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LEFT CENTRALIZERS ON RINGS THAT ARE NOT SEMIPRIME

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ABSTRACT. A (left) centralizer for an associative ring R is an additive map satisfying $T(xy) = T(x)y$ for all x, y in R. A (left) Jordan centralizer for an associative ring R is an additive map satisfying $T(xy+yx) = T(x)y+T(y)x$ for all x, y in R. We characterize rings with a Jordan centralizer T. Such rings have a T invariant ideal I, T is a centralizer on R/I, and I is the union of an ascending chain of nilpotent ideals. Our work requires 2-torsion free. This result has applications to (right) centralizers, (two-sided) centralizers, and generalized derivations.

1. Introduction. Let R be a ring, and let $T : R \to R$ be an additive map such that

$$T(xy + yx) = T(x)y + T(y)x \quad \text{for all } x, y \in R. \tag{1}$$

We define a function $h : R \times R \to R$ by

$$h(x, y) = T(xy) - T(x)y.$$

It is immediate that T is a (left) centralizer if and only if $h(x, y) = 0$ for all x, y in R.

A derivation is an additive map $D : R \to R$ which satisfies $D(xy) = D(x)y + xD(y)$ for all x, y in R. A Jordan derivation is an additive map which satisfies $D(x^2) = D(x)x + xD(x)$ for all x in R.

In [**3**] Herstein showed that in a prime ring of characteristic not 2, any Jordan derivation is actually a derivation. In [**2**] Cusack improved this result by replacing the requirement of prime by semi-prime.

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Hvala [**4**] introduced the concept of a generalized derivation. An additive map $F : R \to R$ is a generalized derivation if $F(xy) = F(x)y + xD(y)$ where $D : R \to R$ is a derivation. For a generalized derivation, the map $G(x) = F(x) - D(x)$ becomes a (left) centralizer. The study of generalized derivations is strongly dependent on the study of (left) centralizers. Similarly, the study of generalized Jordan derivations, i.e., $F(x^2) = F(x)x + xD(x)$, is dependent on the study of Jordan (left) centralizers.

Jordan centralizers have been studied by Zalar [**5**]. He showed that when R is 2-torsion free and semiprime, then any Jordan centralizer is actually a centralizer. This result suggests that if T is a Jordan centralizer on a ring R, I is an ideal of R invariant under T, and R/I is semiprime, then $h(R,R) \subset I$ and T must be a centralizer on R/I.

The intent of our paper is to study Jordan centralizers on rings which are not semiprime. Let $N(R)$ be the (Baer lower) radical of R, i.e., $R/N(R)$ is semiprime. We show that $h(R,R) \subset N(R)$. We do not show that $T(N(R)) \subset N(R)$.

This paper will focus on the T invariant ideal I generated by $H = h(R,R)$. We show that I is the union of a chain of nilpotent ideals. This shows that $I \subset N(R)$. This allows us to prove that T is well defined on the cosets of R/I and that this map is a centralizer on R/I.

2. Preliminaries. In this paper, we use the letter P to represent the set of absolute left annihilators. $P = \{x \in R;\ xR = 0\}$. P is a two sided ideal of R. T will be a left Jordan centralizer. We will be using repeated applications of T. In particular $T^0(x) = x$; $T^{n+1}(x) = T(T^n(x))$. The "$\circ$" product or the "symmetric product" is defined by $x \circ y = xy + yx$.

Lemma 2.1. *Let T be a (left) Jordan centralizer on a ring R. Then*

$$\sum_{i+j=n} T^i(x)T^j(x) = \sum_{i+j=n} T^i(xT^j(x)).$$

Proof. We compute (1) for all possible i, j, k where $i + j + k = n$, $i \geq 1$, $0 \leq j \leq k < n$. The sum of all of these equations will give the conclusion of Lemma 2.1.

