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T-COLORING ON FOLDED HYPERCUBES

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Abstract. Given a graph $G = (V, E)$ and a set T of non-negative integers containing 0, a T-coloring of G is an integer function f of the vertices of G such that $|f(u) - f(v)| \notin T$ whenever $uv \in E$. The edge-span of a T-coloring f is the maximum value of $|f(u) - f(v)|$ over all edges uv, and the T-edge-span of a graph G is the minimum value of the edge-span among all possible T-colorings of G. This paper discusses the T-edge span of the folded hypercube network of dimension n for the k-multiple-of-s set, $T = \{0, s, 2s, \ldots, ks\} \cup S$, where s and $k \geq 1$ and $S \subseteq \{s+1, s+2, \ldots, ks-1\}$.

1. Introduction and Basic Theorem

In the *channel assignment problem*, several transmitters and a forbidden set T (called *T-set*) of non-negative integers containing 0, are given. We assign a non-negative integer channel to each transmitter under a constraint: for two transmitters where potential interference might occur, the difference of their channels does not fall within the given T-set. The interference is due to various reasons such as geographic proximity and meteorological factors. To formulate this problem, we construct a graph G such that each vertex represents a transmitter, and two vertices are adjacent if the potential interference of their corresponding transmitters might occur.

Thus, we have the following definition. Given a T-set and a graph G, a ***T*-*coloring*** of G is a function $f : V(G) \rightarrow Z^+ \cup \{0\}$ such that $|f(x) - f(y)| \notin T$ if $xy \in E(G)$. Note that if $T = \{0\}$, then T-coloring is the same as ordinary vertex-coloring. Hence we may consider the $T\text{-}coloring\ problem$ is a generalized

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graph vertex-coloring problem. T-coloring problem has been studied by several authors, such as [1, 5, 6, 9, 10, 13] and [15].

Let f be a T-coloring for a graph G. There are three important criteria for measuring the efficiency of f: First, the $order$ of a T-coloring, which is the number of different colors used in f; second, the $span$ of f, which is the maximum of $|f(u) - f(v)|$ over all vertices u and v; and third, the $edge$-$span$ of f, which is the maximum of $|f(u) - f(v)|$ over all edges uv. Given T and G, the T-$chromatic$ $number$ $\chi_T(G)$ is the minimum order among all possible T-colorings of G, the T-$span$ $sp_T(G)$ is the minimum span among all possible T-colorings of G, and the T-$edge$-$span$ $esp_T(G)$ is the minimum edge-span among all possible T-colorings of G.

In the case of radio frequency assignment, the forbidden T-sets can be very complex and difficult to model. We focus on a special family T-sets called the *k-multiple-of-s-sets* which has the form T = $\{0,\ s,\ 2s,\ \ldots\ ,\ ks\} \cup S$, where s, $k \geq 1$ and $S \subseteq \{s+1, s+2, \ldots, ks-1\}$. The k-multiple-of-s-sets bas been studied by Raychaudhuri first (see [11, 12]). When $s = 1$, the set $T = \{0, 1, 2, \ldots, k = r\}$ is also called an r-$initial$ set. Some practical forbidden sets, such as those that arise in UHF television problem (see [14]), are very similar to k-multiple-of-s-sets. We denote K_n as the complete graph (or clique) on n vertices and $\omega(G)$ as the maximum size of a clique in G.

Now, we quote some known results about T-spans and T-edge-spans, some of which will be used in next section.

Theorem 1.1. ([2]). *For all graphs G and sets T,*

(1) $\chi_T(G) = \chi(G)$.

(2) $\chi(G) - 1 \leq esp_T(G) \leq sp_T(G)$.

(3) $sp_T(K_{\omega(G)}) \leq esp_T(G) \leq sp_T(K_{\chi(G)})$.

