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PINCHING AND NONEXISTENCE OF STABLE HARMONIC MAPS

TAKASHI OKAYASU

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1. Introduction. Let $f: N^m \to M^n$ be a harmonic map from a compact Riemannian manifold N to a Riemannian manifold M. f is said to be stable if its second variation of the energy is non-negative.

Leung [6] proved that if M^n is a unit sphere S^n ($n \geqq 3$), then constant maps are the only stable harmonic maps for an arbitrary N^m. Considering the above result and some of its generalization (cf. [4], [7] and [8]), we can ask the following:

QUESTION. *Let M^n be a complete simply-connected strictly* (1/4)*-pinched Riemannian manifold of dimension $n(n \geqq 3)$* (i.e., *the sectional curvature K_M satisfies* $1/4 < K_M \leqq 1$). *Let N^m be an arbitrary compact Riemannian manifold. Is every stable harmonic map $f: N^m \to M^n$ a constant map?*

This is a "harmonic-version" of the famous conjecture of Lawson and Simons (cf. [5]) on stable minimal submanifolds (or more generally stable currents). To this question, Howard [3] obtained a partial affirmative answer. He showed that for each $n \geqq 3$ there exists a constant $\delta(n)$ satisfying $1/4 < \delta(n) < 1$ such that if M^n is a simply-connected compact strictly $\delta(n)$-pinched Riemannian manifold of dimension n, then there are no nonconstant stable harmonic maps from any compact Riemannian manifold to M. But unfortunately $\lim_{n\to\infty} \delta(n) = 1$.

The purpose of this paper is to give a dimension-independent pinching constant. We prove:

MAIN THEOREM. *Let M^n be a compact simply-connected* 0.83*-pinched Riemannian manifold* ($n \geqq 3$) (i.e. $0.83 \leqq K_M \leqq 1$). *Then for any compact Riemannian manifold N^m, any stable harmonic map $f: N^m \to M^n$ is a constant map.*

In Section 2 we present some necessary formulas and in Section 3 we prove the main theorem. In Section 4 we use the same technique used in the proof of the main theorem to prove Theorem 3 which is an extension of a theorem of Ohnita [8], and as a corollary we get topological information on minimal submanifolds of sufficiently pinched spheres.

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2. **Preliminaries.** In this section we always assume that M^n is a compact simply-connected δ-pinched Riemannian manifold.

When M^n is a convex hypersurface in the Euclidean space $\boldsymbol{R}^{n+1}$, using the flat connection of $\boldsymbol{R}^{n+1}$ and taking the average of the second variations, Leung [7] proved that for a certain convex hypersurface M^n in $\boldsymbol{R}^{n+1}$ any stable harmonic map $f: N^m \to M^n$ is a constant map. The idea now is to follow the pattern of his calculation. To carry this idea out, we construct a vector bundle E on M and a flat connection ∇' on E instead of $M \times \boldsymbol{R}^{n+1}$ and the flat connection on $M \times \boldsymbol{R}^{n+1}$, respectively. For the construction we follow [1] and [2].

As in [2] we normalize the δ-pinched metric of M by multiplication with $(1+\delta)/2$. Put $E = TM \oplus \varepsilon(M)$, where TM is the tangent bundle of M and $\varepsilon(M)$ is a trivial line bundle on M with a metric. Thus E naturally becomes a Euclidean vector bundle on M. Let e be a cross-section of length one in $\varepsilon(M)$. We define a metric connection ∇'' on E as follows:

$$\nabla''_X Y = \nabla_X Y - \langle X, Y\rangle \cdot e \tag{1}$$

$$\nabla''_X e = X\,, \tag{2}$$

where X and Y are vector fields on M, $\langle\ ,\ \rangle$ and ∇ are the Riemannian metric andt he Riemannian connection of M, respectively. Under the pinching assumption, the curvature R'' of ∇'' is small. We obtain a flat metric connection ∇' close to ∇'' exactly as in [2]. To measure the closeness we define

$$\|\nabla' - \nabla''\| := \operatorname{Max}\{\|\nabla'_X Y - \nabla''_X Y\|; X \in TM, \|X\| = 1, Y \in E, \|Y\| = 1\}\,.$$

