-1}} = -\frac{A_{2n-1}(-1)}{2^{2n-1}}. \tag{2.20}$$

By (1.5) and (2.9), we get

$$\begin{aligned} t\int_{\mathbb{Z}_p} e^{xt}d\mu_{-1}(x) &= \frac{2t}{e^{2t}-1}e^{t} - \frac{2t}{e^{2t}-1} \\ &= \sum_{n=0}^{\infty} B_n\left(\frac{1}{2}\right)2^n\frac{t^n}{n!} - \sum_{n=0}^{\infty}\frac{2^nB_n}{n!}t^n \\ &= \sum_{n=0}^{\infty}\left(B_n\left(\frac{1}{2}\right) - B_n\right)2^n\frac{t^n}{n!}. \end{aligned} \tag{2.21}$$

By (2.21), we get

$$\int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) = \frac{(B_{n+1}(1/2) - B_{n+1})}{n+1}2^{n+1}. \tag{2.22}$$

Thus, from (2.19), Theorem 2.2 and Corollary 2.3, we have

$$\frac{(B_{2n}(1/2) - B_{2n})2^{2n}}{2n} = (-1)^n\frac{T_{2n-1}}{2^{2n-1}} = -\frac{A_{2n-1}(-1)}{2^{2n-1}}. \tag{2.23}$$

Therefore, by (2.23), we obtain the following theorem.

Theorem 2.5. *For $n \in \mathbb{N}$, one has*

$$\frac{(B_{2n}(1/2) - B_{2n})2^{2n}}{n} = (-1)^n\frac{T_{2n-1}}{2^{2n-2}} = -\frac{A_{2n-1}(-1)}{2^{2n-2}}. \tag{2.24}$$

From (1.5), we note that

$$t\int_{\mathbb{Z}_p} e^{xt}d\mu_{-1}(x) = \frac{2t}{e^t+1} = \sum_{n=0}^{\infty} G_n\frac{t^n}{n!} \tag{2.25}$$

(see [13, 14]). Thus, by (2.25), we get

$$G_0 = 0, \qquad (G+1)^n + G_n = 2\delta_{1,n}, \tag{2.26}$$

(see [13, 14]), with the usual convention about replacing G^n by G_n.

From (1.5) and (2.9), one has

$$\begin{aligned} t\int_{\mathbb{Z}_p} e^{xt}d\mu_{-1}(x) &= 2\left(\frac{t}{e^t-1} - \frac{2t}{e^{2t}-1}\right) \\ &= 2\sum_{n=0}^{\infty}(B_n - 2^nB_n)\frac{t^n}{n!}. \end{aligned} \tag{2.27}$$

Thus, by (2.27), we get

$$\int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) = 2\left(\frac{B_{n+1} - 2^{n+1}B_{n+1}}{n+1}\right). \tag{2.28}$$

From (2.28), we have

$$\frac{G_{2n}}{2n} = \int_{\mathbb{Z}_p} x^{2n-1} d\mu_{-1}(x) = \frac{B_{2n} - 2^{2n}B_{2n}}{n}, \quad \text{for} \in \mathbb{N}. \tag{2.29}$$

Therefore, by (2.19), Corollary 2.3 and (2.29), we obtain the following theorem.

Theorem 2.6. *For $n \in \mathbb{N}$, we have*

$$G_{2n} = 2\left(B_{2n} - 2^{2n}B_{2n}\right). \tag{2.30}$$

In particular,

$$\frac{-1}{2^{2n-1}}(A_{2n-1}(-1)) = \left((-1)^n T_{2n-1}\right)\frac{1}{2^{2n-1}} = \frac{G_{2n}}{2n}. \tag{2.31}$$

3. Further Remark

In complex plane, we note that

$$\begin{aligned}
\tan x &= \frac{1}{i}\left(\frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}\right) = \frac{1}{i}\left(1 - \frac{2e^{-ix}}{e^{ix} + e^{-ix}}\right) \\
&= \frac{1}{i}\left(1 - \sum_{n=0}^{\infty}\frac{E_n}{n!}2^n i^n x^n\right) = \frac{1}{i}\left(-\sum_{n=1}^{\infty}\frac{E_n}{n!}2^n i^n x^n\right) \\
&= \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n-1)!}E_{2n-1}2^{2n-1}x^{2n-1}.
\end{aligned} \tag{3.1}$$