Theorem 1.2. ([12]). *If T is a k-multiple-of-s-set, then $sp_T(G) = sp_T(K_{\chi(G)})$. Moreover, if $\chi(G) = st$, for some positive integer t, then $sp_T(G) = st+skt-sk-1$, and if $\chi(G) = st+l$, for some $l \in \{1, 2, \ldots, s-1\}$, then $sp_T(G) = st+skt+l-1$.*

Theorem 1.3. *If T is a k-multiple-of-s-set and $\chi(G) \leq s$, then $sp_T(G)$ = $esp_T(G)$ = $\chi(G) - 1$.*

Proof. By Theorem 1.2 and $\chi(G) \leq s$, we have either $t = 1$ and $\chi(G) = s$, or $t = 0$ and $\chi(G) = l$. Hence, either $sp_T(G) = s + sk - sk - 1 = s - 1 = \chi(G) - 1$ or $sp_T(G) = l - 1 = \chi(G) - 1$. And $\chi(G) - 1 \leq esp_T(G) \leq sp_T(G)$ by Theorem 1.1 (2), it implies $sp_T(G) = esp_T(G) = \chi(G) - 1$. ∎

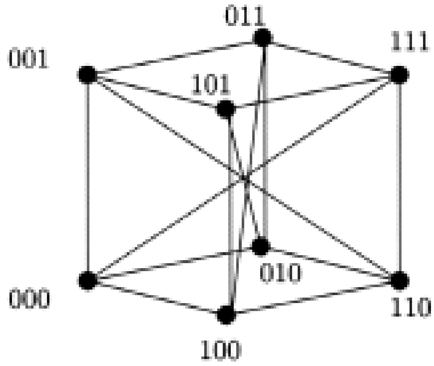

Fig. 1. $FH(3)$.

A $folded\ hypercube\ network\ of\ dimension\ n$ analyzed in [3], denoted by $FH(n)$, is a graph whose vertices are binary sequences $p = (p_0, p_1, \ldots, p_{n-1})$ with $p_i = 0$ or 1 for $0 \leq i \leq n-1$, and two vertices are adjacent if and only if they differ by exactly one coordinate or by all coordinates. Folded hypercube is basically a standard hypercube with some extra links established between its nodes. To compare with hypercube, the hardware overhead of a same dimensional folded hypercube is almost $1/n$, n being the dimensionality of the hypercube, which is negligible for large n. For this new design, optimal routing algorithms are developed and proven to be remarkably more efficient than those of the conventional n-cube. There are many studies on this network (see [4, 7, 16, 17]). In [8], this graph is also called $base$-2 $generalized\ orthogonal\ graph\ of\ dimension\ n$. Figure 1 illustrates the graph $FH(3)$.

2. Main Result

Theorem 2.1. *If G is the folded hypercube network $FH(n)$, then*

(1) $\chi(G) = 2$ *if n is odd,*

(2) $3 \leq \chi(G) \leq 4$ *if n is even.*

(3) *In particular,* $\chi(FH(4)) = 4$.

Proof.

(1) If n is odd, it is easy to check that G is bipartite and we have $\chi(G) = 2$.

(2) There is an $(n+1)$-cycle on G by definition. For example, 0000-1000-1100-1110-1111-0000 is a 5-cycle on $FH(4)$. It is an odd cycle as n is even. Hence, $3 \leq \chi(G)$. Now we give a proper 4-coloring to prove $\chi(G) \leq 4$. Let $p = (p_0, p_1, \ldots p_{n-1})$ be a vertex on G and consider $\alpha_p = \sum_{i=1}^{n-1} p_i$ (mod 2).

Color p as 1 if $p_0 = 0$ and $\alpha_p = 0$. Color p as 2 if $p_0 = 0$ and $\alpha_p = 1$. Color p as 3 if $p_0 = 1$ and $\alpha_p = 0$. Color p as 4 if $p_0 = 1$ and $\alpha_p = 1$. If p and q are both colored as j, then $p_0 = q_0$ and $\alpha_p = \alpha_q$. That is, $p_0 = q_0$ and $\sum_{i=0}^{n-1} p_i$ (mod 2) $= \sum_{i=0}^{n-1} q_i$ (mod 2). Obviously, p and q are not adjacent. The coloring is a proper 4-coloring and $\chi(G) \leq 4$.