Note that our $\|\nabla' - \nabla''\|$ is half of $\|\nabla' - \nabla''\|$ in [1]. We define $k_1(\delta)$, $k_2(\delta)$ and $k_3(\delta)$ as follows:

$$k_1(\delta) = \frac{4}{3}(1-\delta)\delta^{-1}\left[1 + \left(\delta^{1/2}\cdot \sin\frac{1}{2}\pi\delta^{-1/2}\right)^{-1}\right]. \tag{3}$$

$$k_2(\delta) = \left[\frac{1}{2}(1+\delta)\right]^{-1}\cdot k_1(\delta)\,. \tag{4}$$

$$k_3(\delta) = k_2(\delta)\left\{1 + \left[1 - \frac{1}{24}\pi^2(k_1(\delta))^2\right]^{-2}\right\}^{1/2}. \tag{5}$$

By [1, 4.13], we get

$$\|\nabla' - \nabla''\| \leqq \frac{1}{2}k_3(\delta) . \tag{6}$$

3. Proof of the main theorem. Consider a harmonic map $f: N^m \to M^n$. Let e_a ($a = 1, \cdots, m$) be a local orthonormal frame on N. The energy of f is defined as

$$E(f) = \frac{1}{2}\int_N \sum_a \|f_* e_a\|^2 .$$

For any vector field V on M, we denote by ϕ_t the flow generated by V. Then we get the following second variational formula (cf. [6]) for the variational vector field V.

$$\begin{aligned} I(V, V) :&= \frac{d^2}{dt^2}\bigg|_{t=0} E(\phi_t \circ f) \\ &= \int_N \sum_a \{\|\nabla_{f_* e_a} V\|^2 - \langle R(V, f_* e_a) f_* e_a, V\rangle\} , \end{aligned} \tag{7}$$

where ∇ and R denote the Riemannian connection and curvature tensor of M, respectively.

THEOREM 1. *Let M^n be a compact simply-connected δ-pinched n-dimensional Riemannian manifold. Suppose that n and δ satisfy*

$$\frac{n+1}{4}(k_3(\delta))^2 + 1 - \frac{2\delta}{1+\delta}(n-1) + (n+1)^{1/2}k_3(\delta) < 0 . \tag{8}$$

Then the only stable harmonic map $f: N^m \to M^n$ for an arbitrary compact Riemannian manifold N^m is a constant map.

PROOF. First we normalize the metric of M by multiplication with $(1+\delta)/2$. Let E be the vector bundle on M constructed in Section 2. For $W \in E$ we denote by W^T and W^ε the TM-component and the $\varepsilon(M)$-component of W, respectively. Let V be a parallel cross-section of E with respect to ∇'. From (7), the second variation corresponding to V^T is given by

$$I(V^T, V^T) = \int_N \sum_a \{\|\nabla_{f_* e_a} V^T\|^2 - \langle R(V^T, f_* e_a) f_* e_a, V^T\rangle\} . \tag{9}$$

Observe that

$$\begin{aligned} \nabla_{f_* e_a} V^T &= \{\nabla''_{f_* e_a}(V - V^\varepsilon)\}^T = (\nabla''_{f_* e_a} V)^T - \{\nabla''_{f_* e_a}(\langle V, e\rangle e)\}^T \\ &= (\nabla''_{f_* e_a} V)^T - \langle V, e\rangle f_* e_a . \end{aligned}$$

Using (6)

$$\|\nabla_{f_*e_a} V^T\|^2 \leqq (1+k)\|\nabla''_{f_*e_a} V\|^2 + \left(1+\frac{1}{k}\right)\langle V, e\rangle^2 \|f_*e_a\|^2 \tag{10}$$
$$\leqq \frac{1+k}{4}(k_3(\delta))^2 \cdot \|V\|^2 \cdot \|f_*e_a\|^2 + \left(1+\frac{1}{k}\right)\langle V, e\rangle^2 \|f_*e_a\|^2 ,$$

where k is a positive constant fixed later. On the other hand, since we normalized the δ-pinched metric of M by multiplication with $(1+\delta)/2$,

$$\langle R(V^T, f_*e_a)f_*e_a, V^T\rangle \geqq \frac{2\delta}{1+\delta}\{\|V^T\|^2 \cdot \|f_*e_a\|^2 - \langle V^T, f_*e_a\rangle^2\} . \tag{11}$$

