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GOLDIE'S TORSION THEORY AND ITS DERIVED FUNCTOR

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In this paper the global dimension of any complete, well-powered abelian category with injective envelopes in calculated relative to the torsion theory of A. W. Goldie and is found to be always one or zero. The rings R such that the left module category ${}_R\mathscr{M}$ has global dimension zero are precisely those such that every module having zero singular submodule is injective. These rings are characterized as being of the form $T \oplus S$ (ring direct sum) where T is a ring having essential singular ideal and S is semi-simple with minimum condition. The rings with essential singular ideal are precisely those which are torsion as left modules over themselves.

In a recent paper [3] the right derived functors of a torsion subfunctor of the identity were calculated for an abelian category $\mathscr{C}$ sufficiently like the category ${}_R\mathscr{M}$ of left R-modules over a ring R with unit. This leads to a relativized injective dimension for objects of the category for every such torsion subfunctor, and hence to a global dimension for the category which depends on the torsion subfunctor chosen. The torsion sub-functors arise from torsion theories closed under subobjects in the sense of Dickson [2, 3]. In this paper we investigate the global dimension of the category $\mathscr{C}$ relative to a torsion theory introduced by Goldie [7] for modules which grew out of considerations of the singular submodule. This torsion theory enjoys the nice property that the indecomposable injective objects of any sufficiently tame abelian category $\mathscr{C}$ are unmixed, that is, are either torsion or torsion free. This torsion theory also coincides with that considered by Gentile [6] and Jans [8] for ${}_R\mathscr{M}$ whenever the singular ideal of R is zero, and hence coincides with the standard concepts of torsion and torsion free modules over a commutative integral domain.

1. **$\mathscr{G}$-torsion objects in an abelian category.** Throughout this paper, $\mathscr{C}$ shall denote an abelian category which is complete, well-powered, and has for each object A an injective envelope $E(A)$ (see Freyd [5] for this terminology). These conditions on $\mathscr{C}$ are more than sufficient to permit discussion of torsion theories in the sense of [2]. Recall a torsion theory for $\mathscr{C}$ is a pair $(\mathscr{T}, \mathscr{F})$ of classes of objects of $\mathscr{C}$ satisfying the following axioms:

(i) $\mathscr{T} \cap \mathscr{F} = \{0\}$

(ii) $T \to A \to 0$ exact and $T \in \mathscr{T}$ imply $A \in \mathscr{T}$
(iii) $0 \to A \to F$ exact and $F \in \mathscr{F}$ imply $A \in \mathscr{F}$
(iv) For each $X \in \mathscr{C}$ there is an exact sequence

$$0 \to T \to X \to F \to 0$$

with $T \in \mathscr{T}, F \in \mathscr{F}$.
The class $\mathscr{T} \subseteqq \mathscr{C}$ is called a torsion class for $\mathscr{C}$ if there exists a class $\mathscr{F} \subseteqq \mathscr{C}$ such that $(\mathscr{T}, \mathscr{F})$ is a torsion theory. Dually, one defines torsion free class. Torsion classes have been previously called radical classes in other contexts, but we prefer to use the more suggestive terminology when dealing with abelian categories because of the many properties the classes $\mathscr{T}$ and $\mathscr{F}$ have in common with the respective classes of torsion and torsion free abelian groups (see [2] for further analogies and properties of torsion theories). The application of methods of the theory of radicals to the study of torsion theories for abelian categories has been fruitful, however, (see [3] for an account of these methods) and especially so because in abelian categories "normal subobject of" is transitive.

