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# REAL HYPERSURFACES WITH PARALLEL RICCI TENSOR OF A COMPLEX SPACE FORM

By

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**Introduction.**

A Kaehlerian manifold of constant holomorphic sectional curvature $c$ is called a complex space form, which is denoted by $M_n(c)$. The complete and simply connected complex space form consists of a complex projective space $P_nC$, a complex Euclidean space $C_n$ or a complex hyperbolic space $H_nC$, according as $c>0$, $c=0$ or $c<0$. The induced almost contact metric structure of real hypersurfaces of $M_n(c)$ will be denoted by $(J, g, P)$.

Many subjects for real hypersurfaces of a complex projective space have been studied by Cecil and Ryan [1], Kimura [8], [9], Kon [10], Maeda [13], Okumura [15], Takagi [16], [17], [18] and so on. One of those, done by Kimura, asserted the following interesting result.

THEOREM K ([9]). *There are no real hypersurfaces of $P_nC$ with parallel Ricci tensor on which the structure vector $P$ is principal.*

On the other hand, real hypersurfaces of a complex hyperbolic space $H_nC$ have also been investigated from different points of view and there are some studies by Chen [2], Chen, Ludden and Montiel [3], Montiel [12] and Montiel and Romero [14]. In particular, it is proved in [12] the following fact:

THEOREM M. *There are no Einstein real hypersurfaces in $H_nC$.*

A Riemannian curvature tensor is said to be *harmonic* if the Ricci tensor $S$ is of Codazzi type. Although the concept is closely related to a parallel Ricci tensor, it was shown by Derdziński [4] and Gray [5] that it is essentially weaker than the latter one. Nakagawa, Umehara and the present author [6] proved that there exist infinitely many hypersurfaces with harmonic curvature and non-Ricci parallel in a Riemannian space form.

Recently, some studies about the non-existance for real hypersurfaces with

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harmonic curvature of $P_nC$ (resp. $H_nC$) have been made by Kwon and Nakagawa [11] (resp. Kim [7]). Their results are following:

THEOREM KNK. *There are no real hypersurfaces with harmonic curvature of* $M_n(c)$, $c\neq 0$ *on which the structure vector is principal.*

The main purpose of the present paper is to improve Theorem K and Theorem KNK, and study also real hypersurfaces with harmonic curvature of a complex space form $M_n(c)$, $c\neq 0$. We shall prove the followings:

THEOREM A. *There are no real hypersurfaces with parallel Ricci tensor of a complex space form* $M_n(c)$, $c\neq 0$.

THHOREM B. *There are no real hypersurfaces with harmonic curvature of* $M_n(c)$, $c\neq 0$ *satisfying one of the following conditions:*

(1) *P is an eigenvector corresponding to the Ricci tensor,* (2) *the number of Ricci curvatures does not exceed* 2.

## 1. Preliminaries.

We begin by recalling fundamental formulas on real hypersurfaces of a Kaehlerian manifold. Let $N$ be a real $2n$-dimensional Kaehlerian manifold equipped with a parallel almost complex structure $F$ and a Riemannian metric tensor $G$ which is $F$-Hermitian, and covered by a system of coordinate neighborhoods $\{U; x^A\}$. Let $M$ be a real hypersurface of $N$ covered by a system of coordinate neighborhoods $\{V; y^h\}$ and immersed isometrically in $N$ by the immersion $i: M\to N$. Throughout the present paper the following convention on the range of indices are used, unless otherwise stated:

$$A, B, \cdots=1, 2, \cdots, 2n; i, j, \cdots=1, 2, \cdots, 2n-1.$$

The summation convention will be used with respect to those system of indices. When the argument is local, $M$ need not be distinguished from $i(M)$. Thus, for simplicity, a point $p$ in $M$ may be identified with the point $i(p)$ and a tangent vector $X$ at $p$ may also be identified with the tangent vector $i_*(X)$ at $i(p)$ via the differential $i_*$ of $i$. We represent the immersion $i$ locally by $x^A=x^A(y^h)$ and $B_j=(B_j^A)$ are also $(2n-1)$-linearly independent local tangent vectors of $M$, where $B_j^A=\partial_j x^A$ and $\partial_j=\partial/\partial y^j$. A unit normal $C$ to $M$ may then be chosen. The induced Riemannian metric $g$ with components $g_{ji}$ on $M$ is given by $g_{ji}=G(B_j, B_i)$ because the immersion is isometric.