Combining (9), (10) and (11), we get

$$I(V^T, V^T) \leqq \int_N \sum_a \left\{\frac{1+k}{4}(k_3(\delta))^2 \|V\|^2 \cdot \|f_*e_a\|^2 \right. \tag{12}$$
$$\left. + \left(1+\frac{1}{k}\right)\langle V, e\rangle^2 \|f_*e_a\|^2 - \frac{2\delta}{1+\delta}[\|V^T\|^2 \cdot \|f_*e_a\|^2 - \langle V^T, f_*e_a\rangle^2]\right\} .$$

We now define $\mathscr{W} = \{V \in \Gamma(E); \nabla' V = 0\}$, where $\Gamma(E)$ denotes the vector space consisting of all smooth cross-sections of E. Then $\mathscr{W}$ is isomorphic to $\boldsymbol{R}^{n+1}$ and has a natural inner product. We define a quadratic form Q on $\mathscr{W}$ by

$$Q(V) = \text{the right hand side of (12)} := \int_N \sum_a q_a . \tag{13}$$

Take an orthonormal basis $\{W_1, \cdots, W_{n+1}\}$ of $\mathscr{W}$. Then we obtain

$$\sum_{j=1}^{n+1} I(W_j^T, W_j^T) \leqq \sum_{j=1}^{n+1} Q(W_j) = \operatorname{tr} Q = \int_N \sum_a \operatorname{tr} q_a . \tag{14}$$

Since the trace of q_a is independent of the choice of an orthonormal basis for each fiber of E, at each point $x \in M$ we choose an orthonormal basis $\{V_1, \cdots, V_n, e\}$ such that the V_i are tangent to M. Then we get

$$\operatorname{tr} Q = \int_N \sum_a \left\{\frac{(n+1)(1+k)}{4}(k_3(\delta))^2 + 1 + \frac{1}{k} - \frac{2(n-1)\delta}{1+\delta}\right\} \cdot \|f_*e_a\|^2 . \tag{15}$$

Now we take $k = ((n+1)/4)^{-1/2} k_3(\delta)^{-1}$. Then

$$\operatorname{tr} Q = \int_N \sum_a \left\{\frac{n+1}{4}(k_3(\delta))^2 + 1 - \frac{2\delta}{1+\delta}(n-1) \right. \tag{16}$$
$$\left. + (n+1)^{1/2} k_3(\delta)\right\} \|f_*e_a\|^2 .$$

To get the conclusion, we suppose that f is a nonconstant harmonic map and that n and δ satisfy (8). Then we get $\operatorname{tr} Q < 0$. By (14) we obtain $I(W_j^T, W_j^T) < 0$ for some j. Thus f is unstable. q.e.d.

PROOF OF THE MAIN THEOREM. Since we have $k_3(0.83) \fallingdotseq 0.964$, (8) is equivalent to $n \geqq 8$ for $\delta = 0.83$. On the other hand, the constant $\delta(n)$ of Howard [3] satisfies $\delta(n) < 0.83$ for $3 \leqq n \leqq 7$. Thus we get the conclusion. q.e.d.

REMARK. The value δ satisfying $k_3(\delta)^2 = 8\delta/(1+\delta)$ is $0.76\cdots$. So our estimate for δ can be improved up to $0.76\cdots$ if n is large.

4. An extension of a theorem of Ohnita and its application to minimal submanifolds. Ohnita [8] proved the following theorem.

THEOREM 2. *Let M^n be an n-dimensional compact minimal submanifold immersed in a unit sphere $S^{N-1}(1)$. If the Ricci curvature* Ric_M *of M satisfies* $\mathrm{Ric}_M > n/2$, *then M is harmonically unstable. That is, there exists no nonconstant stable harmonic map from M to any Riemannian manifold nor from any compact Riemannian manifold to M.*

Now we prove the following theorem which is a partial extension of Theorem 2.