The form of equation (1) when $i \geq 1$ and $j < k$ is as follows.

$$T^i(T^j(x)T^k(x)) + T^i(T^k(x)T^j(x)) \\ = T^{i-1}(T^{j+1}(x)T^k(x)) + T^{i-1}(T^{k+1}(x)T^j(x)).$$

In this case the equation has exactly four terms. Each side has exactly two terms. The form of equation (1) when $i \geq 1$ and $j = k = l$ is

$$T^i(T^l(x)T^l(x)) = T^{i-1}(T^{l+1}(x)T^l(x)).$$

In this case the equation has exactly two terms. Each side has exactly one term. To help visualize the pattern in the exponents, we use $T^0(x) = x$ and $T^1(x) = T(x)$. In this notation we are trying to show

$$\sum_{i+j=n} T^0(T^i(x)T^j(x)) = \sum_{i+j=n} T^i(T^0(x)T^j(x)).$$

The terms are all of the form $T^a(T^b(x)T^c(x))$, on the left hand side $a \geq 1$, and on the right hand side $b \geq 1$. Any term $T^a(T^b(x)T^c(x))$ with $a \geq 1$ appears on the left hand side in exactly one equation. It appears as the first term if $b < c$ and as the second term if $b > c$. The equation has exactly one term on the left hand side if $b = c$.

Any term $T^a(T^b(x)T^c(x))$ with $b \geq 1$ appears on the right hand side in exactly one equation. It appears as the first term if $b < c+1$ and as the second term if $b > c+1$. The equation has exactly one term on the right hand side if $b = c+1$.

When taking the sum of all of these equations, the terms which appear on both the right and left sides cancel. The terms on the left which do not appear on the right are those of the form

$$T^{n-k}(T^0(x)T^k(x)) \quad \text{for } 0 \leq k < n.$$

The terms on the right hand side which do not appear on the left are those of the form

$$T^0(T^{n-k}(x)T^k(x)) \quad \text{for } 0 \leq k < n.$$

These are the terms appearing in the sums mentioned in Lemma 2.1 with the term $T^0(T^0(x)T^n(x))$ added to both sides to make the notation easier. $\square$

Lemma 2.2. *Let R be a ring and T a Jordan centralizer on R. The function h is defined by $h(x,y) = T(xy) - T(x)y$. The function h satisfies the following.*

(a) $h(x,y) + h(y,x) = 0$

(b) $2h(xy,z) = -h(x,y)z + h(y,z)x - h(z,x)y$

(c) $h([x,y],z) = -h(x,y)z$

(d) $h(x,y)[z,w] + h(z,w)[x,y] = 0.$

Proof. Part (a): $h(x,y)+h(y,x) = T(xy)-T(x)y+T(yx)-T(y)x = 0$ by (1).

Part (b): We use (1) three times and the definition of h four times.

The numbers in parentheses below the terms identify identical terms.

$$\begin{array}{cccccl}
+T(xyz) & +T(zxy) & -T(xy)z & -T(z)xy & =0 & \text{by } (1) \\
(1) & (2) & (3) & (4) & & \\
-T(zxy) & -T(yzx) & +T(zx)y & +T(y)zx & =0 & \text{by } (1) \\
(2) & (5) & (6) & (7) & & \\
+T(yzx) & +T(xyz) & -T(yz)x & -T(x)yz & =0 & \text{by } (1) \\
(5) & (1) & (8) & (9) & & \\
+h(x,y)z & -T(xy)z & +T(x)yz & & =0 & \text{definition of } h \\
(10) & (3) & (9) & & & \\
+h(z,x)y & -T(zx)y & +T(z)xy & & =0 & \text{definition of } h \\
(11) & (6) & (4) & & & \\
-h(y,z)x & +T(yz)x & -T(y)zx & & =0 & \text{definition of } h \\
(12) & (8) & (7) & & & \\
+2h(xy,z) & -2T(xyz) & +2T(xy)z & & =0 & \text{definition of } h \\
(13) & (1) & (3) & & & \\
\hline
2h(xy,z) & +h(x,y)z & -h(y,z)x & +h(z,x)y & =0 & \\
(13) & (10) & (12) & (11) & &
\end{array}$$

Part(c): We use part (b) 2 times and part (a) 3 times.