Let $\mathscr{A} \subseteqq \mathscr{C}$ be a class of objects such that $A \to X \to 0$ exact and $A \in \mathscr{A}$ imply $X \in \mathscr{A}$. Our axioms on $\mathscr{C}$ provide complete subobject lattices for each object (see [2], Prop. 1.1) and for any $X \in \mathscr{C}$ we define $\mathscr{A}^1(X) = \bigcup \{A \subseteqq X \mid A \in \mathscr{A}\}$. If γ is a limit ordinal, define $\mathscr{A}^\gamma(X) = \bigcup_{\beta<\gamma} \mathscr{A}^\beta(X)$ and if $\gamma - 1$ exists, $\mathscr{A}^\gamma(X)$ is defined by the equality $\mathscr{A}^\gamma(X)/\mathscr{A}^{\gamma-1}(X) = \mathscr{A}^1(X/\mathscr{A}^{\gamma-1}(X))$. For any object X, define $\mathscr{A}(X)$ to be the first $\mathscr{A}^\gamma(X)$ of this ascending chain such that $\mathscr{A}^\gamma(X) = \mathscr{A}^{\gamma+1}(X)$, which exists since $\mathscr{C}$ is well-powered. The smallest torsion class $\mathscr{T}$ containing $\mathscr{A}$ is then described as $\mathscr{T} = \mathscr{A}(\mathscr{C}) = \{X \in \mathscr{C} \mid \mathscr{A}(X) = X\}$. Here we have used a construction of Amitsur [1] which can be traced back to the Baer construction of the lower nil radical for rings. By the construction of Kurosch [9] one sees also that $\mathscr{A}(\mathscr{C})$ is the class of all objects X of $\mathscr{C}$ such that each nonzero factor object of X has a nonzero subobject from the class $\mathscr{A}$. A further result is that if $\mathscr{A}$ was originally also closed under subobjects, so is the resulting class $\mathscr{A}(\mathscr{C})$, as is shown in [2]. Objects of the class $\mathscr{A}(\mathscr{C})$ will be called the $\mathscr{A}$-torsion objects of $\mathscr{C}$.

We now turn to the Goldie torsion theory. Let $\mathscr{G}$ be the class of all factor objects B/A (together with all their isomorphic copies in $\mathscr{C}$) such that B is an essential extension of the subobject A. Then $\mathscr{G}$ is clearly closed under subobjects and factor objects. Hence the torsion class $\mathscr{G}(\mathscr{C})$ is closed under subobjects. Further, for any object A of $\mathscr{C}$, $\mathscr{G}(A) = \mathscr{G}^2(A)$. To see this, note first that $\mathscr{G}(A)$ is an essential extension of $\mathscr{G}^1(A)$. For if $H \subseteqq \mathscr{G}(A)$, then $\mathscr{G}(H) = H$ and $0 = H \cap \mathscr{G}^1(A) \supseteqq H \cap \mathscr{G}^1(H)$ shows $\mathscr{G}^1(H) = 0$. But then by a transfinite induction, $\mathscr{G}(H) = 0$. Thus we see that $\mathscr{G}(A)/\mathscr{G}^1(A) \in \mathscr{G}$ or $\mathscr{G}(A) = \mathscr{G}^2(A)$. If A is a $\mathscr{G}$-torsion object, then $E(A)$ is $\mathscr{G}$-torsion since $E(A)/A \in \mathscr{G}$ and $\mathscr{G}(\mathscr{C})$ is closed under extensions. It follows that for any injective object Q of $\mathscr{C}$, $Q = \mathscr{G}(Q) \oplus F$ where F is $\mathscr{G}$-torsion free and injective. These facts and others are summarized in the following theorem.

THEOREM 1.1 *Let $\mathscr{G}(\mathscr{C})$ be the smallest torsion class containing the class $\mathscr{G}$ of factor objects B/A by essential subobjects and their isomorphic copies in $\mathscr{C}$. Then*

(a) *For any $A \in \mathscr{C}$, $\mathscr{G}(A) = \mathscr{G}^2(A)$.*

(b) *$\mathscr{G}(\mathscr{C})$ is closed under taking factor objects, infinite direct sums, and extensions.*

(c) *$\mathscr{G}(\mathscr{C})$ is closed under taking subobjects and injective envelopes provided $\mathscr{C}$ has injective envelopes.*

(d) *Any indecomposable injective is either $\mathscr{G}$-torsion or $\mathscr{G}$-torsion free.*