For the unit normal $C$ to $M$, the following representations are obtained in each coordinate neighborhood:

$$FB_i = J_i^h B_h + p_i C, \quad FC = -p^i B_i, \tag{1.1}$$

where we have put $J_{ji} = G(FB_j, B_i)$ and $p_i = G(FB_i, C)$, $p^h$ being components of a vector field $P$ associated with $P_i$ and $J_{ji} = J_j^r g_{ri}$. By the properties of the almost Hermitian structure $F$, it is clear that $J_{ji}$ is skew-symmetric. A tensor field of type (1, 1) with components $J_i^h$ will be denoted by $J$. By the properties of the almost complex structure $F$, the following relations are then given:

$$J_i^r J_r^h = -\delta_i^h + p_i p^h, \quad p^r J_r^h = 0, \quad p_r J_i^r = 0, \quad p_i p^i = 1,$$

that is, the aggregate $(J, g, P)$ defines an almost contact metric structure. Denoting by $\nabla_j$ the operator of van der Waerden-Bortolotti covariant differentiation formed with $g_{ji}$, the equations of Gauss and Weingarten for $M$ are respectively obtained:

$$\nabla_j B_i = h_{ji} C, \quad \nabla_j C = -h_j^r B_r, \tag{1.2}$$

where $h_{ji}$ are components of a second fundamental form $\sigma$, $A = (h_j^k)$ which is related by $h_{ji} = h_j^r g_{ri}$ being the shape operator derived from $C$. We notice hear that $h_{ji}$ is symmetric. By means of (1.1) and (1.2) the covariant derivatives of the structure tensors are yielded:

$$\nabla_j J_{ih} = -h_{ji} p_h + h_{jh} p_i, \quad \nabla_j p_i = -h_{jr} J_i^r. \tag{1.3}$$

In the sequel, the ambient Kaehlerian manifold $N$ is assumed to be of constant holomorphic sectional curvature $c$ and real dimension $2n$, which is called a complex space form and denoted by $M_n(c)$. Then the components of the curvature tensor $K$ of $M_n(c)$ take the following form:

$$K_{DCBA} = \frac{c}{4}(G_{DA}G_{CB} - G_{DB}G_{CA} + F_{DA}F_{CB} - F_{DB}F_{CA} - 2F_{DC}F_{BA}).$$

Thus, the equations of Gauss and Codazzi for $M$ are respectively obtained:

$$R_{kjih} = \frac{c}{4}(g_{kh}g_{ji} - g_{jh}g_{ki} + J_{kh}J_{ji} - J_{jh}J_{ki} - 2J_{kj}J_{ih}) + h_{kh}h_{ji} - h_{jh}h_{ki}, \tag{1.4}$$

$$\nabla_k h_{ji} - \nabla_j h_{ki} = \frac{c}{4}(p_k J_{ji} - p_j J_{ki} - 2p_i J_{kj}), \tag{1.5}$$

where $R_{kjih}$ are the components of the Riemannian curvature tensor $R$ of $M$.

To be able to write our formulas in a convention form, the components $X_{ji}^m$ of a tensor field $X^m$ and a function $X_m$ on $M$ for any integer $m(\geqq 2)$ are introduced as follows:

$$X_{ji}^m = X_{ji_1} X_{i_2}^{i_1} \cdots X_i^{i_{m-1}}, \quad X_m = \sum_i X_{ii}^m.$$

In our notation, the Gauss equation (1.4) implies

$$S_{ji}=\frac{c}{4}\{(2n+1)g_{ji}-3p_jp_i\}+hh_{ji}-h^2_{ji}, \tag{1.6}$$

where $S_{ji}$ denotes components of the Ricci tensor $S$ of $M$, and $h$ the trace of the shape operator $A$.

REMARK 1. We notice here that the structure vector $P$ cannot be parallel provided that $c\neq 0$. In fact, if $P$ is parallel along $M$, then the second equation of (1.3) becomes $h_{jr}J^r_i=0$. Thus, it is not hard to see that $h_{ji}=hp_jp_j$ because of properties of the almost contact metric structure. Hence it follows that $\nabla_k h_{ji}=(\nabla_k h)p_jp_i$, which together with (1.5) give

$$\frac{c}{4}(p_kJ_{ji}-p_jJ_{ki}-2p_iJ_{kj})=\{(\nabla_k h)p_j-(\nabla_j h)p_k\}p_i.$$

By transvecting $p^iJ^{kj}$, we have $c(n-1)=0$. Thus the assumption $c\neq 0$ will produce a contradiction.

