# **Remembering John W. Tukey**

## Luisa Turrin Fernholz

#### 1. THE LAST WEEK

In the morning of Tuesday, July 25, 2000, I went to the Robert Wood Johnson University Hospital in New Brunswick to visit John Tukey. In that labyrinthine hospital I had to ask several times for directions before I found his room. It was a double room and in one bed there was a young man, semiconscious and complaining. On the other bed John W. Tukey lay totally unconscious. The nurse who was trying to wake him explained that, "He had some tests done this morning and is still under the effect of the drugs given for the tests."

"Do you know who this man is?" I asked.

She replied, "He is a statistician. Yesterday I was joking with him about the mode and the median that I studied when I was in school."

I said, "But he is much more than a statistician. Please make sure that he'll be well cared for."

"Oh, he should be OK," she assured me, "His condition is reversible."

But John was not reacting well to the treatment, and other doctors and nurses came in to assist. I left the room and called John's home in Princeton. Mary Bittrich, his secretary from Bell Labs, was at the house and answered the phone. I told her that John did not look very well, and that it would be a good idea for someone else to come to the hospital to watch over him. Mary said that Khris Quicksall, the woman who had been assisting John since 1998, was on her way to the hospital, and that Phyllis Anscombe, John's sisterin-law, and other members of his family would soon be going.

After Khris arrived at the hospital I left. For the rest of that day I was afraid to call Mary back. Then at 1:30 AM my phone rang. It was Khris to tell me the bad news: at 1 AM on July 26, John Wilder Tukey had passed away.

I had talked to John the previous week when he was staying at the Merwick Rehabilitation Center in Princeton. There he seemed to have regained some strength after the stroke he had suffered 2 weeks earlier; we had had some statistical discussions, he was starting to walk slowly, and the prognosis looked good. John was eager to go home. Mary Bittrich had been busy with the contractors to have some remodeling done in the house so that John could have his bedroom and studio on the same floor. John had hoped to return home soon and to continue his work in the newly configured house with Khris as his housekeeper. Khris was a friendly and warm person, and Elizabeth Tukey had been fortunate to have hired her to assist them after Elizabeth became ill. After Elizabeth passed away in January 1998, Khris had been taking care of John. Khris' devotion to both John and Elizabeth had earned her John's respect and affection, and made John's last few years quite positive in spite of Elizabeth's death. During that time, John was able to work with statisticians in the Princeton area as well as to correspond with other statisticians all around the world, and he also continued some of his consulting work. It seemed as if this could have gone on indefinitely, but it did not happen that way. It all stopped in the first hour of July 26, 2000.

### 2. MATHEMATICS AND STATISTICS

I started working with John Tukey in 1995. Our collaboration may have been unusual, since I had not been a student of his, and my graduate training had been in mathematics and theoretical statistics, which according to many observers, Tukey apparently disliked. I first met him in the spring of 1980 when I gave a seminar talk at the Statistics Department of Princeton University. In this talk I presented some mathematical results that used Frechet and Hadamard derivatives to derive certain asymptotic results. John was in the audience, and I had heard that he loathed both mathematics and asymptotics. Nevertheless, my lecture proceeded without incident, and in fact he did not even fall asleep (as was his custom, although he always seemed to hear everything that went on). At one point I mentioned that the empirical distribution functions  $F_n$  were not measurable on the space D(0, 1), so they had to be smoothed to make them measurable, to which he commented in reference to the  $F_n$ , that, "if these things

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don't work, it is because we are not defining them correctly." In spite of the functional-analytic nature of my talk, I was invited to join the Princeton Statistics Department as an instructor in the fall of 1980. While at Princeton, I had the good fortune to be able to audit John's lectures of Statistics 411, Data Analysis, where I became aware of a new reality of statistics. There I discovered that the "over-utopian" world of the textbooks was something we should beware of and that a random sample is indeed a "batch of values" which "fail to be utopian" most of the time. I soon understood why the jackknife can be seen as "a meat-grinder with some reusable pieces of meat." His way of looking at uncertainty made me reflect on my statistical training and has enlarged, deepened and changed my approach to statistics in general and my own research in particular.

Of course, I was not the only person affected by Tukey's ideas in statistics: a revolution had already begun. Since the early 1950s the world of the academic statisticians had been challenged by his writings that questioned the validity of the comfortable Gaussian assumptions. One of the strongest confrontations with the statistics establishment occurred with the presentation of "The Future of Data Analysis" (Tukey, 1962) in which Tukey essentially proposed a rupture with the traditional statistical thinking of the time and a restructuring of the field along the lines of data analysis. In this provocative article he wrote, "To the extent that pieces of mathematical statistics fail to contribute, or are not intended to contribute, even by a long and tortuous chain, to the practice of data analysis, they must be judged as pieces of pure mathematics, and criticized according to its purest standards."

