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ON DIFFERENTIABLY SIMPLE ALGEBRAS

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The main contribution of the present paper is a direct and simple approach to a special case of a recent definitive theorem of Block. Block's theorem settles the problem of determination of differentiably simple algebras. The main result of the present paper relates to an allied part of this determination which answers the question: When is a differentiably semisimple algebra a direct sum of differentiably simple algebras?

Let A be a finite dimensional nonassociative algebra over a field F and D be a set of derivations (linear mappings d of A into A such that $(xy)d = (xd)y + x(yd)$ for all x, y in A) of A. An ideal of A invariant under all the derivations in D is called a *D-ideal*. We shall call the maximal solvable D-ideal R of A the *D-radical* of A (see [14]). For an alternative or Jordan algebra (but not for a Lie algebra), R is also the maximal nilpotent D-ideal (*D-nilradical*). Further, corresponding to any notion of a radical for the algebra A, there exists an associated notion of D-radical, and the latter is, in general, the maximal D-ideal of A contained in the radical. For instance, the D-radical in a power-associative algebra A is its maximal nil D-ideal. The algebra A is said to be *D-semisimple* if its D-radical R is the zero ideal. One can easily see that the algebra A/R is $\bar{D}$-semisimple, where $\bar{D}$ is the set of derivations induced in A/R by D. (This result is not true (see [7, p. 26]) for the D-nilradical of a Lie algebra. The notions of D-nilsemisimplicity and D-semisimplicity are, however, equivalent for a Lie algebra.) By abuse of language, we shall speak of D-ideals of a quotient A/R, or those of a D-ideal B, instead of ideals invariant under the respectively induced derivations. Thus A/R is D-semisimple. A is said to be *D-simple* if $AA \neq 0$ and if it has no proper D-ideals. A D-simple algebra is D-semisimple; a simple (semisimple) algebra is D-simple (D-semisimple). Now, a recent result of Anderson [5] asserts that any decent radical of an algebra over a field of characteristic zero is a D-ideal. Thus, for a wide class of algebras (including power-associative algebras, Lie algebras) over a field of characteristic zero, the notions of semisimplicity and D-semisimplicity are equivalent. Also, the notions of D-simplicity and simplicity are equivalent for any algebra over a field of characteristic zero [12, Lemma 1].

One can define (analogous to [14]) an algebra A to be completely semisimple if it is a direct sum of ideals A_i which are D-simple as algebras. In this case, A_i are even characteristic ideals of A (since

$A_iA_i = A_i$). Further, for any nonzero D-ideal B of A, $BA_i + A_iB$ can be seen to be a D-ideal of A_i; D-simplicity of A_i can be used to deduce easily that B is a direct sum of some of the A_i's; B cannot be solvable. Thus *a completely semisimple algebra is D-semisimple* (a generalization of [14, Th. 3]). The main result of the present paper, from one point of view, partially answers the question as to when a D-semisimple algebra is completely semisimple (converse to the result just stated and not having a positive unconditional answer as is evidenced by the Lie case; see [7, p. 73]).

Another point of view of the main result of this paper is related to a deduction from Lemma 1.2 that, for many classes of algebras (including the power-associative, Lie classes), a semisimple D-simple algebra is simple. In particular, for an algebra of one of these classes, a decomposition of a D-semisimple algebra into D-simple ideals would imply the corresponding decomposition for a semisimple algebra of this class into simple ideals. This latter decomposition is known to hold for flexible strictly power-associative algebras. The main result of the paper asserts the existence of the former decomposition too in this case. We start with the following easily-proved characterisation of the D-radical.

PROPOSITION 1.1. *Let N be the solvable radical (maximal solvable ideal) of an algebra A over a field F. Then the D-radical R of A is precisely the set of elements x in N such that $xD_1D_2 \cdots D_r$ belongs to N for every finite collection $D_1, D_2, \cdots, D_r$ of derivations in D.*

REMARK. Proposition 1.1 holds for the D-radical associated to any notion of radical of an algebra defined via ideals for the reason that what goes into its proof is essentially that the D-radical R is the maximal D-ideal of A contained in the radical.