## 2. Real hypersurfaces with harmonic curvature.

Let $M$ be a real hypersurface with harmonic curvature of a complex space form $M_n(c)$, $c\neq 0$, that is, the Ricci tensor $S$ satisfies $\nabla_k S_{ji}=\nabla_j S_{ki}$. Then, we easily, using the second Bianchi identity, see that the scalar curvature $r$ of $M$ is constant everywhere. Moreover, the Ricci formula for $S_{ji}$ gives rise to

$$\nabla_m\nabla_k S_{ji}=\nabla_j\nabla_i S_{mk}-R_{mjkr}S^r_i-R_{mjir}S^r_k,$$

which together with the first Bianchi identity and the Ricci formula imply that

$$R_{mkir}S^r_j+R_{kjir}S^r_m+R_{jmir}S^r_k=0, \tag{2.1}$$

where $S^h_j=S_{ji}g^{ih}$, $g^{ji}$ being the contravariant components of $g_{ji}$. Therefore, it follows that

$$J^{kj}R_{kjih}S^h_m+2J^{rk}R_{kmih}S^h_r=0$$

and hence, in consequence of (1.4),

$$\Big(-n+\frac{3}{2}\Big)cS_{jr}J^r_i+\frac{c}{2}\{S_{ir}J^r_j-(r-A_1)J_{ji}-p_i(S_{rt}p^r)J^t_j-2p_j(S_{tr}p^r)J^t_i\}$$
$$+2h_{tr}h_{is}J^{rs}S^t_j-2h_{jt}h_{ir}J^{sr}S^t_s=0,$$

where we have put $A_1=S_{ji}p^jp^i$. By the way, the last two terms of this reduces to $-\frac{3}{2}cp_j(h_{rt}p^t)h_{is}J^{rs}$ by virtue of (1.6). Accordingly we have

$$S_{ir}J^r_j-(2n-3)S_{jr}J^r_i-(r-A_1)J_{ji}-S_{tr}p^r(p_iJ^t_j+2p_jJ^t_i)-3h_{rt}p^th_{is}J^{rs}p_j=0$$

because of the fact that $c\neq 0$ is assumed, which implies

$$3h_{rt}p^t h_{is}J^{rs}+(2n-1)S_{rt}p^t J_i^r=0.$$

Thus, the last equation can be written as

$$(2.2)\qquad (2n-3)\{S_{jr}J_i^r-(S_{tr}p^r)p_j J_i^t\}-S_{ir}J_j^r+(S_{rt}p^t)p_i J_j^r+(r-A_1)J_{ji}=0,$$

from which, taking the symmetric parts,

$$S_{jr}J_i^r+S_{ir}J_j^r=S_{tr}p^r(p_j J_i^t+p_i J_j^t).$$

Hence, the relationship (2.2) turns out to be

$$2(n-1)\{S_{jr}J_i^r-(S_{tr}p^r)p_j J_i^t\}+(r-A_1)J_{ji}=0.$$

Transforming this by $J_k^i$ and utilizing properties of the almost contact metric structure, it is reduced to

$$(2.3)\qquad 2(n-1)\{S_{ji}-p_iS_{jr}p^r-p_jS_{ir}p^r\}-(r-A_1)g_{ji}+\{r+(2n-3)A_1\}p_jp_i=0,$$

which implies immediately that

$$(2.4)\qquad 2(n-1)(S_2-2A_2+A_1^2)=(r-A_1)^2,$$

where $A_2=S_{ji}^2p^jp^i$.

PROPOSITION 2.1. *Let $M$ be a real hypersurface with harmonic curvature of a complex space form $M_n(c)$, $c\neq 0$. If the structure vector $P$ is an eigenvector of the Ricci tensor, namely, if*

$$(2.5)\qquad S_{jr}p^r=A_1p_j,$$

*then $M$ is Ricci parallel.*

PROOF. By means of (2.5), the relationship (2.3) reduces to

$$(2.6)\qquad 2(n-1)S_{ji}-(r-A_1)g_{ji}+\{r-(2n-1)A_1\}p_jp_i=0,$$

which implies

$$(2.7)\qquad 2(n-1)S_{ji}^2-\{r+(2n-3)A_1\}S_{ji}+A_1(r-A_1)g_{ji}=0.$$

Differentiating (2.6) covariantly, we find

$$(2.8)\qquad \begin{aligned}2(n-1)\nabla_kS_{ji}+(\nabla_kA_1)g_{ji}-(2n-1)(\nabla_kA_1)p_jp_i\\ +\{r-(2n-1)A_1\}\{(\nabla_kp_j)p_i+(\nabla_kp_i)p_j\}=0\end{aligned}$$

because the scalar curvature $r$ is constant. Since the Ricci tensor $S$ is of Codazzi type, it is seen that

