ERRATA

Addendum to

SEQUENCES OF HOMEOMORPHISM WHICH CONVERGE TO HOMEOMORPHISMS

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We observe that Theorem 1 remains valid when we omit the relative compactness requirement on the U_i, and add the hypothesis that M be complete. The proof of the alternative theorem is the same as that of Theorem 1, except that some conclusions which were previously based on relative compactness arguments now follow from completeness.

The alternative Theorem 1 can be used to extend certain results of the paper to the Hilbert space l_2. In what follows, M, N shall be locally convex topological (real) vector spaces, and U a locally convex subset of M having no isolated points. Given a continuous map $f: U \rightarrow N$, a point $x \in U$ is called a *spiral point of* f if given *any* hyperplane (=translate of a codimension one linear subspace) Π containing $f(x)$, and *any* point $y \neq x$ in U, then $f([x, y]) \cap \Pi$ is an infinite set, where $[x, y]$ denotes the (closed) line segment joining x and y.

PROPOSITION. *Given a continuous map $f: U \rightarrow N$, with U, N as above, the set of nonspiral points of f is dense in U.*

Proof. Let x and y be any two points such that $[x, y] \subset U$. To prove the proposition, it suffices to show that there is a point z in the open segment (x, y) such that z is *not* a spiral point of f. Note that if f is constant on $[x, y]$, then $[x, y]$ consists entirely of nonspiral points of f. Hence, we can assume that there is a point $w \in (x, y)$ such that $f(x) \neq f(w)$. Let Π be a hyperplane in N which separates $f(x)$ and $f(w)$. Then there is a (unique) point $z \in (x, w)$ such that $f([x, z]) \cap \Pi = f(z)$, so that z is not a spiral point of f, and the proposition is proved.

In spite of the existence of a dense set of nonspiral points for any continuous map $f: U \rightarrow N$, when $M = N =$ the Hilbert space l_2, we have the following

THEOREM. *Let U be a locally convex subset, having no isolated points, of the Hilbert space l_2. Then those topological imbeddings of U into l_2 which have a dense set (in U) of spiral points form* *a dense subset, in the fine C^0 topology, of the set of topological imbeddings of U into l_2.*

The proof of this theorem, which requires the alternative form of Theorem 1, is similar to the proof of Theorem 2 and is therefore omitted. The principal modification needed consists in allowing the maps $F_{c,r,i,j,m}$, (which are now defined on l_2 in the obvious way using (9)-(9)'''), to act now on the *left* of the imbeddings via a suitably defined infinite left composition, and where the positive integer j is not subject to the condition $j \leqq n$ of Theorem 2.

Correction to

DIMENSION THEORY IN POWER SERIES RINGS

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While recently answering a letter of inquiry of T. Wilhelm, I discovered an error in Corollary 4.6. The result, as originally stated, clearly requires that $P\cdot V[[X]] \subset P[[X]]$. However, if P is not branched, it is possible that $P\cdot V[[X]] = P[[X]]$; a counterexample can be obtained from Proposition A.

The following modification of Corollary 4.6 is sufficient for the proof of Theorem 4.7.

COROLLARY 4.6′. *Let V be a valuation ring having a proper prime ideal P which is branched. If P is idempotent, then there is a prime ideal Q of $V[[X]]$ which satisfies $P\cdot V[[X]] \subsetneqq Q \subset P[[X]]$.*

Proof. Since P is branched, there is a prime ideal $\bar{P}$ of V with $\bar{P} \subset P$ and such that there are no prime ideals of V properly between $\bar{P}$ and P [1; 173]. By passing to $V[[X]]/\bar{P}[[X]]$ $(\cong (V/\bar{P})[[X]])$, we may assume that P is a minimal prime ideal of V.

Since P is idempotent, PV_P is idempotent by Lemma 4.1; hence V_P is a rank one nondiscrete valuation ring. By Theorem 3.4, there is a prime ideal Q of $V_P[[X]]$ such that $(PV_P)\cdot V_P[[X]] \subsetneqq Q \subset (PV_P)[[X]]$. But then we see that $Q \subset (PV_P)[[X]] = P[[X]] \subsetneqq V[[X]]$. Hence $Q \cap V[[X]] = Q$ and Q is a prime ideal of $V[[X]]$ with $P\cdot V[[X]] \subsetneqq Q \subset P[[X]]$.

The following result is now of interest.