Let B be a subspace of an algebra A. Define $B^{\langle n\rangle}$ to be sums of all products of elements from A involving n or more elements from B (irrespective of parentheses). Then $B^{\langle n\rangle}$ is an ideal of A. If further B is an ideal of A, $B = B^{\langle 1\rangle} \supseteqq B^{\langle 2\rangle} \supseteqq \cdots \supseteqq B^{\langle n\rangle} \supseteqq \cdots$ and $B^{\langle i\rangle}B^{\langle j\rangle} \subseteqq B^{\langle i+j\rangle}$. The ideal B can be defined to be *strongly nilpotent* if $B^{\langle k\rangle} = 0$ for some k. This notion is in general stronger than (equivalent in the case of associative algebras) the usual notion of nilpotency. For this notion and the following lemma the author is indebted to Professor McCrimmon.

LEMMA 1.2 (cf. [6, Lemma 2.2]). *Any proper ideal of a D-simple algebra is strongly nilpotent.*

The above lemma generalises [14, Th. 2] and its proof is similar to that of this particular case. Further, the maximal strongly nilpotent ideal N exists uniquely for any algebra and in the case of a D-simple nonsimple algebra A, N is the maximal proper ideal of A (by Lemma 1.2). In the latter case, N is also the radical of A for a class of algebras including power-associative and Lie algebras; A/N is simple (see also [6, Lemma 2.2]).

Passing on to power-associative algebras, the D-radical of such an algebra is its maximal nil D-ideal; further, a D-simple algebra is already assumed to be nonnil. Proposition 1.1 holds for this case too. For an algebra A over a field of characteristic $\neq 2$ (an assumption we make throughout this section), A^+ denotes the algebra got by introducing the multiplication $x \circ y = xy + yx/2$ in the vector space A. When A is power-associative, so is also A^+; any derivation of A is also a derivation of A^+. A is said to be flexible if it satisfies the identity $(xy)x = x(yx)$ for all x, y in A. Flexibility of A is equivalent to the condition that the map $x \rightarrow xy - yx \equiv xD_y$ is a derivation of A^+ for fixed y in A (see [13, p. 146]). We shall for convenience denote the collection of derivations $D \cup \{D_y\}_{y \in A}$ of A^+ by $\bar{D}$. Then, any $\bar{D}$-ideal B of A^+ is a D-ideal of A. These observations yield

PROPOSITION 1.3. *If A is a flexible power-associative algebra over a field of characteristic $\neq 2$, $\bar{D}$-radical of $A^+ \subseteqq D$-radical of A. In particular, if A is D-semisimple (D-simple), then A^+ is $\bar{D}$-semisimple ($\bar{D}$-simple).*

The following result is easily deduced using Lemma 1.2.

LEMMA 1.4. *Let A be a D-semisimple flexible power-associative algebra over a field of characteristic $\neq 2$ such that A^+ is D'-simple for some set D' of derivations of A^+. Then A is D-simple.*

We recall that a power-associative algebra A over a field F is said to be strictly power-associative, if for every extension field K of F, A_K (the algebra obtained from A by extending F to K) is power-associative. Let now A be a commutative strictly power-associative algebra over a field of characteristic $\neq 2$. Then, for an idempotent e of A let $A = A_e(0) + A_e(1/2) + A_e(1)$ be the associated Peirce decomposition of A relative to e. For x in A,

$$(*) \qquad x = x_e(0) + x_e(1/2) + x_e(1)$$

with $x_e(\lambda)$ in $A_e(\lambda)$; $x_e(\lambda)e = \lambda x_e(\lambda) (\lambda = 0, 1/2, 1)$. The components $x_e(\lambda)$ are given by (see [2, Chapter I, (23)]): $x_e(0) = x - 3xe + 2(xe)e$; $x_e(1/2) =$

$4xe - 4(xe)e$; and $x_e(1) = 2(xe)e - xe$. For a principal idempotent e of A the elements of $A_e(0) + A_e(1/2)$ are contained in the radical of A [8, Th. 5]. Moreover, we have

PROPOSITION 1.5. *If e is a principal idempotent of a commutative strictly power-associative algebra A over a field of characteristic $\neq 2$, $T \equiv A_e(0) + A_e(1/2)$ is contained in the D-radical R of A.*

