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Comment

Hirotugu Akaike

Professor Shafer's paper shows his concern for the future of statistics. He considers that the present situation of statistics is alarming and, assuming that mathematical statistics is a child of mathematical probability, attributes this situation to the popularization and diversification of the use of probability. I completely agree with Professor Shafer on the recognition of the problematical status of statistics and would like to add some observations on the nature of statistics and probability.

STATISTICS FOR PLANNING AND PROBABILITY FOR DECISION

It is almost certain that the original concept of statistics started with the description of the state of a nation by counting and classifying its people. Any country appearing in the history must have used some kind of statistics for the management of the country. Along with this very old origin of the concept of statistics was also the use of probabilistic mechanisms or randomizers by ancient kings.

A typical example of the use of a randomizer is given by the *I Ching*, or the Book of Changes, which shows the wisdom of ancient Chinese people for the handling of uncertainties. With this book there is an advice that recommends the minimum use of the book to attain a proper objective.

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Consider a king who was going to declare a war against another country. It is almost certain that he used statistics for the planning of the war. But there must have remained some uncertainty. If he intended to consult the *I Ching*, then the advice would have forced him to make utmost effort to minimize the uncertainty before he turned to the randomizer. This means that the process of setting up a probability distribution for a particular purpose must be based on a fully efficient use of available information which is often supplied by the related statistical data in the case of the decision related to the future of a nation.

Here we can see a typical example of the use of statistics for planning and probability for decision. This example also demonstrates the inherent connection of probability and statistics with the proper use of information.

PROBABILITY OF A SINGLE EVENT

Consider a situation where probability $p(A)$ of the occurrence of an event A is given. When $p(A)$ is greater than 0.5, according to the interpretation of probability as described by Shafer, it would seem reasonable to bet on the occurrence of A. However, since probability does not tell anything about actual occurrence of a particular event, some justification is required for the decision to bet on A.

This problem is deeply related to the argument of objectivity or subjectivity of probability. If the probability is considered to be objective, in the sense that it is accepted by most of the members of a society, the

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decision will be considered to be rational. The evaluation is based on the totality of past experiences of the society, and hopefully the society will maintain its stability by such behavior of each of its members.

However, we must note here that further wisdom is contained in the actual use of a randomizer. Even when $p(A)$ is very close to 1, an actual flipping of a coin with $p(A)$ for its head may produce a tail and lead to the rejection of A. This is sometimes important for the society, as the process of evaluation of $p(A)$ may not have been perfect. Some important aspects may have been ignored that could only be confirmed by betting on the nonoccurrence of A; for example, in the case of a new discovery obtained by performing an experiment which was originally considered extremely unpromising.

These observations show that the actual use of probability for decision can only be justified within the context of society.

MONEY AS A PREDECESSOR OF PROBABILITY

Advocates of the subjective or personal theory of probability sometimes place too much emphasis on the subjective aspect of probability. However, the argument for the justification of subjective probability is usually based on the concept of fair price for a bet.

The concept of price is certainly based on the concept of money. Money is not a natural product but is a product of human society. However, it is not simple subjective evaluation of a value. It has an objective background that assures its use within a society. No one is going to pay an arbitrarily set price. The same is true for an arbitrarily chosen subjective probability. It is the acceptability of the argument for the adoption of a probability that justifies the use of a subjective probability. The adoption of a subjective probability must then be accompanied by an effort to secure the acceptance of society, which is almost the synonym of objectivity.

One important use of statistical method is the evaluation and comparison of various probability models by a set of data. The use of likelihood for this purpose has been well developed. In particular, the concept of entropy or information provides a basis for the comparison of probability models. The recognition of the objectivity or intersubjectivity of log likelihood as a criterion, as discussed in Akaike (1985), provides a basis for the objectivity of statistical evaluation of subjectively chosen models.

DEPARTMENT OF STATISTICAL SCIENCE

Professor Shafer argues that the department of statistics must be revitalized to keep its position as the intellectual center of probability. As was touched upon earlier in this comment, the concept of statistics seems to be more broadly connected with the concept of state of a system defined by a collection of constituents, as in the case of a nation. Only a collection of data shows the necessity of such basic information processing as classification or clustering. An abstract theory of probability does not presuppose any collection of data. Thus it may not be quite appropriate to characterize statistics department solely by the use of probability, though this is certainly a dominant characteristic.

When we pay attention to the uncertainties and complexities of the environment that surrounds us, we feel that we live in a statistical world. The statistical method may be viewed as having been developed for the handling of the problems of this world and statistical science as the science of this world. Such a science naturally permeates through other sciences.