THEOREM 3. *Let $(\bar{M}^{N-1}, h)$ be a complete simply-connected δ-pinched $(N-1)$-dimensional Riemannian manifold with $(k_3(\delta))^2 \leqq 4(2n+\delta-1)/N(1+\delta)$. Suppose that $f\colon (M^n, g) \to (\bar{M}^{N-1}, h)$ is an isometric minimal immersion of a complete n-dimensional Riemannian manifold (M^n, g) with $\rho > c(N, n, \delta) := (2n+\delta-1)/4 + \{[(2n+\delta-1)/4]^2 - [(2n+\delta-1)/4 - N(1+\delta)k_3(\delta)^2/16]^2\}^{1/2}$, where ρ is the infimum of the Ricci curvature of M. Then for any compact Riemannian manifold M′, any stable harmonic map $\phi\colon M' \to M$ is a constant map.*

PROOF. We normalize the metrics g and h by multiplication with $(1+\delta)/2$. We use the same letters g, h for the normalized metrics. Let $\bar{\nabla}$, $\bar{R}$ be the Riemannian connection and the curvature tensor of $(\bar{M}^{N-1}, h)$. We construct a Euclidean vector bundle E on $(\bar{M}^{N-1}, h)$ and also construct metric connections ∇', ∇'' on E as in Section 2. Let $\langle\ ,\ \rangle$ be the metric on E. Thus we have

$$\nabla''_X Y = \bar{\nabla}_X Y - h(X, Y)\cdot e$$
$$\nabla''_X e = X\,,$$

where X and Y are vector fields on $(\bar{M}^{N-1}, h)$. *Let* σ be the second fundamental form of M^n in $(\bar{M}^{N-1}, h)$ and let ∇ be the Riemannian connection of M. Set $N(M) := \{X \in f^*E;\ X \perp TM\}$. Then we obtain

$$\bar{\nabla}_X Y = \nabla_X Y + \sigma(X, Y)$$
$$\bar{\nabla}_X \xi = -A^\xi X + \nabla^\perp_X \xi\,,$$

where $X, Y \in TM$, $\xi \in N(M) \cap T\bar{M}^{N-1}$ and A^{ξ}, $\nabla^{\perp}$ are the Weingarten map in the direction of ξ and the normal connection of M in $(\bar{M}^{N-1}, h)$ respectively. Let V be a parallel cross-section of E with respect to ∇'. Let V^T and V^N be the TM-component and the $N(M)$-component of V, respectively. Thus

$$V^N = \langle V, e\rangle e + \sum_j \langle V, \xi_j\rangle \xi_j ,$$

where $\{\xi_1, \cdots, \xi_{N-1-n}\}$ is an orthonormal basis of $N(M) \cap T\bar{M}^{N-1}$. The second variation of $E(\phi)$ corresponding to V^T is given by

$$I(V^T, V^T) = \int_{M'} \sum_a \{\|\nabla_{\phi_* e_a} V^T\|^2 - \langle R(\phi_* e_a, V^T)V^T, \phi_* e_a\rangle\} , \tag{17}$$

where $\{e_a\}$ is a local orthonormal frame of M' and R is the curvature tensor of M. For $W \in E$ we denote by W^{TM} and $W^{T\bar{M}^{N-1}}$ the TM-component and the $T\bar{M}^{N-1}$-component of W, respectively. Observe that

$$\begin{aligned} \nabla_{\phi_* e_a} V^T &= (\bar{\nabla}_{\phi_* e_a} V^T)^{TM} = \{(\nabla''_{\phi_* e_a} V^T)^{T\bar{M}^{N-1}}\}^{TM} \\ &= \{(\nabla''_{\phi_* e_a} V)^{T\bar{M}^{N-1}} - (\nabla''_{\phi_* e_a} V^N)^{T\bar{M}^{N-1}}\}^{TM} \\ &= (\nabla''_{\phi_* e_a} V)^{TM} - (\nabla''_{\phi_* e_a} V^N)^{TM} \end{aligned} \tag{18}$$