(3) By Figure 2, $FH(4)$ contains the Grötzsch graph. Then we have $\chi(FH(4)) = 4$. ■

Theorem 2.2. *If G is the folded hypercube network $FH(n)$ with n is odd and T is a k-multiple-of-s set. Then*

(1) $esp_T(G) = sp_T(G) = k + 1$ *as* $s = 1$,

(2) $esp_T(G) = sp_T(G) = 1$ *as* $s \geq 2$.

Proof. Since n is odd, we have $\chi(G) = 2$ by Theorem 2.1.

(1) If $s = 1$, T is a k-initial set. By Theorem 1.1 (3), we have $sp_T(K_{\omega(G)}) = sp_T(K_{\chi(G)}) = sp_T(K_2) = k+1$. It's easy to see $esp_T(G) = sp_T(G) = k+1$.

(2) Because $s \geq 2 = \chi(G)$, it can be obtained by Theorem 1.3. ■

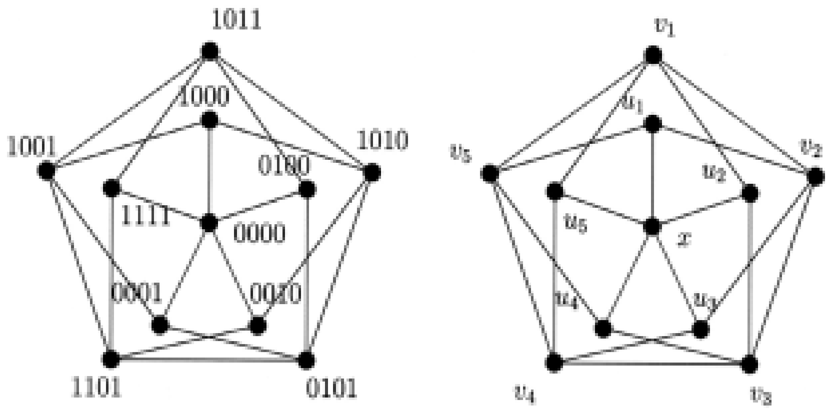

Fig. 2. There is a Grötzsch subgraph in $FH(4)$.

Theorem 2.3. *If G is the folded hypercube network $FH(n)$ with n is even, $\chi(G) = 3$ and T is a k-multiple-of-s-set, then*

(1) $k + 1 \leq esp_T(G) \leq sp_T(G) = 2k + 2$ *as* $s = 1$,

(2) $esp_T(G) = sp_T(G) = 2k + 2$ *as* $s = 2$,

(3) $esp_T(G) = sp_T(G) = 2$ *as* $s \geq 3$.

Proof. Consider $s = 2$, that is, $\{0, 2, 4, \ldots, 2k\} \subseteq T$. Let f be a T-coloring of G. Obviously, if $f(u)$ and $f(v)$ are both even (or odd), then $|f(u) - f(v)| \geq 2k + 2$. On an odd cycle, there are two adjacent vertices which are both even (or odd). It means, $esp_T(G) \geq 2k + 2$ if G is not a bipartite graph. Hence, $esp_T(FH(n)) \geq 2k + 2$ if $s = 2$. And $sp_T(FH(n)) = sp_T(K_3) = 2k + 2$. We have $2k + 2 \leq esp_T(G) \leq sp_T(G) = 2k + 2$.

