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# THE BRAUER GROUP OF POLYNOMIAL RINGS

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**Let $R$ be a commutative ring and $S$ a commutative $R$-algebra. The induced homomorphism $B(R) \rightarrow B(S)$ of Brauer groups is studied for the following choices of $S$. First, $S = R/I$ where $I$ is an ideal in the radical of $R$. Second, $S = R[x]$ the ring of polynomials in one variable over $R$. Third, $S = K$ the quotient field of $R$ when $R$ is a domain.**

In [3] M. Auslander and O. Goldman introduced the Brauer group $B(R)$ of a commutative ring $R$. If $S$ is a commutative $R$-algebra there is a homomorphism $B(R) \rightarrow B(S)$ induced by the homomorphism from $R$ to $S$. Some of the choices for $S$ considered in [3] are $S = R/I$ for an ideal $I$ of $R$, or $S = K$ the quotient field of $R$ when $R$ is a domain, or $S = R[x]$ the ring of polynomials in one variable over $R$.

We observe here relationships between the homomorphisms of Brauer groups induced from these choices for $S$. We show that if $I$ is an ideal in the radical of $R$ and $R$ is complete in its $I$-adic topology then $B(R) \cong B(R/I)$. This answers a question raised in [11]. If $I$ is a nil ideal in $R$ then $B(R) \cong B(R/I)$. If $R[[x]]$ is the ring of formal power series over $R$ then $B(R[[x]]) \cong B(R)$. If we assume $R$ is a domain with quotient field $K$ an algebraic number field and $t_1, \cdots, t_n$ are indeterminates the homomorphism $B(R[t_1, \cdots, t_n]) \rightarrow B(K(t_1, \cdots, t_n))$ is a monomorphism where $K(t_1, \cdots, t_n)$ is the function field in $n$-variables over $K$. Let $B'(R[x])$ be the kernel of the natural homomorphism $B(R[x]) \rightarrow B(R)$ where $x$ is an indeterminate. If $R$ is a domain there is a procedure given in [13] for calculating $B'(R[x])$ in terms of $B'(\bar{R}[x])$ where $\bar{R}$ is the integral closure of $R$. In [3] it is shown that $B'(R[x]) = 0$ if $R$ is a regular domain of characteristic $= 0$. We fill in the gap between these two results in the Noetherian case.

If $R$ is an integrally closed Noetherian domain, let $\mathrm{Ref}\,(R)$ denote the isomorphism classes of finitely generated reflexive $R$-modules $M$ with $\mathrm{End}_R\,(M)$ projective over $R$ and let $\mathrm{Pro}\,(R)$ be the projective elements in $\mathrm{Ref}\,(R)$. Under the multiplication $|M|\cdot|N| = |(M \otimes N)^{**}|$ $\mathrm{Ref}\,(R)$ is a monoid, $\mathrm{Pro}\,(R)$ is a submonoid and $\mathrm{Ref}\,(R)/\mathrm{Pro}\,(R)$ is a group (see [6]). There is a split exact sequence.

$0 \rightarrow \mathrm{Ref}'\,(R[x]) \rightarrow \mathrm{Ref}\,(R[x])/\mathrm{Pro}\,(R[x]) \rightarrow \mathrm{Ref}\,(R)/\mathrm{Pro}\,(R) \rightarrow 0$ where $\mathrm{Ref}'\,(R[x]) = \mathrm{Ref}\,(R[x])/(\mathrm{Pro}\,(R[x]) + \mathrm{Ref}\,(R))$. Utilizing results in [1] we show that the sequence.

$0 \rightarrow \mathrm{Ref}'\,(R[x]) \rightarrow B'(R[x]) \rightarrow B'(K[x])$ is exact. If $R$ is any von

Neumann regular ring then $B'(R[x])$ is trivial if and only if $R/m$ is a perfect field for each maximal ideal $m$ of $R$. If $R$ is a Boolean ring, $B(R[x]) = (0)$.

We adapt an example from [14] to obtain a Noetherian domain $R$ with quotient field $K$ such that $B(R) \to B(K)$ is monomorphism but $B(R[x]) \to B(K(x))$ has an infinite kernel. We also give some examples of domains $R$ containing a prime ideal $I$ such that the homomorphism $B(R) \to B(R/I)$ is not onto.

Throughout $R$ denotes a commutative ring and unless otherwise specified $\otimes$ means $\otimes_R$. An Azumaya algebra is called central separable in [3] and [10].

Otherwise all our undefined terminology, conventions, and notations are as in [10]. This paper was written while the author visited at the Forschungsinstitut für Mathematik in Zürich. The author thanks the people of the Institute, and especially Professor M. Knus for a sympathetic ear.