Proof. We observe first that (because of $(*)$) an element x of A belongs to the radical N of A if and only if ex belongs to N. For x in $A_e(0)$, $xe = 0$; hence for a derivation D_1 of A, $0 = (xD_1)e + x(eD_1)$; $(xD_1)e = -x(eD_1) \in N$. From what was observed just now $xD_1 \in N$. Let, by induction, $xD'_1D'_2 \cdots D'_{r-1} \in N$ for any $(r-1)$ derivations D'_i of A. Then, for any r derivations $D_1, D_2, \cdots, D_r$ of A,

$$(xD_1D_2 \cdots D_r)e + (xD_1D_2 \cdots D_{r-1})eD_r + \cdots + x(eD_1D_2 \cdots D_r) = 0 .$$

The induction hypothesis and the first observation shows that $xD_1D_2 \cdots D_r \in N$. In other words, $x \in R$, by Proposition 1.1. For y in $A_e(1/2)$, $2ye = y$, and for a derivation D_1 of A, $2(yD_1)e + 2y(eD_1) = yD_1$, i.e., $(yD_1)_e(1) = 2((yD_1)e)e - (yD_1)e = -(2y(eD_1))e \in N$. From the decomposition $(*)$, it follows in particular, that $yD_1 \in N$. An inductive argument similar to the preceding case shows that $y \in R$. Thus $A_e(0)$, $A_e(1/2)$ and hence T are contained in the D-radical R of A.

It follows immediately from Proposition 1.5 that a D-semisimple commutative strictly power-associative algebra A contains an identity element. Further it is known (see [8, p. 371]) that the radical of an ideal B of A is contained in the radical of A. Proposition 1.1 shows immediately that the D-radical of a D-ideal B of A is contained in the D-radical of A. Thus we have

LEMMA 1.6 (cf. [14, Lemma 1]). *A D-ideal of a D-semisimple commutative strictly power-associative algebra over a field of characteristic $\neq 2$ is D-semisimple.*

PROPOSITION 1.7. *A D-semisimple commutative strictly power-associative algebra A over a field of characteristic $\neq 2$ is completely semisimple.*

Proof. If A were itself D-simple, there is nothing to prove. Hence, let B be a proper D-ideal of A. B is D-semisimple (by Lemma 1.6) and contains an identity element e. Then it is easily seen that $A = B \oplus A_e(0)$ with $B = A_e(1)$ and $A_e(1/2) = 0$. $A_e(1)$ and $A_e(0)$ being orthogonal subalgebras of A, $A_e(0)$ is an ideal of A. Further, for a derivation D_1 of A, $eD_1 \in A_e(1/2) = 0$. This means that $A_e(0)$ is also a characteristic ideal of A. Iteration of this procedure starting from B and $A_e(0)$ etc., yields the required decomposition of A into D-simple ideals, in view of the finite dimensionality of A.

Let now A be a flexible strictly power-associative algebra A over a field of characteristic $\neq 2$. For an idempotent e of A, $A = A_e(0) + A_e(1) + A_e(2)$, where $x \in A_e(\lambda)$ if and only if $xe + ex = \lambda x$ ($\lambda = 0, 1, 2$). If e is a principal idempotent of A (also of A^+), then $T = A_e(0) + A_e(1)$ is contained in the D'-radical of A^+, for any collection D' of derivations of A^+. This observation, in conjunction with Proposition 1.3, immediately leads to

PROPOSITION 1.8. *Let A be a flexible strictly power-associative algebra over a field of characteristic $\neq 2$ and e be a principal idempotent of A. Then,* [*the ideal generated by* $T \equiv A_e(0) + A_e(1)$ *in* A^+] $\subseteqq$ [*the ideal generated by T in A*] $\subseteqq$ [*the D-ideal generated by T in A*] $\subseteqq$ [*the $\bar{D}$-ideal generated by T in A^+*] $\subseteqq$ [*$\bar{D}$-radical of A^+*] $\subseteqq$ [*D-radical of A*] $\subseteqq$ [*radical of A*] $\subseteqq$ [*radical of A^+*].

REMARK. The above chain of inclusions incorporates a result of Oehmke [10, Lemma 3.3].

We can now prove the main result of this paper.