(e) *The corresponding torsion free class $\mathscr{F}$ is closed under taking subobjects, infinite direct products, and extensions.*

(f) *$\mathscr{F}$ is closed under injective envelopes provided $\mathscr{C}$ has injective envelopes.*

(g) *$\mathscr{G}$ is an idempotent, left exact, additive subfunctor of the identity of $\mathscr{C}$ satisfying $\mathscr{G}(A/\mathscr{G}(A)) = 0$ for all $A \in \mathscr{C}$.*

By property (g) of the functor $\mathscr{G}$ there is a connected sequence $\{Gold^n(A)\}_{n=1}^{\infty}$ of right derived functors [10, p. 389] which can be obtained by first taking a resolution of A in injectives, applying $\mathscr{G}$, and then taking homology. These were computed for a general left exact subfunctor of the above type in [3], using an economical resolution by injective envelopes:

$$0 \longrightarrow A \longrightarrow E(A) \longrightarrow E(E^1(A)) \longrightarrow E(E^2(A)) \longrightarrow \cdots$$

where $E^{n+1}(A) = E(E^n(A))/E^n(A)$ for $n = 0, 1, 2, \cdots$, and $E^0(A) = A$. However, since all the injective objects $E(E^n(A))$ in this resolution are $\mathscr{G}$-torsion for $n \geqq 1$, the sequence remains exact after applying $\mathscr{G}$ at the spots $E(E^n(A))$ for $n \geqq 2$. It follows that $\mathrm{Gold}^n(A) = 0$ for $n \geqq 2$, and it is easily checked that $\mathrm{Gold}(A) = \mathscr{G}(E(A)/A)/(\mathscr{G}(E(A)) + A/A)$ (where we have now dropped the superscript). This formula is simplified by the observation that $\mathrm{Gold}(\mathscr{G}(A)) = 0$, so that the short exact sequence

$$\begin{gathered} 0 \longrightarrow \mathscr{G}(A) \longrightarrow A \longrightarrow A/\mathscr{G}(A) \longrightarrow 0 \text{ yields} \\ 0 \longrightarrow \mathrm{Gold}(A) \longrightarrow \mathrm{Gold}(A/\mathscr{G}(A)) \longrightarrow 0 \end{gathered}$$

exact, with the resulting simplification of the expression for $\mathrm{Gold}(A)$

in the following result.

THEOREM 1.2. *For any short exact sequence*

$$0 \to A \to B \to C \to 0$$

there is a corresponding exact sequence

$$0 \to \mathscr{G}(A) \to \mathscr{G}(B) \to \mathscr{G}(C) \to Gold(A) \to Gold(B) \to Gold(C) \to 0$$

where $Gold(A) = E(A/\mathscr{G}(A))/(A/\mathscr{G}(A))$.

COROLLARY 1.3. *The category* $\mathscr{C}$ *has global dimension zero with respect to the subfunctor* $\mathscr{G}$ *if and only if each* $\mathscr{G}$*-torsion free object is injective. In this case we say that* $\mathscr{C}$ *has* $\mathscr{G}$*-global dimension zero.*

Let $\mathscr{T}$ be a torsion class. After Freyd [5] we say that an object L is $\mathscr{T}$-absolutely pure, if L is $\mathscr{T}$-torsion free and whenever $L \subseteqq F$ with F a $\mathscr{T}$-torsion free object, it follows that F/L is $\mathscr{T}$-torsion free. The next result establishes a link between the right derived functor of $\mathscr{G}$ and the $\mathscr{G}$-absolutely pure objects of $\mathscr{C}$.

PROPOSITION 1.4. The following statements are equivalent for $L \in \mathscr{C}$.

(i) L is $\mathscr{G}$-absolutely pure

(ii) L is injective and $\mathscr{G}$-torsion free

(iii) L is $\mathscr{G}$-torsion free and $Gold(L) = 0$.

We first prove a lemma which is valid for any torsion theory $(\mathscr{T}, \mathscr{F})$ for $\mathscr{C}$ such that $\mathscr{T}$ is closed under subobjects. In this situation $\mathscr{F}$ must be closed under taking injective envelopes [2].