By (1.10) and (3.1), we also get

$$T_{2n-1} = (-1)^n E_{2n-1} 2^{2n-1}, \quad \text{for } n \in \mathbb{N}. \tag{3.2}$$

From (1.5), we have

$$\begin{aligned}\sum_{n=1}^{\infty}\frac{t^{2n}}{(2n)!}G_{2n} &= \sum_{n=1}^{\infty}\frac{(it)^{2n}}{(2n)!}(-1)^n G_{2n} = \frac{2it}{1+e^{it}} - it \\ &= \frac{it\left(1-e^{it}\right)}{1+e^{it}} = \frac{it\left(e^{-it/2}-e^{it/2}\right)}{e^{it/2}+e^{-it/2}} = t\left(\frac{\left(e^{it/2}-e^{-it/2}\right)/2i}{\left(e^{it/2}+e^{-it/2}\right)/2}\right) \\ &= t\tan\left(\frac{t}{2}\right).\end{aligned} \tag{3.3}$$

Thus, by (1.10) and (3.3), we get

$$\sum_{n=1}^{\infty}\frac{t^{2n}}{(2n)!}G_{2n} = t \quad \tan\left(\frac{t}{2}\right) = t\sum_{n=1}^{\infty}\frac{(t/2)^{2n-1}}{(2n-1)!}T_{2n-1} = \sum_{n=1}^{\infty}\frac{t^{2n}}{(2n-1)!2^{2n-1}}T_{2n-1}. \tag{3.4}$$

From (3.4), we have

$$nT_{2n-1} = 2^{2n-2}G_{2n} = 2^{2n-1}\left(1-2^{2n}\right)B_{2n}. \tag{3.5}$$

By (1.1), we see that

$$\frac{2}{1+e^{-2it}} = \sum_{n=0}^{\infty}A_n(-1)\frac{i^n t^n}{n!}. \tag{3.6}$$

Thus, we note that

$$\begin{aligned}\sum_{n=1}^{\infty}i^{n-1}A_n(-1)\frac{t^n}{n!} &= \frac{1}{i}\left(\frac{2}{1+e^{-2it}}-1\right) = \frac{1-e^{-2it}}{(1+e^{-2it})i} = \frac{\left(\left(e^{it}-e^{-it}\right)/2\right)}{\left(\left(e^{it}+e^{-it}\right)/2\right)i} \\ &= \tan t = \sum_{n=1}^{\infty}T_{2n-1}\frac{t^{2n-1}}{(2n-1)!}.\end{aligned} \tag{3.7}$$

From (3.7), we have

$$A_{2n}(-1) = 0, \quad A_{2n-1}(-1) = (-1)^{n-1}T_{2n-1}, \quad (n \geq 1). \tag{3.8}$$

It is easy to show that

$$\sum_{k=1}^{m}k^n(-1)^k = (-1)^m\sum_{k=0}^{n}\binom{n}{k}\frac{A_k(-1)}{2^{k+1}}m^{n-k} - \frac{\left\{(-1)^m-1\right\}}{2^{n+1}}A_n(-1). \tag{3.9}$$

For simple calculation, we can derive the following equation:

$$i \tan x = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} = 1 - \frac{2}{e^{2ix} - 1} + \frac{4}{e^{4ix} - 1}. \tag{3.10}$$

By (3.10), we get

$$x \tan x = -ix + \frac{2ix}{e^{2ix} - 1} - \frac{4ix}{e^{4ix} - 1} = \sum_{n=1}^{\infty} \frac{(-1)^n B_{2n} 4^n (1 - 4^n)}{(2n)!} x^{2n}. \tag{3.11}$$

Thus, from (3.11),we have

$$\tan x = \sum_{n=1}^{\infty} \frac{(-1)^n B_{2n} 4^n (1 - 4^n)}{(2n)!} x^{2n-1}. \tag{3.12}$$

By (1.10) and (3.12), we get

$$T_{2n-1} = \frac{(-1)^n B_{2n} 4^n (1 - 4^n)}{2n}, \quad \text{for } n \in \mathbb{N}. \tag{3.13}$$

From Corollary 2.3 and (3.13), we can derive the following identity:

$$A_{2n-1}(-1) = -\frac{B_{2n} 2^{2n-1} (1 - 4^n)}{n}. \tag{3.14}$$

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