## 1.

LEMMA 1. *Let $I$ be an ideal in the radical of a commutative ring $R$. If for any Azumaya $R$-algebra $A$ idempotents can be lifted from $A/IA$ to $A$ then the induced homomorphism $B(R) \to B(R/I)$ is a monomorphism.*

In [9] we showed that if $I$ is nilpotent then $B(R)$ is isomorphic to $B(R/I)$. The proof there that $B(R) \to B(R/I)$ is a monomorphism also proves Lemma 1. See also [16].

THEOREM 2. *Let $I$ be an ideal in the radical of a commutative ring $R$. If $R$ is complete in its $I$-adic topology then the homomorphism $B(R) \to B(R/I)$ is an isomorphism.*

*Proof.* Let $A$ be a Azumaya $R$-algebra. Since $R$ is $I$-adically complete and $A$ is finitely generated as an $R$-module, we have $A$ is complete with respect to the ideal $IA$. Thus idempotents can be lifted from $A/IA$ to $A$ so by Lemma 1 $B(R) \to B(R/I)$ is a monomorphism.

Next let $A_0$ be a Azumaya $R/I$-algebra. We showed in [9] in proving that $B(R) \to B(R/I)$ is onto when $I$ is nilpotent that one can construct a sequence $A_i$ of Azumaya algebras over $R/I^{2^i}$ with $A_i/I^{2^{i-1}}A_i \cong A_{i-1}$ by a natural homomorphism $\varphi_i$. We let $A = \varprojlim (A_i)$. Then $A$ is an $R$-algebra with $A/I^{2^i} \cong A_i$. If $a \in A$ we let the natural image of $a$ in $A_i$ be $a^i$. Let $a_1, \cdots, a_n$ be elements of $A$ with $a_1^0, \cdots, a_n^0$ generating $A/I = A_0$ as an $R$-module. By Nakayama's lemma (Prop.

2.2 p. 85 [5]) the elements $a_1^i, \cdots, a_n^i$ generate $A_i$ as an $R$-module for each $i$. Let $a \in A$. Then

$$a^0 = \sum_{j=1}^{n} \alpha_{j,0} a_j^0 \qquad \alpha_{j,0} \in R .$$

Thus

$$a^1 = \sum_{j=1}^{n} \alpha_{j,0} a_j^1 + x^1 \quad \text{where} \quad x^1 \in I(A/I^2) ,$$

we can write

$$x^1 = \sum_{j=1}^{n} \gamma_{j,0} a_j^1 \quad \text{where} \quad \gamma_{j,0} \in I .$$

Thus

$$a^1 = \sum_{j=1}^{n} (\alpha_{j,0} + \gamma_{j,0}) a_j^1 .$$

Let $\alpha_{j,1} = \alpha_{j,0} + \gamma_{j,0}$, then $a^1 = \sum_{1=1}^{n} \alpha_{j,1} a_j^{1}$ and $\alpha_{j,1} - \alpha_{j,0} \in I$. Continuing inductively we find

$$a^i = \sum_{j=1}^{n} \alpha_{j,i} a_j^i \quad \text{with} \quad \alpha_{j,i} \in R,\ \alpha_{j,i} - \alpha_{j,i-1} \in I^{2^{i-1}} .$$

Let $\alpha_j = \lim_{i \to \infty} \alpha_{j,i}$ and let $b = \sum_{j=1}^{n} \alpha_j a_j$. For any $i$, $a^i = b^i$ so $b = a$ and $A$ is finitely generated as an $R$-module.

For any maximal ideal $m$ of $R$, $R/m \otimes A = R/m \otimes R/I \otimes A = R/m \otimes A_0$. Thus $R/m \otimes A$ is separable over $R/m$. By Theorem 2.7.1. of [10], $A$ is separable over $R$. Let $Z(A)$ be the center of $A$, then $Z(A)$ is a direct summand of $A$ so $Z(A)$ is finitely generated over $R$. Moreover, $0 \to R/m \otimes Z(A) \to R/m \otimes A$ is exact so for every maximal ideal $m$ of $R$, $R/m \otimes Z(A)/R = 0$. Thus $Z(A) = R$ and $A$ is an Azumaya $R$-algebra completing the proof.

COROLLARY 3. *Let $N$ be a nil ideal in the commutative ring $R$, then $B(R) \cong B(R/N)$.*

*Proof.* The hypothesis of Lemma 1 are satisfied since idempotents can be lifted modulo a nil ideal so the map from $B(R)$ to $B(R/N)$ is one-to-one.