(2.9) $$(\nabla_k A_1)g_{ji}-(\nabla_j A_1)g_{ki}-(2n-1)\{(\nabla_k A_1)p_j-(\nabla_j A_1)p_k\}p_i$$
$$+\{r-(2n-1)A_1\}\{(\nabla_k p_j-\nabla_j p_k)p_i+(\nabla_k p_i)p_j-(\nabla_j p_i)p_k\}=0.$$

If we transvect this with $g^{ji}$, then we obtain

$$\nabla_k A_1-(2n-1)(p^r\nabla_r A_1)p_k+\{r-(2n-1)A_1\}p^r\nabla_r p_k=0$$

and hence $p^r\nabla_r A_1=0$. Thus, it follows that $\nabla_k A_1+\{r-(2n-1)A_1\}p^r\nabla_r p_k=0$. Transvecting (2.9) with $p^j p^i$ and taking account of the last equation, we can verify that $A_1$ is constant everywhere. Therefore, by differentiating (2.7) covariantly, we find

$$2(n-1)\nabla_k S^2_{ji}-\{r+(2n-3)A_1\}\nabla_k S_{ji}=0,$$

which shows that $S^2_{ji}$ is of Codazzi type. Thus, the Ricci tensor $S$ is parallel because the scalar curvature of $M$ is constant (see Umehara, Theorem 1.3 of [19]). This completes the proof of Proposition 2.1.

REMARK 2. If the structure vector $P$ is principal, that is, $h_{jr}p^r=\alpha p_j$, we can see from (1.6) that $P$ is the eigenvector of the Ricci tensor and hence the Ricci tensor is parallel.

Now, transforming (2.3) by $S^i_k$, we obtain

(2.10) $$2(n-1)\{S^2_{jk}-(S_{kt}p^t)(S_{jr}p^r)-p_j S^2_{kr}p^r\}-(r-A_1)S_{jk}$$
$$+\{r+(2n-3)A_1\}p_j S_{kr}p^r=0,$$

which enables us to obtain

$$(2(n-1)S^2_{kr}p^r-\{r+(2n-3)A_1\}S_{kr}p^r)p_j-(2(n-1)S^2_{jr}p^r$$
$$-\{r+(2n-3)A_1\}S_{jr}p^r)p_k=0.$$

Thus, it is seen that

(2.11) $$2(n-1)S^2_{kr}p^r-\{r+(2n-3)A_1\}S_{kr}p^r=(2(n-1)A_2-A_1\{r+(2n-3)A_1\})p_k.$$

Making use of the last equation, (2.10) turns out to be

(2.12) $$2(n-1)\{S^2_{jk}-(S_{jt}p^t)(S_{kr}p^r)\}-(r-A_1)S_{jk}+\mu p_j p_k=0,$$

where $\mu=A_1(r-A_1)-2(n-1)(A_2-A_1^2)$. Transforming (2.12) by $S^k_i$ and utilizing (2.3), (2.11) and (2.12), we get

(2.13) $$4(n-1)^2S^3_{ji}-4(n-1)\{r+(n-2)A_1\}S^2_{ji}$$
$$+\{(r-A_1)(r+(4n-5)A_1)-4(n-1)^2(A_2-A_1^2)\}S_{ji}-\mu(r-A_1)g_{ji}=0,$$

or, equivalently

$$\left(S_j^r-\frac{r-A_1}{2(n-1)}\delta_j^r\right)\{2(n-1)S_{ir}^2-\lambda S_{ir}+\mu g_{ir}\}=0,$$

where we have put $\lambda=r+(2n-3)A_1$. Thus the minimal polynomial for $S$ tells us that there exist at most three Ricci curvatures of $M$: $(r-A_1)/2(n-1)$, $(\lambda\pm\sqrt{D})/4(n-1)$, where

(2.14) $$D=\{r-(2n-1)A_1\}^2+16(n-1)^2(A_2-A_1^2).$$

And their multiplicities are respectively denoted by $2n-1-l_1-l_2$, $l_1$ and $l_2$. Therefore the scalar curvature $r$ of $M$ satisfies

(2.15) $$(l_1+l_2-2)\{r-(2n-1)A_1\}=\sqrt{D}(l_1-l_2).$$

We also have

$$4(n-1)^2S_2=\frac{1}{4}(\lambda^2+D)(l_1+l_2)+\frac{1}{2}\lambda\sqrt{D}(l_1-l_2)+(r-A_1)^2(2n-1-l_1-l_2),$$

which together with (2.4), (2.14) and (2.15) imply that

(2.16) $$(A_2-A_1^2)(l_1+l_2-2)=0.$$

Now, suppose that the number of distinct Ricci curvatures does not exceed 2. Then we can easily see that $A_2=A_1^2$ because of (2.15). Thus, it follows that $S_{jr}p^r=A_1p_j$.