$$\begin{array}{rcccl}
2h(xy,z) = & -h(x,y)z & +h(y,z)x & -h(z,x)y & \quad \text{Lemma } 2.2(b) \\
& (1) & (2) & (3) &
\end{array}$$

$$\begin{array}{rlr}
-2h(yx,z) = & \underset{(4)}{+h(y,x)z}\underset{(5)}{-h(x,z)y}\underset{(6)}{+h(z,y)x} & \text{Lemma } 2.2(b)\\
0 = & \underset{(1)}{-h(x,y)z}\underset{(4)}{-h(y,x)z} & \text{Lemma } 2.2(a)\\
0 = & \underset{(2)}{-h(y,z)x}\underset{(6)}{-h(z,y)x} & \text{Lemma } 2.2(a)\\
0 = & \underset{(3)}{+h(z,x)y}\underset{(5)}{+h(x,z)y} & \text{Lemma } 2.2(a)
\end{array}$$

Adding these five equalities gives:

$$2h([x,y],z) = -2h(x,y)z.$$

Using 2-torsion free, we get part (c).

The proof of part (d) is from part (a) and part (c).

$$\begin{array}{rl}
+h([x,y],[z,w]) + h([z,w],[x,y]) = 0 & \text{Lemma 2.2 (a)}\\
-h([x,y],[z,w]) = h(x,y)[z,w] & \text{Lemma 2.2 (c)}\\
-h([z,w],[x,y]) = h(z,w)[x,y] & \text{Lemma 2.2 (c)}
\end{array}$$

Adding these three equalities gives:

$$0 = h(x,y)[z,w] + h(z,w)[x,y]. \qquad \square$$

Lemma 2.3. *More properties of h.*

(a) $h(xy,z) = -h(xz,y)$

(b) $h(xy,z) = +h(y,xz)$

(c) $h([x,y]z,w) = 0$.

Proof. Part (a): Uses Lemma 2.2 (b) with $y = z$ once and Lemma 2.2 (a) twice.

$$\begin{array}{rl}
2h(xy,y) = -h(x,y)y + h(y,y)x - h(y,x)y & \text{Lemma 2.2 (b)}\\
0 = +h(x,y)y + h(y,x)y & \text{Lemma 2.2 (a)}\\
0 = -h(y,y)x & \text{Lemma 2.2 (a)}\\
 & \text{and 2-torsion free.}
\end{array}$$

Adding gives:

$$2h(xy,y)=0 \text{ and 2-torsion free gives } h(xy,y)=0.$$

Linearizing this gives Lemma 2.3 (a).

Part (b): Uses Lemma 2.3 (a) and Lemma 2.2 (a)

$$\begin{aligned} h(xy,z) &= -h(xz,y) \text{ Lemma 2.3 (a)} \\ &= +h(y,xz) \text{Lemma 2.2 (a).} \end{aligned}$$

Part (c): Uses Lemma 2.2 (a) and Lemma 2.3 (a)

$$\begin{aligned} h([x,y]z,w) &= h(xyz,w)-h(yxz,w) && \text{definition of } [x,y] \\ &= h(xyz,w)+h(yxw,z) && \text{Lemma 2.3 (a)} \\ &= h(yz,xw)+h(xw,yz) && \text{Lemma 2.3 (b)} \\ &= 0 && \text{Lemma 2.2 (a).} \end{aligned}$$

□

Lemma 2.4. *Further properties of* h.

(a) $h(x,y)[z,w]R=0$

(b) $T^n(h(x,y))[z,w]R=0$

(c) $h(x,y)T^n(h(z,w))\subset T(P)+P$.

Proof. Part (a):

$$\begin{aligned} h(x,y)[z,w]r &= -h([x,y],[z,w])r && \text{Lemma 2.2 (c)} \\ &= +h([[x,y],[z,w]],r) && \text{Lemma 2.2 (c)} \\ &\subset h([R,R]R,R) && \\ &= 0 && \text{Lemma 2.3 (c)} \end{aligned}$$

Part (b): The case $n=0$ is part(a).

Assume the result is true for n.

$$\begin{aligned} &T^{n+1}(h(x,y))[z,w]r= \\ &T(T^n(h(x,y)))[z,w]r= \\ &T(T^n(h(x,y))[z,w]r)-\ h(T^n(h(x,y)),[z,w]r)=0, \end{aligned}$$

by induction and Lemma 2.3 (c) and Lemma 2.2 (a).