LEMMA 1.5. *Let* $(\mathscr{T}, \mathscr{F})$ *be a torsion theory for* $\mathscr{C}$ *such that* $\mathscr{T}$ *is closed under subobjects. Then* $L \in \mathscr{F}$ *is* $\mathscr{T}$*-absolutely pure if and only if* $E(L)/L$ *is* $\mathscr{T}$*-torsion free.*

Proof. If L is $\mathscr{T}$-absolutely pure, then $L \in \mathscr{F}$ so that $E(L) \in \mathscr{F}$, hence $E(L)/L \in \mathscr{F}$. On the other hand, if $L \in \mathscr{F}$ and $E(L)/L \in \mathscr{F}$ let $L \subseteqq F$ for $F \in \mathscr{F}$. Then the sequence

$$0 \to E(L)/L \to E(F)/L \to E(F)/E(L) \to 0$$

is exact with each and in $\mathscr{F}$ so that $E(F)/L \in \mathscr{F}$ since $\mathscr{F}$ is closed under extensions. Thus $F/L \in \mathscr{F}$.

The proof of Proposition 1.4 is now immediate.

2. **$\mathscr{G}$-torsion R-modules.** Let R be a ring with unit. In [7]

Goldie has considered for a left R-module M the submodules

$$Z_1(M) = \{m \in M \mid (0 : m) \text{ is an essential left ideal of } R\}$$
$$Z_2(M) = \{m \in M \mid (Z_1(M) : m) \text{ is an essential left ideal of } R\}.$$

The module $Z_1(M)$ is known as the singular submodule of M. The connection of $Z_2(M)$ with the previous section is expressed in

PROPOSITION 2.1. Let $\mathscr{G}$ be the class of factor modules B/A such that B is an essential extension of the submodule A, along with all isomorphic copies of these. Then $\mathscr{G}^1(M) = Z_1(M)$ and $\mathscr{G}(M) = \mathscr{G}^2(M) = Z_2(M)$ for any left R-module M.

Proof. It suffices to show that $\mathscr{G}^1(M) = Z_1(M)$ and it is clear that $Z_1(M) \subseteqq \mathscr{G}^1(M)$. Now let $H \subseteqq \mathscr{G}^1(M)$ where $H \approx B/A$ for A essential in B. Then for any $x \in H$, $(0 : x)$ is essential in R so that $x \in Z_1(M)$. This concludes the proof.

Gentile [6] has shown that for rings R with zero singular ideal, $Z_1(M) = M$ if and only if $\mathrm{Hom}_R(M, E(R)) = 0$, where $E(R)$ is the injective envelope of R considered as a left R-module. It follows that those modules M with $Z_1(M) = M$ form a torsion class when R has zero singular ideal and hence $Z_1(M) = Z_2(M)$ in this case. This fact has also been pointed out by Goldie [7, Prop. 2.3]. Jans [8] has *investigated the $E(R)$-torsion theory that results for ${}_R\mathscr{M}$ by taking* for the torsion modules those modules having only the zero homomorphism into $E(R)$. In general this torsion does not coincide with Goldie's torsion. To see this consider the ring S of polynomials in the indeterminate x over any field F modulo x^4. Findlay and Lambek [4] have noted that the subring R of S generated by 1, x^2 and x^3 has the R-module S as an essential extension which is not a rational extension, i.e., there is an R-homomorphism $f : S \to S$ such that $f(R) = 0$ but $f \neq 0$. This map f can be extended to $E(R)$ by injectivity, producing a nonzero map $g : E(R)/R \to E(R)$ which shows that $E(R)/R$ is not torsion in the $E(R)$-theory, but clearly is torsion in the Goldie theory.