Once this view is accepted, it would be more reasonable to contemplate the future of the statistics department as the department of statistical science. This idea is actually being discussed by the National Committee for Statistics in Japan as a possibility for a new type of department within a graduate school.

HOW TO ATTAIN INDEPENDENCE

I have learned statistics through contact with real problems. My friends who brought me real problems were in other areas of science or engineering. They acted as my teachers. In return, I helped them with my knowledge gained from my former experiences of statistical problems. Only through this kind of contact could I develop my own view of statistical world.

Thus it seems to me that the most important thing in establishing a center for statistical studies is to find people in other disciplines who would be willing to come and teach us statisticians about their challenging problems. Only the charm and productivity of statistical science and its related technology will attract these people to statistical science. Successful development of useful methods realized through the cooperation with people from outside is most important to secure the independence of statistics or statistical science as a respectable discipline.

This last idea was actually adopted as a guiding principle for the restructuring of the Institute of Statistical Mathematics as an interuniversity research institute of Japan in 1986. Our experience in the past five years seems to confirm the validity of this principle.

CONCLUDING REMARK

The above observations only confirm the importance of the subject discussed in the paper by Professor Shafer. I feel that the paper should be read and discussed by all the statisticians who really care about the future of statistics.

I am very grateful to the Executive Editor, Professor Carl N. Morris, for inviting me to join the discussion.

Comment

David Aldous

Though asserting no interest in the foundational side of probability (thereby inviting a Keynesian riposte about being "enslaved to a philosophy long discredited"!), I found Professor Shafer's article interesting and thought-provoking. Professor Shafer uses the word "probability" in a wise sense; reflecting my own interests, I use it mean "probability and its applications, *excluding* statistics." Readers may judge for themselves whether my comments are relevant to statistics proper. Many of Professor Shafer's comments concern teaching issues, whereas mine mostly address research.

1. RESEARCH-LEVEL APPLIED PROBABILITY

It is curious that there is no phrase "[adjective] mathematics" which adequately conveys the idea

(M) research whose conclusion is the statement and proof of a theorem

as opposed to

(A) research whose goal is answering a science question, using mathematics as a means rather than an end.

(I use "science" very broadly to mean some academic discipline in which mathematics can be used.) Though making distinctions between theory and applications is unfashionable and politically incorrect, I do see a distinction between seeking to make money at blackjack or the stock market and proving optimal strategy theorems; between designing airplane wings and proving theorems about air flow; between building reliable systems of components and proving theorems about increasing failure rates; between understanding molecular evolution and proving theorems about measure-valued diffusions.

Although an applied mathematician or statistician might claim to be doing both (A) and (M)—posing an extra-mathematical question and then answering it by proving a theorem—the proportion of research papers that actually do both is extremely small. Most papers in (for example) *Annals of Statistics, Journal of Applied Probability, IEEE Transactions on Information Theory, SIAM Journal of Control and Optimization,* and *Journal of the Association for Computing Machinery* and much that is usually called "applied mathematics," are plainly (M) but not (A). Good applied mathematics is like the unicorn: something we can all recognize but seldom actually see.

The part of (M) that is not traditional "pure mathematics" needs a name: I call it "theory-motivated-by-applications" (TMA) mathematics. Of current research involving probability, much more is TMA than is either (A) or pure mathematics. The key problem with research-level applied probability is the lack of agreed standards for evaluating TMA research. While this is not a pressing issue for most of us, it is for Mike Steele (as Editor of the new *Annals of Applied Probability*) and his associate editors. It would be unreasonable and divisive to erect high threshold standards for "serious math" and for "serious science" and insist that research exceeds one threshold or the other. There is a spectrum: at one extreme is serious math theory (at the level of *Annals of Probability*) with a rather vague connection to an application; at the other extreme is a serious science question which is solved by (to an expert theoretician) rather routine mathematics. Linear interpolation between these extremes is fine and constitutes what I regard as worthwhile applied probability. What concerns me is that, once a dozen people write papers on "probability methods in subject S," a continuing subdiscipline is likely to be established. At best, this subdiscipline will produce results of interest to both the mainstream nonmathematical scholars in subject S and to workers in broader areas of theoretical and applied probability. At worst, it becomes an inward-looking clique ignored by everyone else. Of course this worst case also happens within theoretical disciplines, but there it is easier to detect. A cynic might say that applied probabilists can get away with claiming to theoreticians that they are solving science questions, and claiming

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