Part (c):

$h(x,y)T^n(h(z,w))$

$= -h([x,y],T^n(h(z,w)))$ by Lemma 2.2 (c)

$= +h(T^n(h(z,w)),[x,y])$ by Lemma 2.2 (a)

$= T(T^n(h(z,w))[x,y]) - T^{n+1}(h(z,w))[x,y]$ by definition of h

$\subset T(P) + P$ by Lemma 2.4 (b). □

An extremely important result follows from Lemma 2.4 (b). Suppose we have a product of the form $T^n(h(x,y))z_1z_2z_3\cdots z_nw$. It has a $T^n(h(x,y))$ on the left end. It ends with an element w of R on the right end. The product is independent of the order of the $z_1z_2\cdots z_n$.

$$T^n(h(x,y))z_1z_2\cdots z_nw = T^n(h(x,y))z_{i_1}z_{i_2}z_{i_3}\cdots z_{i_n}w$$

for any permutation $(i_1i_2i_3\cdots i_n)$ of $(123\cdots n)$.

We will eventually prove that, for z_i's of a certain type, sandwiched strings of large enough n are zero. The proof will involve replacing terms of the form

$$\cdots 2T^i(x)T^i(y)\cdots,$$

with

$$\cdots T^i(x+y)T^i(x+y)\cdots - \cdots T^i(x)T^i(x)\cdots - \cdots T^i(y)T^i(y)\cdots.$$

This allows us to reduce the problem to the case where the arguments of the two adjacent elements are equal. This is necessary when we apply Lemma 2.1. We shall refer to the elements which are sandwiched between $T^n(h(x,y))$ on the left and an element of R on the right as a sandwiched string. Remember that the order of the elements of the sandwiched string is immaterial.

Lemma 2.5. *Using the "$\circ$" to denote the "symmetric product" $x \circ y = xy + yx$.*

(a) $(h(R,R)\circ h(R,R))R = 0$,

(b) $h(x,y)h(z,w)h(u,v)R = 0$.

Proof. Part (a): We must show that $h(x,y)h(z,w)u + h(z,w)h(x,y)u = 0$.

$$\begin{aligned} h(x,y)h(z,w)u &= -h([x,y], h(z,w)u) && \text{by Lemma 2.2 (c)} \\ &= +h(u, h(z,w)[x,y]) && \text{by Lemma 2.3 (a)} \\ & && \text{and Lemma 2.2(a)} \\ &= -h(u, h(x,y)[z,w]) && \text{by Lemma 2.2 (d)} \\ &= +h([z,w], h(x,y)u) && \text{by Lemma 2.3 (a)} \\ & && \text{and Lemma 2.2(a)} \\ &= -h(z,w)h(x,y)u && \text{by Lemma 2.2 (c).} \end{aligned}$$

Therefore $(h(x,y) \circ h(z,w))R = 0$.

Part (b):

$$\begin{aligned} h(x,y)[h(z,w), h(u,v)]r = 0 \quad &\text{by Lemma 2.4 (a)} \\ h(x,y)(h(z,w) \circ h(u,v))r = 0 \quad &\text{by Lemma 2.5 (a).} \end{aligned}$$

Adding these gives $2h(x,y)h(z,w)h(u,v)r = 0$. The 2-torsion free assumption says that $h(x,y)h(z,w)h(u,v)r = 0$. This finishes the proof of Lemma 2.5. $\square$

Lemma 2.6. *Let h_1 and h_2 be elements of $H = h(R,R)$. Then $T^n(h_1) \circ T^n(h_2)$ is equal to a linear combination of terms of the form*

$$T^i(H)T^j(H) \quad or \quad T^k(P)$$

where $i + j = 2n$, $i \neq j$ and $0 \leq k \leq 2n+1$. Remember that P is the set of absolute left zero divisors of R. In particular we can replace the expression $T^n(h_1) \circ T^n(h_2)$ which has two terms of exponent n by other terms at least one of which has exponent $< n$. At the same time we may have to add some additional terms involving $T^k(P)$.