For other cases, it is easy to observe that by using Theorem 1.1 (3), 1.2 and 1.3. ■

Theorem 2.4. *If G is the folded hypercube network $FH(n)$ with n is even, $\chi(G) = 4$ and T is a k-multiple-of-s-set, then*

(1) $k + 1 \leq esp_T(G) \leq sp_T(G) = 3k + 3$ *as* $s = 1$,

(2) $2k + 2 \leq esp_T(G) \leq sp_T(G) = 2k + 3$ *as* $s = 2$,

(3) $3 \leq esp_T(G) \leq sp_T(G) = 3k + 3$ *as* $s = 3$,

(4) $esp_T(G) = sp_T(G) = 3$ *as* $s \geq 4$.

Proof. It is similar to the proof of above theorem, there exists an odd cycle on G. Therefore, $esp_T(FH(n)) \geq 2k + 2$ as $s = 2$.

For other cases, it is easy to check that by using Theorems 1.1, 1.2 and 1.3. ■

Now, consider $s = 2$, $esp_T(FH(4))$ is either $2k + 2$ or $2k + 3$ by Theorems 2.1 (3) and 2.4 (2). We prove the T-edge-span of the Grötzsch graph which is a subgraph of $FH(4)$ is at least $2k + 3$. This implies that $esp_T(FH(4)) = 2k + 3$ when $s = 2$. Let x be the vertex with degree 5 in the Grötzsch graph (see Figure 2). Due to the vertex transitivity of $FH(n)$, without loss of generality, we may assume $f(x) = 0$ for the vertex x. First, we have the following properties and lemmas for sets $T = \{0, 2, 4, ..., 2k\}$ and T-coloring f with T-edge-span $2k + 2$, for a graph G.

Lemma 2.5. *If f is a T-coloring with T-edge-span $2k + 2$ for a graph G where $T = \{0, 2, 4, ..., 2k\}$ and P is a path of length i from x to y on G, then*

$$|f(x) - f(y)| \leq i(2k + 2).$$

Proof. The result follows immediately by the property $|f(u) - f(v)| \leq 2k + 2$ for all edges uv and the triangle inequality: $|a + b| \leq |a| + |b|$ for any integers a and b. ■

If f is a T-coloring with T-edge-span $2k + 2$ for a graph G where $T = \{0, 2, 4, ..., 2k\}$ and $xy \in E(G)$, then $f(x) = f(y) \pm (2k + 2)$ if and only if $f(x)$ and $f(y)$ are both odd (or even). In fact, the equality of Lemma 2.5 holds only if the colors of all vertices on P are all odd (or even). Furthermore, if P is a path of length i from x to y and the colors of all vertices on P are all odd (or even), then $|f(x) - f(y)|$ = $(i - 2t)(2k + 2)$ for some integer t with $0 \leq t \leq \lfloor \frac{i}{2} \rfloor$. Therefore, if i is odd, $|f(x) - f(y)|$ is an odd multiple of $2k + 2$; if i is even, $|f(x) - f(y)|$ is an even multiple of $2k + 2$.

Lemma 2.6. *If G is an odd cycle C_n and f is a T-coloring of G with T-edge-span $2k + 2$ where $T = \{0, 2, 4, ..., 2k\}$, then*

(1) *the colors of vertices are not all even (or odd),*

(2) *not all $n - 1$ vertices color odd as one colors 0.*

Proof.

(1) Assume x and y are two vertices on C_n and the colors of all vertices to be all odd (or even). Obviously, there are two (x,y)-paths P and $C_n \backslash P$ with length i and $n - i$, respectively. Because n is odd and the above observation, we obtain that $|f(x) - f(y)|$ is both an odd multiple and an even multiple of $2k + 2$. That implies a contradiction.