THEOREM 1.9. *A D-semisimple flexible strictly power-associative algebra A over a field of characteristic $\neq 2$ contains an identity element. Any such algebra is either D-simple or is a direct sum of ideals which are D-simple as algebras.*

Proof. That A contains an identity element is immediate from Proposition 1.8 and the fact that A is nonnil. A^+ is $\bar{D}$-semisimple (by Proposition 1.3) and is a direct sum of ideals A_i' of A^+ which are $\bar{D}$-simple as algebras (by Proposition 1.7). Now A_i' are characteristic ideals of A^+ (and hence also of A) and are of the form A_i^+ for D-ideals A_i of A. A_i being direct summands of A are themselves D-semisimple as flexible algebras. An appeal to Lemma 1.4 shows that A_i are D-simple, thus completing the proof of the theorem.

REMARKS. (i) Theorem 1.9 is proved by Block differently as a deduction from a more general result (see [6, Th. 8.2 and Corollary 8.4]). His Theorem 8.2 for algebras can be described as a decomposition theorem for decomposition of D-semisimple algebras into D-simple components, his notion of D-radical being the one associated (see the beginning of this section) to the general notion of radical of an algebra due to Albert [1].

(ii) The remarks preceding Proposition 1.1 show that Theorem 1.9 generalises Oehmke's results [10, Lemma 3.4, Th. 3.5] in particular including the characteristic 3 case (of the base field) not considered by him.

(iii) One can see directly from Theorem 1.9 that Lemma 1.6 holds for the case of a flexible algebra. Strict power-associativity has been used in our arguments only in the cases of characteristic 3 or 5 of the base field. Moreover, we see that using the two-sided Peirce decomposition, *key results of this section hold for alternative algebras over a field of arbitrary characteristic.*

COROLLARY 1.10. *The radical (maximal nilideal) of a flexible power-associative algebra over a field of characteristic zero is a D-ideal of A for any collection of derivations D of A.*

The above corollary can be easily deduced from Theorem 1.9 by noting that A/R is a direct sum of simple ideals (for the D-radical R of A) using a result of Sagle and Winter [12, Lemma 1].

REMARKS. (i) Anderson [5, Th. 2.2] has proved Corollary 1.10 more generally for the hereditary radical of an algebra.

(ii) The method of proof of Corollary 1.10 sketched above can be employed to deduce from Block's more general decomposition theorem ([6, Th. 8.2]) the result: *The radical of an algebra* (*in the sense of Albert* [1], *if it exists*; see also Remark (i) following Theorem 1.9) *is always a characteristic ideal, when the base field is of characteristic zero.*

(iii) A suggestion made to the author by Professor McCrimmon can be amplified further to prove that a D-simple algebra A with identity is simple, also when the characteristic of the base field F is greater than the dimension (over F) of the algebra A. In other words, for a D-simple nonsimple algebra A with identity over a field F of characteristic p, $p \leqq n =$ the dimension of A. The proof suggested by him uses the maximal strongly nilpotent ideal N of A (which exists, and is the maximal proper ideal of A), and concludes that $N' = 0$, when $p > n$. For brevity, we omit the details of this proof and note that this assertion also follows (without the assumption of existence of the identity) directly from the Main Theorem of Block [6]. The assertion in particular implies that Corollary 1.10 holds also for algebras over a field of characteristic p with $p > \dim A$.

We now briefly consider one class of power-associative algebras, viz. trace-admissible algebras. Let A be a trace-admissible algebra over a field F with the bilinear trace form f. (See [13, p. 136] for definition etc.) The radical (maximal nilideal) of A is then known to be the set $\{x \in A \mid f(x, y) = 0 \text{ for all } y \text{ in } A\}$. A subalgebra of a trace-admissible algebra is trace-admissible. Proposition 1.1 can be used to deduce from a known result [2, Chapter II, Th. 2] that a D-ideal of a D-semisimple trace-admissible algebra is again D-semisimple. We also have

PROPOSITION 1.11. *A D-semisimple trace-admissible algebra A over a field F of characteristic* $\neq 2$ *is flexible. If further the characteristic of F is* $\neq 5$, *A is also a noncommutative Jordan algebra.*

Proof. It is known (see the proof of [13, Th. 5.4]) that $(x, y, x) \equiv (xy)x - x(yx)$ belongs to the radical N of A for all x, y in A. By linearisation $(x, y, z) + (z, y, x)$ belongs to N for all x, y, z in A. Now, the subspace B spanned by all associators (x, y, x) and $\{(x, y, z) + (z, y, x)\}$ is a characteristic subspace of A contained in the radical N of A; the ideal generated by B in A is a D-ideal contained in N (and thus is zero). In other words, A is flexible. The second part is similarly proved from the known fact that $(x^2, y, x)^\circ$ (associator in A^+) belongs to the radical of A^+.