Proof. Using Lemma 2.1 with $h \in H$,

$$\sum_{i+j=2n} T^i(h)T^j(h) = \sum_{i+j=2n} T^i(hT^j(h)).$$

Each term on the left hand side involves an i or a j which is less than n except for the term with $i = j = n$. Each term on the right hand side involves $hT^j(h)$. By Lemma 2.4 (c), $hT^j(h) \subset T(P) + P$. Transposing all terms on the left hand side except the one with $i = j = n$ gives

$$T^n(h)T^n(h) \subset -\sum_{\substack{i+j=2n \\ i\neq j}} T^i(h)T^j(h) + \sum_{k=0}^{2n+1} T^k(P).$$

Now applying this three times to the right hand side of

$$\begin{aligned} &T^n(h_1) \circ T^n(h_2) \\ &\qquad = T^n(h_1+h_2)T^n(h_1+h_2) - T^n(h_1)T^n(h_1) - T^n(h_2)T^n(h_2), \end{aligned}$$

we get the result. □

Theorem 2.7. *Any sandwiched product involving at least 2^n terms of the form $T^i(P)$ with $0 \leq i \leq n$ is zero.*

Proof. The proof is done by showing that such a sandwiched product multiplied by an appropriate power of 2 is zero. Then 2-torsion free implies the sandwiched product itself is zero.

Using the commutativity of the sandwiched string, we can collect the 2^n terms of the form $T^i(P)$ with $0 \leq i \leq n$ into at least 2^{n-1} pairs. Either a pair contains two terms of the form $T^n(P)$ or at least one of the pairs has exponent less than n. Using Lemma 2.1, the commutativity within the sandwiched string, and an additional factor of two, we can replace any pair of $T^n(p_1)T^n(p_2)$ with a sum of products of the form $T^i(P)T^j(P)$ where $i < n$. This creates a sandwiched string that has at least 2^{n-1} terms of form $T^i(P)$ with $i \leq n-1$. Continuing in this way, we reduce each string to have at least 2^0 terms of the form $T^0(P)$. Since $T^0(P) = P$, and P is the set of left absolute zero divisors, the product of the sandwiched string together with the end terms is zero. □

Theorem 2.8. *Any product with at least $2 + 3 \times 2^{3n+1}$ terms of the form $T^i(H)$ with $i \leq n$ is zero.*

Proof. The proof is done by showing that such a product multiplied by an appropriate power of 2 is zero. Then 2-torsion free implies the string itself is zero.

A product with at least $2+3\times 2^{3n+1}$ terms of the form $T^i(H)$ with $i\le n$, contains a sandwiched string containing at least $3\times 2^{3n+1}$ terms of the form $T^i(H)$ with $i\le n$. Partition this sandwiched string into $3\times 2^{2n+1}$ substrings each containing at least 2^n terms of the form $T^i(H)$ with $i\le n$. Within each of these substrings, collect 2^n of the terms of the form $T^i(H)$ with $i\le n$ into pairs. If both terms in a pair are of the form $T^n(h_1)T^n(h_2)$, we use Lemma 2.6 to rewrite this product as a sum. There are two types of terms in this sum. One type is $T^i(P)$ for $0\le i\le 2n+1$. The other type is $T^i(H)T^j(H)$ where either i or j is less than n. In this way we replace the original substring by one that has at least one instance of $T^i(P)$ for $0\le i\le 2n+1$, or else there are at least 2^{n-1} terms of the form $T^i(H)$ with $i\le n-1$. We continue in this way by repeatedly pairing and using Lemma 2.6 when necessary. Eventually the substring will either have a term of the form $T^i(P)$ where $0\le i\le 2n+1$, or else it will have a term of the form $T^0(H)=H$.

Now group these substrings into triples. If each member of the triple has an element of the form $T^0(H)$, then the product of that triple will be zero by Lemma 2.5 (b). If not all of the elements of the triple have an element of $T^0(H)$, then among the three elements of that triple exists an element of $T^i(P)$ with $0\le i\le 2n+1$. Since we started with $3\times 2^{2n+1}$ substrings, there were 2^{2n+1} triples. Since we have a sandwiched string with at least 2^{2n+1} terms of the form $T^i(P)$ with $0\le i\le 2n+1$, the product is zero by Theorem 2.7. $\square$

Applications.