According to Proposition 2.1, we have

PROPOSITION 2.2. *Let $M$ be a real hypersurface with harmonic curvature of a complex space form $M_n(c)$, $c\neq 0$. Then the number of distinct Ricci curvature is at most* 3. *In particular, it does not exceed* 2, *then $M$ is Ricci parallel.*

## 3. Real hypersurfaces with parallel Ricci tensor.

In this section we devote to investigate the real hypersurfaces with parallel Ricci tensor of a complex space form $M_n(c)$, $c\neq 0$. Since the Ricci tensor $S$ is assumed to be parallel, we have (2.13) and hence

$$4(n-1)^2S_3-4(n-1)rS_2-4(n-1)(n-2)S_2A_1+r(r-A_1)^2+4(n-1)rA_1(r-A_1)$$
$$+2(n-1)r(A_2-A_1^2)-2(n-1)(2n-1)A_1(A_2-A_1^2)-(2n-1)A_1(r-A_1)^2=0,$$

which together with (2.4) yield

$$\frac{1}{2(n-1)}(r-A_1)^3+2(n-1)A_1^3+3rA_1(r-A_1)-3(2n-3)S_2A_1-3rS_2$$
$$+4(n-1)S_3=0.$$

Thus, $A_1$ is a root of the cubic equation with constant coefficients because $S_i$ is constant for each number $i$. Accordingly $A_1$ is constant. By the definition of $A_1$, it is not hard to see that

$$S_{ir}p^i\nabla_k p^r=0 \tag{3.1}$$

because the Ricci tensor is parallel. By differentiating (2.3) covariantly, we find

$$\begin{aligned}2(n-1)\{(\nabla_k p_i)S_{jr}p^r+(\nabla_k p_j)S_{ir}p^r+p_iS_{jr}\nabla_k p^r+p_jS_{ir}\nabla_k p^r\}\\ =\{r+(2n-3)A_1\}\{(\nabla_k p_j)p_i+(\nabla_k p_i)p_j\}.\end{aligned} \tag{3.2}$$

If we apply $p^j$ to this and sum for $j$, and make use of (3.1), we obtain

$$2(n-1)S_{ir}\nabla_k p^r=(r-A_1)\nabla_k p_i.$$

Thus, (3.2) turns out to be

$$(\nabla_k p_i)S_{jr}p^r+(\nabla_k p_j)S_{ir}p^r=A_1(p_i\nabla_k p_j+p_j\nabla_k p_i).$$

Transvecting the last equation with $S_t^j p^t$ and utilizing (3.1), we get

$$(A_2-A_1^2)\nabla_k p_i=0. \tag{3.3}$$

By means of Remark 1, it follows that $A_2=A_1^2$ and hence $S_{jr}p^r=A_1p_j$. Therefore, the relationship (2.3) is reduced to

$$2(n-1)S_{ji}=(r-A_1)g_{ji}-\{r-(2n-1)A_1\}p_jp_i.$$

The Ricci tensor of $M$ being parallel, it is seen that

$$\{r-(2n-1)A_1\}(p_i\nabla_k p_j+p_j\nabla_k p_i)=0$$

and hence $r-(2n-1)A_1=0$. Thus, $M$ is Einstein. But, there are no Einstein real hypersurfaces of $M_n(c)$, $c\neq 0$ because of Theorem K and Theorem M (see also [10]). Hence Theorem A is completely proved.

Proof of Theorem B. Due to Theorem A, Proposition 2.1 and Proposition 2.2.

By means of (2.16), Theorem A and Proposition 2.2, it is clear that $l_1=l_2=1$. Therefore we can state the following fact:

Remark 3. Let $M$ be a real hypersurface with harmonic curvature of $M_n(c)$, $c\neq 0$. Then $M$ has three distinct Ricci curvatures: $(r-A_1)/2(n-1)$, $(\lambda+\sqrt{D})/4(n-1)$, $(\lambda-\sqrt{D})/4(n-1)$ with multiplicities $2n-3$, 1, 1 respectively.

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