3. Rings having $\mathscr{G}$-global dimension zero. We say that the ring R has $\mathscr{G}$-global dimension zero if the category ${}_R\mathscr{M}$ does, i.e., if the right derived functor of $\mathscr{G}$ vanishes identically on ${}_R\mathscr{M}$. Our main result in this connection follows:

THEOREM 3.1. *The ring R has $\mathscr{G}$-global dimension zero if and only if $R = T \oplus S$ (ring direct sum) where T is a ring with essential* *singular ideal and S is semi-simple with minimum condition.*

Proof. First assume that each $\mathscr{G}$-torsion free R-module is injective. Let F be $\mathscr{G}$-torsion free and let $s(F)$ be the sum of all simple submodules of F ($s(F) = 0$ if none exist). Then $s(F)$ is $\mathscr{G}$-torsion free, hence injective, so that $F = s(F) \oplus K$, say. If $K \neq 0$ choose $x \neq 0 \in K$. Then Rx has a simple homomorphic image V, but the kernel of this epimorphism is $\mathscr{G}$-torsion free, hence injective, and it splits off, showing that K would have a simple submodule isomorphic to V. Hence $F = s(F)$. Now all simple submodules which are $\mathscr{G}$-torsion free are projective (their order ideals are maximal left ideals of R which are not essential). Hence the $\mathscr{G}$-torsion free modules are projective and completely reducible, and are closed under homomorphic images, by the complete reducibility. Hence the canonical exact sequences $0 \to \mathscr{G}(A) \to A \to A/\mathscr{G}(A) \to 0$ are all split exact. In particular we have $R = \mathscr{G}(R) \oplus S$, with S completely reducible. This is a two-sided decomposition, observing that right multiplications in R are left R-homomorphisms and that the classes of $\mathscr{G}$-torsion modules as well as the $\mathscr{G}$-torsion free modules are each closed under homomorphic images. Thus S is a semi-simple ring with minimum condition and $T = \mathscr{G}(R)$ is a subring of R which is $\mathscr{G}$-torsion as an R-module. To see that T is torsion as a T-module, it suffices to show that for any essential left ideal L of $R, L \cap T$ is an essential left ideal of T. Let L be an essential left ideal of R, and H a left ideal of T with $H \cap (L \cap T) = 0$. But then $H \cap L = 0$ so $H = 0$ as H is also a left ideal of R. Hence T is torsion as a T-module, and therefore has essential singular ideal (see Lemma 3.5 below).

Conversely, assume that $R = T \oplus S$, where T has essential singular ideal and S is semi-simple with minimum condition. Then T and S are two-sided ideals of R, and S is a completely reducible R-module whereas T is a torsion R-module, for if L is an essential left ideal of T then $L \oplus S$ is an essential left ideal of R. We show first that a module A is $\mathscr{G}$-torsion free if and only if $TA = 0$. Let $\mathscr{G}(A) = 0$ and $x \in A$. Then Tx is $\mathscr{G}$-torsion, for given $t \in T$, $(0 : tx) \supseteq S$ and so $R/(0 : tx) \approx Rtx$ is a left R-homomorphic image of $R/S \approx T$. Hence $Tx = 0$ and so $TA = 0$. Now assume $TA = 0$. If $\mathscr{G}(A) \neq 0$, choose $x \neq 0 \in \mathscr{G}(A)$. Since $TRx = 0$, $(0 : x) \supseteq T$. Hence $R/(0 : x) \approx Rx$ is a homomorphic image of S and is therefore isomorphic to a submodule of S by complete reducibility. But then of course $Rx = 0$, a contradiction. Hence $\mathscr{G}(A) = 0$. Now let $\mathscr{G}(A) = 0$. Then $\mathscr{G}(E(A)) = 0$, and the above characterization shows $\mathscr{G}(E(A)/A) = 0$ so that $E(A) = A$. We are through by Corollary 1.3.