Theorem 2.9. *Let T be a (left) Jordan centralizer on a ring R. Let $h(x,y)=T(xy)-T(x)y$ for all x,y in R. Let I be the T-invariant ideal generated by $h(R,R)$. Then I is the union of nilpotent ideals, T is well defined on R/I, and T is a centralizer on R/I.*

Proof. We use the notation $\langle X\rangle$ for the ideal of R generated by the set X. Let $H_0=\langle h(R,R)\rangle$. Inductively, define $H_{n+1}=H_n+\langle T(H_n)\rangle$. It is clear that each H_n is an ideal, and by Theorem 2.8, each H_n is nilpotent of index $\le 2+3\times 2^{3n+1}$. Letting $I=\cup_{i=0}^{\infty}H_i$, I will be an invariant ideal because $T(H_n)\subset H_{n+1}$. I is a radical ring because it is the union of nilpotent ideals. T is well defined on R/I because $T(I) \subset I$. T is a centralizer on R/I because $h(x,y) \subset H_0 \subset I$. $\square$

Remark 2.1. If R is a semiprime ring, then we can easily give a short proof for Zalar's results in [**5**] and also prove that any generalized Jordan derivation is a generalized derivation.

Theorem 2.10. *Let R be a 2-torsion free semiprime ring and $T : R \to R$ a left Jordan centralizer. That is, T is an additive mapping and satisfies $T(x^2) = T(x)x$, for all $x \in R$. Then T is a left centralizer, i.e., $T(xy) = T(x)y$, for all $x, y \in R$.*

Proof. Let $h(x,y) = T(xy) - T(x)y$. It is clear $h(x,x) = 0$. Our goal is to prove that $h(x,y) = 0$, for all $x, y \in R$.

We will show that the ideal generated by $h(R,R)$ is a nilpotent ideal, i.e., we will prove that the ideal

$$\langle h(R,R)\rangle = h(R,R) + Rh(R,R) + h(R,R)R + Rh(R,R)R,$$

is nilpotent. Consider the fourth power of the ideal generated by $h(R,R)$. Lemma 2.5 (b) gives $h(R,R)h(R,R)h(R,R)x = 0$. The fourth factor of the ideal generated by $h(R,R)$ is only used to provide the right hand factor which is called "R" in Lemma 2.5 (b). Using Lemma 2.2 (c), we have $h(x,y)z = -h([x,y],z) \subseteq h(R,R)$, for all $x, y, z \in R$. Therefore $h(R,R)R \subseteq h(R,R)$. That is, $h(R,R)$ acts like a right ideal. Therefore

$$\begin{aligned}\langle h(R,R)\rangle\langle h(R,R)\rangle\langle h(R,R)\rangle\langle h(R,R)\rangle &\\ \subset \cdots h(R,R)h(R,R)h(R,R)\langle h(R,R)\rangle &\\ = 0 \text{ by Lemma 2.5 (b).}&\end{aligned}$$

It follows that $\langle h(R,R)\rangle\langle h(R,R)\rangle$ is an ideal that squares to zero. Applying semiprime once again, we get $\langle h(R,R)\rangle = 0$, which gives $h(R,R) = 0$ and so

$$T(xy) = T(x)y \quad \text{for all } x, y \in R,$$

that is, T is a left centralizer. $\square$

Corollary 2.11. *Let R be a 2-torsion free semiprime ring, and let $G : R \to R$ be a Jordan generalized derivation, i.e., G is an additive mapping satisfying the relation $G(x^2) = G(x)x + xD(x)$, for all $x \in R$ and some derivation D of R. Then G is a generalized derivation, i.e., G satisfies the relation $G(xy) = G(x)y + xD(y)$, for all $x, y \in R$ and some derivation D of R.*

Proof. If $G(x^2) = G(x)x + xD(x)$, where D is a derivation, then $G - D$ is a left Jordan centralizer. So $G - D$ is a left centralizer, by the above theorem, and so G is a generalized derivation.

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