(2) Assume x is the vertex colored 0 and the colors of other vertices are all odd. Let a and b be the two neighbors of x, $1 \leq f(a), f(b) \leq 2k + 1$. There is a ($a$,$b$)-path P of length $n - 2$ and the colors of the vertices on P are all odd. Since $n - 2$ is odd, $|f(a) - f(b)|$ is an odd multiple of $2k + 2$. Hence $2k + 2 \leq |f(a) - f(b)|$, a contradiction. ■

Lemma 2.7. *If G is an even cycle C_n and f is a T-coloring of G with T-edge-span $2k + 2$ where $T = \{0, 2, 4, ..., 2k\}$, one vertex x colors 0 and all $n - 1$ vertices color odd, then the two neighbors of x have the same color.*

Proof. Let a and b be the two neighbors of x, $1 \leq f(a), f(b) \leq 2k + 1$. There is a ($a$,$b$)-path P of length $n - 2$ and the colors of the vertices on P are all odd. Since $n - 2$ is even and by the above observation, $|f(a) - f(b)|$ is an even multiple of $2k + 2$. Hence, we have $|f(a) - f(b)| = 0$ or $|f(a) - f(b)| \geq 2(2k + 2)$. Hence, $f(a) = f(b)$. ■

Property 2.8. *If f is a T-coloring with T-edge-span $2k + 2$ for a graph G where $T = \{0, 2, 4, ..., 2k\}$.*

(1) *If there are two edges uv, $u'v' \in E(G)$, such that $f(u) = c$,$f(v) = d + 2k + 2$, $f(u') = d$, $f(v') = c + 2k + 2$, then $c = d$.*

(2) *If w is a common neighbor of x and y, $0 < f(x) - f(y) < 4k + 4$ and $f(x) - f(y)$ is even, then both $f(x) - f(w)$ and $f(w) - f(y)$ are odd.*

(3) *If w is a common neighbor of x and y, and $f(x) - f(y) > 0$ is odd, then $f(w)$ is either $f(x) - (2k + 2)$ or $f(y) + (2k + 2)$.*

Proof. It is easy to check these properties. ∎

Theorem 2.9. *For a k-multiple-of-2-sets T, $esp_T(FH(4)) = 2k+3$.*

Proof. Let $T = \{0, 2, 4, \ldots, 2k\}$. By Theorem 2.4 (2), $2k+2 \leq esp_T(FH(4)) \leq 2k+3$. To prove that the upper bound is the exact value of $esp_T(FH(4))$, we assume to the contrary that there exists a T-coloring f for $FH(4)$ with T-edge-span $2k+2$. Since $FH(4)$ is vertex transitive, without loss of generality, let x be the vertex with $f(x) = 0$. Let G be a subgraph of $FH(4)$ such that G is isomorphic to the Grötzsch graph and x is the vertex of G with degree 5. Because there are an odd cycle on G, f is a T-coloring function of G with edge-span $2k+2$, too. For any vertex y of G, $f(y) \leq 4k+4$ since $d(x, y) \leq 2$ and by Lemma 2.5.

On Figure 2, the vertices v_i are on a 5-cycle. We consider the colors $f(v_i)$, $1 \leq i \leq 5$. They are (1) all even (or odd), (2) four even numbers and one odd number, (3) four odd numbers and one even number, (4) three even numbers and two odd numbers or (5) three odd numbers and two even numbers. We discuss all these conditions and verify that they are all impossible.