and

$$\begin{aligned} (\nabla''_{\phi_* e_a} V^N)^{TM} &= \{\nabla''_{\phi_* e_a}(\langle V, e\rangle e)\}^{TM} + \sum_j \{\nabla''_{\phi_* e_a}(\langle V, \xi_j\rangle \xi_j)\}^{TM} \\ &= \langle V, e\rangle \phi_* e_a + \sum_j \langle V, \xi_j\rangle \{\nabla''_{\phi_* e_a} \xi_j\}^{TM} \\ &= \langle V, e\rangle \phi_* e_a + \sum_j \langle V, \xi_j\rangle \{\bar{\nabla}_{\phi_* e_a} \xi_j\}^{TM} \\ &= \langle V, e\rangle \phi_* e_a - \sum_j \langle V, \xi_j\rangle A^j(\phi_* e_a) , \end{aligned} \tag{19}$$

where we abbreviate $A^j = A^{\xi_j}$. Hence we obtain from (18), (19) and (6) that

$$\begin{aligned} \|\nabla_{\phi_* e_a} V^T\|^2 &\leqq (1+k)\|\nabla''_{\phi_* e_a} V\|^2 + \left(1 + \frac{1}{k}\right)\left\|\langle V, e\rangle \phi_* e_a - \sum_j \langle V, \xi_j\rangle A^j(\phi_* e_a)\right\|^2 \\ &\leqq \frac{(1+k)}{4}(k_3(\delta))^2 \cdot \|V\|^2 \cdot \|\phi_* e_a\|^2 \\ &\quad + \left(1 + \frac{1}{k}\right)\left\|\langle V, e\rangle \phi_* e_a - \sum_j \langle V, \xi_j\rangle A^j(\phi_* e_a)\right\|^2 , \end{aligned}$$

where k is a positive constant fixed later. Therefore from (17)

$$\begin{aligned} I(V^T, V^T) \leqq \int_{M'} \sum_a \Big\{ &\frac{(1+k)}{4}(k_3(\delta))^2 \cdot \|V\|^2 \cdot \|\phi_* e_a\|^2 \\ &+ \left(1 + \frac{1}{k}\right)\left\|\langle V, e\rangle \phi_* e_a - \sum_j \langle V, \xi_j\rangle A^j(\phi_* e_a)\right\|^2 \\ &- \langle R(\phi_* e_a, V^T)V^T, \phi_* e_a\rangle \Big\} . \end{aligned} \tag{20}$$

Denote by $Q(V)$ the right hand side of (20). Then Q is a quadratic form on $\mathscr{W} = \{V \in \Gamma(E); \nabla' V = 0\}$. We take the trace of Q on $\mathscr{W}$. Then we obtain

$$\text{(21)} \qquad \operatorname{tr} Q = \int_{M'} \sum_a \Big\{ \frac{(1+k)N}{4}(k_3(\delta))^2 \cdot \|\phi_* e_a\|^2 + \Big(1 + \frac{1}{k}\Big)\|\phi_* e_a\|^2 + \Big(1 + \frac{1}{k}\Big) \sum_j \|A^j(\phi_* e_a)\|^2 - \operatorname{Ric}_M(\phi_* e_a, \phi_* e_a) \Big\} .$$

Let $\{V_1, \cdots, V_n\}$ be a local orthonormal basis of TM. Then we get

$$\text{(22)} \qquad \begin{aligned} \sum_j \|A^j(\phi_* e_a)\|^2 &= \sum_{j,i} \langle A^j(\phi_* e_a), V_i \rangle^2 \\ &= \sum_{j,i} \langle \xi_j, \sigma(\phi_* e_a, V_i) \rangle^2 = \sum_i \|\sigma(\phi_* e_a, V_i)\|^2 . \end{aligned}$$