Since Tukey maintained this philosophical antagonism to mathematical statistics throughout his life, the perception remained that he was "antimathematical." However, this conclusion is rather simplistic when one considers the full scope of his philosophical writings, most of them published in Volumes III and IV of his collected works, Philosophy and Principles of Data Analysis (Tukey, 1986). As we all know, Tukey was a prominent mathematician before becoming a statistician. He had a thorough understanding of the strengths and limitations of mathematics in statistics as well as in other sciences. It is clear that he was deeply committed to the truth, and his careful exploration of concepts, such as "scientific statistics," "mathematical statistics," "theory of statistics" and "theory of data analysis," shows his concern regarding these issues and his intellectually honest approach to them.

Tukey knew that mathematics had its uses; in his revolution he did not propose to eliminate mathematics from statistics, but rather to avoid being bound by the limitations that mathematics imposed. When Stephan Morgenthaler and I, in a conversation with him in 1997 (Fernholz and Morgenthaler, 2000, page 83), asked whether formal mathematical proofs are necessary, he answered:

No, because I know too much about the anomaly of what is constructible in such a way to want to go that way. On the other hand, I think I've always been willing to take the mathematical structures and mathematical proofs as part of the story, and to expect that there were situations where one wouldn't have a feel for how to understand.

#### 3. WORKING WITH TUKEY

My working experience with John Tukey falls into the realm of surrealism. I remember showing him pages of computer outputs with hundreds of numbers in which, after a few seconds of inspection, he would find a small discrepancy that I doubt anybody else would have detected so quickly, if at all. At other times he would find a pattern that was inconspicuously lurking among the numbers, but probably would have gone unnoticed by another person. The variety and abundance of his thoughts so overwhelmed me that I sometimes felt drowned in an ocean of exuberant new ideas. An army of statisticians would have been needed to carry out all the research that he envisioned.

Until I had discussed with John the outlier detection properties of the multihalver, I would not have believed that any human being could have mentally kept track of the intricacies of such complicated algorithms. Although he invented the word "software," he did not use computers. He was able to anticipate and visualize the results of complex algorithms, and he carried out his computations either in his head or sometimes with pencil and paper, but usually without using a calculator. On one extreme occasion, confronted with a particularly intractable set of numbers, he started using an antique calculator that he happened to have, but soon abandoned it. The calculator was too tedious; using his mind would provide the answer more quickly.

John's devotion and passion for his work are legendary. He would sometimes phone me at unconventional hours of day or night when he had a new idea regarding one or another of our research prob-



FIG. 1. Suggestions for using the multihalver jackknife. Handwritten by J. W. Tukey, 1999.

lems. Elizabeth Tukey mentioned in Fernholz and Morgenthaler (2000, page 90) that her father had asked her "whether while he was waiting for [her] at the altar, he would whip out a yellow pad and not waste any time!" I remember John sitting at Elizabeth's bedside with his yellow pad when she was ill at the hospital. Until the last days of his life he had statistical discussions with me and others at Princeton, as well as with several other statisticians around the world with whom he was corresponding and doing research. His nephew, Francis Anscombe, observed that John wanted to die with "his working boots on," and this he certainly achieved.

John was in no sense a conventional thinker. To understand the total extent of his ideas, every word of every sentence of his had to be carefully considered. The following paragraph, which was taped in his house in March 1998 in one of our work meetings, demonstrates the multidimensional nature of his thinking: Jacknife technology is analogical. We do things that would be sensible for a situation that is a simpler analog of the one we face. All general results so far known are on asymptotics, and the situations where the technology is used are now mostly very unasymptotic. Trusting the analogy, has, so far, in situations where the behavior is known, led to appropriate asymptotic results. What is needed to complement the analogy is empirical results for specific farfrom-asymptotic situations. This may seem un-mathematical, but it is the best that we can do today and almost certainly the best that we will be able to do tomorrow.

The notation that he used to describe his concepts was so unique that it must be seen in order to be imagined. I reproduce here a page that he wrote in 1999 (see Figure 1).

#### 4. JOHN AND ELIZABETH

John's wife, Elizabeth, played a vital role in John's life and career. She attended many of John's conferences and participated in several interviews with him. Her love, devotion and understanding allowed John to pursue his work without interruption or distraction. In her speech for John's 80th birthday she wrote, "From the day of our marriage, John and I have been 'a team' and therefore feel we have accomplished more than either of us could have done individually" (Brillinger, Fernholz and Morgenthaler, 1997).