1. They are all even (or odd).
   By Lemma 2.6 (1), it is obviously impossible.
2. They are four even numbers and one odd number.
   Without loss of generality, let $f(v_1)$ be odd and color the vertices v_2, v_3, v_5 as d, $d+(2k+2)$, $d+(2k+2)$, respectively, where $0 \leq d \leq 2k+2$ is even. If both $f(u_2)$ and $f(u_5)$ are odd, it implies that $f(u_2) = f(u_5) = c$ and $f(v_1) = c + (2k+2)$ where $0 \leq c \leq 2k+2$ by Lemma 2.7. With v_1v_2 and $u_2v_3 \in E(G)$, we have $c = d$ by Property 2.8 (1), a contradiction. If both$f(u_2)$ and $f(u_5)$ are even, it implies that the colors on the 5-cycle $x - u_2 - v_3 - v_4 - u_5$ are all even, a contradiction to Lemma 2.6 (1). If $f(u_2) = 2k+2$ and $f(u_5) = a$ are odd, then $d = 2k+2$ because $u_2v_3 \in E(G)$ and d is even. Thus $f(u_1)$ is odd because u_1 is a common neighbor of x and v_2. However, $f(u_1) = 2k+2$ because u_1 is a common neighbor of x and v_5. It is a contradiction. The proof is similar as $f(u_5) = 2k+2$ and $f(u_2) = a$ is odd.
3. They are four odd numbers and one even number.
   Without loss of generality, let $f(v_1)$ be even and color the vertices v_2, v_3, v_4, v_5 as a, $a+(2k+2)$, a, $a+(2k+2)$, respectively, where $1 \leq a \leq 2k+1$ is odd. If both $f(u_2)$ and $f(u_5)$ are odd, it implies that the colors on the 5-cycle $x - u_2 - v_3 - v_4 - u_5$ are all odd but one is 0, a contradiction to Lemma 2.6 (2). If one of $f(u_2)$ and $f(u_5)$ is even, i.e. $2k+2$, thus $f(v_1) = 0$ or $4k+4$. However, $a < f(v_1) < a + (2k+2)$ since v_1 is a common neighbor of v_2 and v_5. It is a contradiction.
4. They are three even numbers and two odd numbers.

(a) The three even-colored nodes are consecutive.
Without loss of generality, let $f(v_1)$, $f(v_2)$ and $f(v_3)$ be even and $f(v_4)$ and $f(v_5)$ be odd. (i) If $0 < f(v_2) < 4k+4$, then $f(u_1)$ and $f(u_3)$ are both odd by Property 2.8 (2). It implies that the colors on the 5-cycle $x - u_1 - v_5 - v_4 - u_3$ are all odd but one 0. It is a contradiction to Lemma 2.6 (2). (ii) If $f(v_2) = 0$ or $4k+4$, then $f(v_1) = f(v_3) = 2k+2$. Hence $f(u_4)$ and $f(u_5)$ are both odd by Property 2.8 (2). It implies that the colors on the 5-cycle $x - u_4 - v_5 - v_4 - u_5$ are all odd but one 0. It is a contradiction again.

(b) The three even-colored nodes are not consecutive.
Without loss of generality, let $f(v_1)$, $f(v_3)$ and $f(v_4)$ be even and $f(v_2)$ and $f(v_5)$ be odd. Let $0 \leq f(v_3) = c \leq 2k+2$ and $f(v_3) < f(v_4) = c+(2k+2)$. (i) If $0 \leq c < 2k+2$, then $2k+2 \leq c+(2k+2) < 4k+4$. Hence $f(u_3) = d$ is odd by Property 2.8 (2) and $f(v_2) = d+(2k+2)$. By Property 2.8 (1), $c = d$. It is a contradiction. (ii) If $c = 2k+2$, it implies that $f(v_4) = 4k+4$, $f(u_5) = 2k+2$, $f(u_2)$ is odd and $f(v_1) = 0$ or $4k+4$. If $f(v_1) = 0$, it forces $f(v_5) = 2k+2$, a contradiction. If $f(v_1) = 4k+4$, $f(u_2)$ must be $2k+2$ because u_2 is a common neighbor of x and v_1. It is a contradiction.

5. They are three odd numbers and two even numbers.

(a) Three odd-colored nodes are consecutive.
Without loss of generality, let $f(v_1)$, $f(v_2)$ and $f(v_3)$ be odd and $f(v_4)$ and $f(v_5)$ be even and $0 < f(v_4) < 4k+4$. Hence $f(u_3)$ and $f(u_5)$ are both odd by Property 2.8(2). It implies that the colors on the 5-cycle $x - u_5 - v_1 - v_2 - u_3$ are odd but one 0, a contradiction to Lemma 2.6 (2).