Since a noncommutative Jordan algebra is strictly power-associative [13, p. 141], Proposition 1.11 along with Theorem 1.9 gives

PROPOSITION 1.12. *A D-semisimple trace-admissible algebra over a field of characteristic prime to* 10 *is completely semisimple.*

REMARKS. (i) Block has deduced Propositions 1.11 and 1.12 from his Main Theorem and his Theorem 8.2. His deduction covers the characteristic 5 case also of Proposition 1.12.

(ii) Schafer's definition of trace-admissibility coincides with that of Albert [3] when the algebra contains an identity, and we note that the above results for trace could have been deduced also from Albert's [3] as in the flexible case dealt with earlier.

(iii) The center of a D-semisimple (D-simple) trace-admissible algebra can be easily seen to be again D-semisimple (D-simple). Further the above results for trace hold good also for the variant notion of trace due to Albert [4].

2. This section is devoted to a brief consideration of certain

individual classes of algebras relating to the notions of D-simplicity etc..

Let A be an alternative algebra with identity 1, and $1 = e_1 + e_2 + \cdots + e_r$ be a decomposition of 1 into pairwise orthogonal primitive idempotents. Then we have

PROPOSITION 2.1 (cf. [13, Lemma 3.15]). *A D-simple alternative algebra of degree $r \geqq 3$ is associative.*

Proposition 2.1 can be proved either by using the Main Theorem of Block [6] along with [13, Lemma 3.15] or directly from the latter as follows: when A is D-simple alternative, and N is the radical of A, A/N is simple (see the remarks following Lemma 1.2). When $r \geqq 3$ or $r = 1$, A/N is associative, i.e., all associators of A belong to N. The associator ideal of A is a D-ideal of A contained in N, implying (in view of D-simplicity of A) that A is associative.

PROPOSITION 2.2. *If A is a D-simple power-associative algebra over a field F of the form $F1 + N$ (1, the identity of A and N, an ideal of A), then N is the radical of A. A is commutative and associative.*

The above proposition is easily proved by noting that $(x, y, z) \equiv (xy)z - x(yz)$ and $(x, y) \equiv xy - yx$ belong to the radical N (see Lemma 1.2) and in fact to the D-radical of A (using Proposition 1.1). Evidently, this suggests a simple proof of the result of Kokoris (see [13, pp. 144–145]): If A is a nodal noncommutative Jordan algebra over a field of characteristic $\neq 2$, A^+ is associative. In fact, [13, Th. 5.6] *holds verbatim for D-simple algebras, with the same conclusion.*

We now briefly sketch the invariance properties of D-simplicity of an algebra under scalar extensions. Let A be a D-simple algebra over a field F. A is said to be *normal D-simple* over F, if the algebra A_k (obtained by extending the base field F to k) is D_k-simple ($D_k =$ the set of all derivations extended to A_k from D of A) for every extension field k of F. The associative subalgebra (not necessarily containing the identity) $M(A)$ of linear transformations on A generated by the left, right multiplications in A and the derivations in D can be called the *D-multiplication algebra* of A (see [13, p. 14]), and the centralizer $C(A)$ of $M(A)$ in the algebra of all linear transformations of A, the *D-centroid* of A. The D-centroid is precisely the set of those elements of the usual centroid of A that commute with the derivations in D; when A is D-simple, $C(A)$ is a field and A, regarded as an algebra over $C(A)$, is normal D-simple. We define the

D-center of an algebra A to be the set of elements x in the center of A such that $xD_t = 0$ for every derivation D_t in D and note that the usual centroid-center relations hold in the present case too, the details being omitted for brevity.