I remember visiting John and Elizabeth at their home in Princeton, a colonial on Arreton Road, surrounded by several acres of woodland, and furnished with early American style furniture, of which Elizabeth was a dedicated collector. I started meeting with John at their house in 1997 when Elizabeth became ill, since he did not want to leave her alone and go to his office at Fine Hall. Usually we worked in the dining room, where we used the large table to display the many sheets of paper that we needed for our work. However sometimes we met in the library or the kitchen, which was of 50s vintage, and included the original refrigerator, bought in 1950 when they were married, but still working perfectly. "There is no need to buy a new refrigerator since this is quite good and works well," John once commented. After the statistics work, there were always interesting discussions with John and Elizabeth.

My family enjoyed gatherings with John and Elizabeth. It was truly an extraordinary experience to participate in that unique atmosphere of informal surroundings, intellectual reflections, and stimulating discussions. My children also were privileged to be able to participate in discussions with John on a wide range of topics, from poison ivy and poison oak, to the different kinds of berries in New England, to quantum computing. It seemed that in every topic John was an expert.

John and Elizabeth were both active in community affairs in Princeton. Phyllis Marchand, the Mayor of Princeton, recalled (Marchand, 2000) that at a neighborhood meeting, when residents were concerned about the effect on the traffic of a proposed development in the Arreton Road/Route 206 area, it was decided to carry out some traffic counts. To avoid unnecessary expense, it was suggested that a volunteer from the neighborhood do the counts. This might have been a perfect job for a teenager, but Marchand was surprised that "John Tukey with all his fame, volunteered his expertise, and without calculators, computers or fancy technology, sat on a chair at a designated spot on Route 206 and with a pad of paper and a pencil counted the cars and trucks as they traveled past." There is no doubt that the batch of numbers that John collected that day was quite relevant! (And well analyzed.) Mayor Marchand (2000) added, referring to John, "He along with Elizabeth were active in raising money and raising consciousness about environmental sensitivity, quality of life, and historic preservation." In fact, Elizabeth was First Chair of the Princeton Township Historic Preservation Commission.

Tukey was always able to convey much information in few words, never more so than when, after Elizabeth's death in January 1998, in the eulogy he stated: "One is so much less than two" (Anscombe, 1999).

#### 5. JOHN W. TUKEY, THE HUMAN BEING

In addition to being an eminent scientist, John had a warm human dimension that complemented his profound intellect. Those who had the privilege of knowing him personally would share the image of a kind, warm and compassionate John Tukey. He enjoyed working with others, and many of us had the privilege to participate in his genius. His extreme generosity with his ideas is witnessed by the long list of his Ph.D. students (more than 50) whom he referred to as "his children" as well as "his grandchildren" (his students' students). Kafadar (2002) stated that, "It touched me deeply to know that he thought of us as his children-because, to me, he was in fact so much more than an 'academic father'." Brillinger (2002) noted that, "It is clear that he gave his students a substantial head start on their research careers." Brillinger (2002) also commented that John acted as a cushion to the vicissitudes of graduate student life: "My personal experience was that he put students down a bit when they were over-cocky, and built them up when they were down." Another student of John's, Thompson (2001) wrote, "Whenever I think of John Tukey it is first as the best of professors and only second as the best of statisticians."

John had a particular wit and subtle sense of humor. Brillinger (2002) recalled some humorous stories and most everyone who interacted with John has amusing stories to tell. When I was pregnant with my second child and I was attending his Statistics 411—Data Analysis lectures he commented to me that, "I'm glad to see that you are training your baby to be good in Data Analysis." The Princeton University undergraduate newspaper *The Daily Princetonian* (2000) published the article "Professor Emeritus Remembered for Lively Humor and Passion," which mentions the mathematician Robert Gunning's comments that Tukey, "was very lively and energetic, and in some ways idiosyncratic." He added, "In the dim, dark days of the past, when [Tukey] threw a party, instead of a keg of beer, he would have a keg of milk." But perhaps Elizabeth best captured his human dimension when she said that "[John] is a New Englander through and through" and his individuality "is one of the things that John has in spades" (Fernholz and Morgenthaler, 2000, page 88).

#### SOME FINAL WORDS

Among the many influential positions held by John Tukey was that of vice president of the American Philosophical Society from 1974 to 1977. His deep philosophical interests were always present in his scientific work and formed a basis for his intellectual contributions. The concept of uncertainty was central to his philosophical thoughts, as he wrote (Tukey, 1997, page 23):

We live in a paradoxical world, where the only true safety, true though limited, comes from admitting both our uncertainty and the incompleteness with which we are able to meet it.

John chose the following epitaph to be written on his tomb:

Stone walls do not a prison make, Nor iron bars a cage; Minds innocent and quiet take That for an hermitage; If I have freedom in my love And in my soul am free, Angels alone, that soar above Enjoy such liberty.

**Richard Lovelace** 

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