(b) Three odd-colored nodes are not consecutive.
Without loss of generality, let $f(v_1)$, $f(v_3)$ and $f(v_4)$ are odd and $f(v_2)$ and $f(v_5)$ are even, and $0 < f(v_3) = c < 2k+2$ and $2k+2 < f(v_4) = c+(2k+2) < 4k+4$. By Property 2.8 (3), $f(u_4) = 2k+2$. Then $f(v_5) = 4k+4$ because $f(v_5)$ is even and hence $f(u_1) = 2k+2$. It implies that $f(v_2) = 0$ because $f(u_1) - f(v_3) = (2k+2) - c \geq 0$ is odd and $f(v_2)$ is even. Because $f(v_2) = 0$ and $f(v_5) = 4k+4$, it forces $f(v_1) = 2k+2$ is not odd. It is a contradiction.

Hence, if $T = \{0, 2, 4, ..., 2k\}$, there is not a T-coloring function of G satisfying that $f(x) = 0$ and the edge span of f is $2k+2$. Then, for any k-multiple-of-2-sets T, there is not a T-coloring function of the Grötzsch graph G with edge-span $2k+2$. It implies that $esp_T(FH(4)) = 2k+3$. ∎

Corollary 2.10. *If $G = FH(4)$ and T is a k-multiple-of-s-set, then*

(1) $esp_T(G) = sp_T(G) = 3$ *as* $s \geq 4$,

(2) $esp_T(G) = sp_T(G) = 3k + 3$ *as* $s = 3$,

(3) $esp_T(G) = sp_T(G) = 2k + 3$ *as* $s = 2$.

Proof. As noted before Theorems 2.1, 2.4 and 2.9, we need only to consider $3k + 3 \leq esp_T(G)$ as $s = 3$. In the following, we denote x mod 3 by $(x)_3$.

If $s = 3$, $3 \leq esp_T(G) \leq sp_T(G) = 3k + 3$ by Theorem 2.4 (3). Suppose $3k + 3 > esp_T(G)$ and f is a T-coloring function with T-edge-span less than $3k+3$ for graph G. Observe that $(a-b)_3 = ((a)_3 - (b)_3)_3$ for any integers a, b and $\{0, 3, 6, \ldots, 3k\} \subseteq T$, then $(f(u) - f(v))_3 \neq 0$ for any two adjacent vertices u, v. We have $(f(u))_3 \neq (f(v))_3$ for any two adjacent vertices u, v. Let $(f(x))_3 = g(x)$ for all vertices x of G, then the function g will be a proper 3-coloring of G, a contradiction to $\chi(G) = 4$. Thus we have $3k + 3 \leq esp_T(G)$. ■

3. Conclusion

In this paper, we discuss the T-edge-span and T-span of the folded hypercube network of dimension n for a k-multiple-of-s-set $T = \{0,\ s,\ 2s,\ \ldots\ ,\ ks\} \cup S$, where s and $k \geq 1$ and $S \subseteq \{s+1, s+2, \ldots, ks-1\}$. Note that when $s = 1$, T is also called a k-initial set. Our results are shown in Table 1.

Table 1. $esp_T(G)$ and $sp_T(G)$ for $G = FH(n)$ and a k-multiple-of-s-sets T

n	χ	s	$esp_T(G)$, $sp_T(G)$ where $G = FH(n)$
n is odd	2	1	$esp_T(G) = sp_T(G) = k + 1$
		≥ 2	$esp_T(G) = sp_T(G) = 1$
n is even	3	1	$k + 1 \leq esp_T(G) \leq sp_T(G) = 2k + 2$
		2	$esp_T(G) = sp_T(G) = 2k + 2$
		≥ 3	$esp_T(G) = sp_T(G) = 2$
	4	1	$k + 1 \leq esp_T(G) \leq sp_T(G) = 3k + 3$
		2	$2k + 2 \leq esp_T(G) \leq sp_T(G) = 2k + 3$
		3	$3 \leq esp_T(G) \leq sp_T(G) = 3k + 3$
		≥ 4	$esp_T(G) = sp_T(G) = 3$

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