Now, let A be a normal D-simple trace-admissible algebra (in the variant sense of Albert [4]) of degree one over a field F. (i.e., 1 is the only idempotent of A_k, k being the algebraic closure of F.) A_k is trace-admissible (in this sense) so that A_k is of the form $k1 + N$ for the radical N of A_k. Since A_k is D_k-simple, it is commutative and associative (by Proposition 2.2). Thus we have

PROPOSITION 2.3. *A normal D-simple trace-admissible algebra (in the sense of* [4]) *of degree one is commutative and associative.*

3. In this section we briefly record the study of Lie triple systems in the light of the notions of D-simplicity etc., again omitting the details.

Let T be a Lie triple system (L.t.s.) over a field F with the trilinear composition $[x, y, z]$ (we refer to [9] for details regarding the L.t.s.). The concepts of D-radical, D-semisimplicity, D-simplicity and complete semisimplicity do make sense for T. Proposition 1.1 is true; *any proper ideal of a D-simple L.t.s. is solvable* (cf. Lemma 1.2). Over a field of characteristic zero, the radical of an L.t.s. is a D-ideal [9, Lemma 5] so that D-semisimplicity and semisimplicity are equivalent concepts in this case; D-simplicity and simplicity are also equivalent (in view of the validity of Lemma 1.2 for L.t.s.). However, over a field of characteristic $p \neq 0$, these notions are distinct (as can be seen from the example of the L.t.s. associated to the Lie algebra L considered by Seligman [14, p. 164]). Block has noted in a postscript to his paper [6] that his Main Theorem remains valid for L.t.s. too. However, the L.t.s. identities do not by themselves seem to suffice for a study of D-semisimple systems (as in the Lie case). Consequently we consider the Lie triple system T_A that is associated to a Malcev algebra A (see [9] and [11] for relevant definitions etc.). Such an L.t.s. has the properties: (i) The left multiplication L_x in A is a derivation of T_A. (ii) Any derivation of A is a derivation of T_A; any $D \cup \{L_x\}_{x \in A} \equiv \bar{D} -$ ideal of T_A is a D-ideal of A. These and other properties of such systems (see [9, Lemma 2 and Satz 2] for instance) can be used to prove the following results.

PROPOSITION 3.1. *Let A be a Malcev algebra over a field F of characteristic* $\neq 2, 3$. (*This assumption on F is made throughout this section.*) *If A is D-semisimple* (*D-simple*), *T_A is a $\bar{D}$-semisimple* ($\bar{D}$-*simple*) *L.t.s.. If A is D-semisimple and T_A is D'-simple* (*for* *some D'), then A itself is D-simple as an algebra.*

PROPOSITION 3.2. *If A is a D-semisimple Malcev algebra, any $\bar{D}$-ideal of T_A is $\bar{D}$-semisimple as L.t.s.. A minimal proper $\bar{D}$-ideal of T_A is $\bar{D}$-simple. Sum of any two completely semisimple ideals of T_A* (cf. [14, Lemma 2]) *is a completely semisimple $\bar{D}$-ideal of T_A. A minimal D-ideal of A is a minimal $\bar{D}$-ideal of T_A and is hence $\bar{D}$-simple as L.t.s..*

Combining some of these results, we have

PROPOSITION 3.3. *If A is a finite dimensional D-semisimple Malcev algebra, then A has finitely many minimal D-ideals T_i and their sum T is direct. T_i are $\bar{D}$-simple as L.t.s.*

REMARK. Proposition 3.3 can be essentially described as analogue of Lemma 9.1 of [6]. Further, in this case the set $\{x \in T_A \mid [T, T, x] = 0\}$ is the zero ideal. The preceding results do not seem to be direct consequences of Block's.

We conclude this section by observing that most of the results of [11, §5] for Malcev algebras remain valid for the present situation, with D-ideal, D-simplicity, complete semisimplicity respectively replacing ideal, simplicity, semisimplicity, of course with suitable modifications. We omit the details and proofs in view of the triviality of the adaptation. Among these modified results is the following important one: *If A is a completely semisimple Malcev algebra over a field of characteristic $\neq 2$, 3, the center of the Lie algebra L generated by the multiplications in A and the derivations in D is the zero ideal.* We also note that these modifications are treated independently of the results of Block—a use of which does not seem to be advantageous in these cases.

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