COROLLARY 3.2. *If R has the property that all $\mathscr{G}$-torsion free* *modules are injective, then all $\mathscr{G}$-torsion free modules are projective.*

COROLLARY 3.3. *If R has $\mathscr{G}$-global dimension zero, then any R-module M splits as $M \approx \mathscr{G}(M) \oplus M/\mathscr{G}(M)$.*

COROLLARY 3.4. *If R is a (not necessarily commutative) integral domain, then R has $\mathscr{G}$-global dimension zero if and only if R is a division ring.*

REMARK. If R has $\mathscr{G}$-global dimension zero, then the class $\mathscr{F}$ of $\mathscr{G}$-torsion free modules is closed under homomorphic images and hence is a TTF class in the sense of Jans [8]. Applying Theorem 2.4 of [8], we get that $(\mathscr{F}, \mathscr{G})$ is also a torsion theory and thus if R has $\mathscr{G}$-global dimension zero, an arbitrary infinite product of $\mathscr{G}$-torsion modules is $\mathscr{G}$-torsion.

We now consider those rings R which are $\mathscr{G}$-torsion as left R-modules.

LEMMA 3.5. *Let R be a ring with unit. Then $R = Z_2(R)$ if and only if there exists an essential left ideal L of R with $(0:x)$ essential in R for all $x \in L$.*

Proof. If $R = R_t$, then $Z_1(R)$ satisfies the condition.
Conversely, let L satisfy the condition of the lemma. Then $L \subseteq Z_1(R)$ and so $Z_1(R)$ is essential in R. Hence $(Z_1(R):x)$ is essential in R for all $x \in R$ and so $Z_2(R) = R$.

Notation. Let R be a ring with unit and L a left ideal of R. Denote by R_n the ring of all $n \times n$ matrices with entries in R and by L_n the left ideal of R_n consisting of all matrices with entries in L.

LEMMA 3.6. *If L is essential in R, L_n is essential in R_n.*

Proof. Let I be a nonzero left ideal of R_n. Left multiplication by the matrix units e_{ij} of R_n yields a nonzero $n \times n$ matrix A in I with top row $(r_1, r_2, \cdots, r_n)$ and all other entries zero. There is an $r \in R$ such that $rr_1, \cdots, rr_n \in L$ and $rr_k \neq 0$ for some k. Thus $0 \neq re_{11}A \in I \cap L_n$ and so L_n is essential in R_n.

THEOREM 3.7. *If R is $\mathscr{G}$-torsion as an R-module, then R_n is $\mathscr{G}$-torsion as an R_n-module.*

Proof. By Lemma 3.5, there is an essential left ideal L of R with $(0:x)$ essential in R for all $x \in L$. By Lemma 3.6, L_n is essential in R_n. Let $(\alpha_{ij}) \in L_n$. Then $(0:\alpha_{ij})$ is essential in R for all i, j and so $\bigcap_{i,j}(0:\alpha_{ij}) = K$ is essential in R. Now $(0:(\alpha_{ij})) \supseteqq K_n$ and K_n is essential in R_n, by Lemma 3.6, so $(0:(\alpha_{ij}))$ is essential in R_n. The theorem now follows by Lemma 3.5.

EXAMPLE 1. Let Z be the ring of integers and $p \in Z$ prime. Let $R = Z/(p^2)$. The ideal $L = (p)/(p^2)$ is essential in R and $L_x = 0$ for all $x \in L$. Hence by Lemma 3.5, $R = \mathscr{G}(R)$. By applying the previous theorem one gets noncommutative examples of $\mathscr{G}$-torsion rings.

Note also that since $Z_1(R/L) = 0$, the class of $\mathscr{G}$-torsion rings is not closed under ring homomorphisms. Hence there is no hope that taking $Z_2(R)$ for a ring R would in general produce a radical for rings (see [**1**]).

EXAMPLE 2. If R is any quasi-frobenius ring, then R has $\mathscr{G}$-global dimension zero. To see this just note that the projective part of the left socle of R is injective, hence a summand. The complementary summand is a $\mathscr{G}$-torsion ring. It is easily checked that a ring with minimum condition on left ideals is $\mathscr{G}$-torsion if and only if no simple module is projective.

E. P. Armendáriz has kindly pointed out to us that the first half of the proof of Theorem 3.1 is valid for any torsion theory $(\mathscr{T}, \mathscr{F})$ when $\mathscr{F}$ consists entirely of injectives.

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