On the other hand, since

$$\begin{aligned} \langle R(\phi_* e_a, V_i) V_i, \phi_* e_a \rangle &= \langle \bar{R}(\phi_* e_a, V_i) V_i, \phi_* e_a \rangle + \langle \sigma(\phi_* e_a, \phi_* e_a), \sigma(V_i, V_i) \rangle \\ &\quad - \langle \sigma(\phi_* e_a, V_i), \sigma(\phi_* e_a, V_i) \rangle \\ &\leqq \frac{2}{1+\delta}\{\|V_i\|^2 \cdot \|\phi_* e_a\|^2 - \langle V_i, \phi_* e_a \rangle^2\} \\ &\quad + \langle \sigma(\phi_* e_a, \phi_* e_a), \sigma(V_i, V_i) \rangle - \|\sigma(\phi_* e_a, V_i)\|^2 , \end{aligned}$$

from the assumption that M is a minimal submanifold of $(\bar{M}^{N-1}, h)$, we obtain

$$\text{(23)} \qquad \operatorname{Ric}_M(\phi_* e_a, \phi_* e_a) \leqq \frac{2(n-1)}{1+\delta}\|\phi_* e_a\|^2 - \sum_i \|\sigma(\phi_* e_a, V_i)\|^2 .$$

From (22) and (23) we get

$$\text{(24)} \qquad \begin{aligned} \sum_j \|A^j(\phi_* e_a)\|^2 &\leqq \frac{2(n-1)}{1+\delta}\|\phi_* e_a\|^2 - \operatorname{Ric}_M(\phi_* e_a, \phi_* e_a) \\ &\leqq \frac{2(n-1)}{1+\delta}\|\phi_* e_a\|^2 - \frac{2\rho}{1+\delta}\|\phi_* e_a\|^2 . \end{aligned}$$

Thus from (21) and (24)

$$\text{(25)} \qquad \operatorname{tr} Q \leqq \int_{M'} \sum_a \Big\{ \frac{(1+k)N}{4}(k_3(\delta))^2 + 1 + \frac{1}{k} + \Big(1 + \frac{1}{k}\Big)\Big[\frac{2(n-1)}{1+\delta} - \frac{2\rho}{1+\delta}\Big] - \frac{2\rho}{1+\delta} \Big\} \|\phi_* e_a\|^2 .$$

Now we set $k = (N/4)^{-1/2} k_3(\delta)^{-1}\{1 + 2(n-1)/(1+\delta) - 2\rho/(1+\delta)\}^{1/2}$. Then

$$\operatorname{tr} Q \leqq \int_{M'} \sum_a \Big\{ \frac{N}{4}(k_3(\delta))^2 + \frac{2n+\delta-1}{1+\delta} - \frac{4}{1+\delta}\rho + \Big[\frac{N}{1+\delta}(2n - 2\rho + \delta - 1)\Big]^{1/2} k_3(\delta) \Big\} \|\phi_* e_a\|^2 .$$

When $(k_3(\delta))^2 \leqq 4(2n+\delta-1)/N(1+\delta)$, $c(N, n, \delta)$ is a unique solution of the following equation for t.

$$\frac{N}{4}(k_3(\delta))^2 + \frac{2n+\delta-1}{1+\delta} - \frac{4}{1+\delta}t + \left[\frac{N}{1+\delta}(2n-2t+\delta-1)\right]^{1/2} k_3(\delta) = 0\,.$$

Thus $\operatorname{tr} Q < 0$, from which the theorem follows.

We obtain the following corollary as in [8].

COROLLARY. *Let $(\bar{M}^{N-1}, h)$ be a complete simply-connected δ-pinched Riemannian manifold with $(k_3(\delta))^2 \leqq 4(2n+\delta-1)/N(1+\delta)$.*

Suppose that $f\colon M^n \to (\bar{M}^{N-1}, h)$ is a minimal immersion of a complete Riemannian manifold. If the Ricci curvature of M satisfies $\mathrm{Ric}_M > c(N, n, \delta)$, then $\pi_1 M = \{1\}$ and $\pi_2 M = \{1\}$.

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DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
HIROSAKI UNIVERSITY
HIROSAKI, 036
JAPAN