Let $\bar{A}$ be an Azumaya algebra over $RN = \bar{R}$. There is a finitely generated subring $\bar{S}$ of $\bar{R}$ and an Azumaya $\bar{S}$-algebra $\bar{B}$ with $\bar{B} \otimes_{\bar{S}} \bar{R} = \bar{A}$. Let $\bar{s}_1, \cdots, \bar{s}_m$ be the generators of $\bar{S}$ over its prime ring and let $s_1, \cdots, s_m$ be corresponding elements in $R$. Let $S$ be the subring of $R$ generated by $s_1, \cdots, s_m$. Now $N$ consists of nilpotent elements in $R$ so $N \cap S$ is a nil-ideal in $S$ and $S/N \cap S \cong \bar{S}$. Since $S$ is Noetherian, $N \cap S$ is nilpotent. As we saw in the proof of Theorem 2 there is an Azumaya $S$-algebra $B$ with $\bar{S} \otimes_S B = \bar{B}$. Let $A = R \otimes_S B$, then

$$\bar{R} \otimes A \cong \bar{R} \otimes R \otimes_S B \cong \bar{R} \otimes_S B \cong \bar{R} \otimes_{\bar{S}} \bar{S} \otimes_S B \cong \bar{R} \otimes_{\bar{S}} \bar{B} \cong \bar{A}$$

thus the homomorphism from $B(R)$ to $B(R/N)$ is onto.

COROLLARY 4. *Let $R[[x]]$ be the ring of formal power series in $x$ over $R$, then $B(R) \cong B(R[[x]])$.*

*Proof.* The ideal $(x)$ generated by $x$ is in the radical of $R[[x]]$ and $R[[x]]$ is complete in the topology induced by this ideal. Thus by Theorem 2, $B(R) \cong B(R[[x]])$.

LEMMA 5. *Let $R$ be a commutative ring and $S$ a commutative $R$-algebra. Then the sequence*

$$0 \longrightarrow B(S[x]/R[x])/B(S/R) \longrightarrow B'(R[x]) \longrightarrow B'(S[x])$$

*is exact and $B(S[x]/R[x]) \cong B(S/R) \oplus L$ for an abelian group $L$.*

*Proof.* Consider the diagram of Brauer groups where the maps are all natural.

$$\begin{tikzcd}
 & 0 & 0 & 0 \\
0 \arrow[r] & B(S/R) \arrow[u] \arrow[r] & B(R) \arrow[u] \arrow[r] & B(S) \arrow[u] \\
0 \arrow[r] & B(S[x]/R[x]) \arrow[u] \arrow[r] & B(R[x]) \arrow[u] \arrow[r] & B(S[x]) \arrow[u] \\
0 \arrow[r] & L \arrow[u] \arrow[r] & B'(R[x]) \arrow[u] \arrow[r] & B'(S[x]) \arrow[u] \\
 & 0 \arrow[u] & 0 \arrow[u] & 0 \arrow[u]
\end{tikzcd}$$

The first two horizontal rows are exact by definition and the last two vertical rows are split exact. The exactness of the third horizontal row follows as does the split exactness of the first vertical row. This gives the lemma.

Let $R$ be a domain with quotient field $K$. Let $K(x)$ be the field of fractions of $K[x]$. If $K$ is perfect then $L$ in Lemma 5 is $B'(R[x])$

since $B'(K[x]) = 0$ by Theorem 7.5 of [3]. Thus $B(K[x]/R[x]) \cong B(K/R) \oplus B'(R[x])$. Later we give an example where $B'(R[x])$ is infinite and $B(K/R) = 0$. If $K$ is any field then $0 \rightarrow B(K[x]) \rightarrow B(K(x))$ is exact so $B(K(x)/R[x]) \cong B(K[x]/R[x])$.

THEOREM 6. *Let $R$ be a domain whose quotient field $K$ is an algebraic number field. Then* $0 \rightarrow B(R[x_1, \cdots, x_n]) \rightarrow B(K(x_1, \cdots, x_n))$ *is exact.*

*Proof.* The case $n = 0$ is in [7]. By Corollary 3.9 of [14] $B'(R[x_1, \cdots, x_{n-1}][x_n]) = 0$. Now $K(x_1, \cdots, x_n)$ is a perfect field thus $B(K(x_1, \cdots, x_n)/R[x_1, \cdots, x_n]) = B(K(x_1, \cdots, x_{n-1})/R[x_1, \cdots, x_{n-1}])$ the proof is complete by induction.

Next let $R$ be a Noetherian integrally closed domain. Let Ref $(R)$ denote the isomorphism classes of finitely generated reflexive $R$-modules $M$ with $\mathrm{End}_R(M)$ projective. Multiply two such classes by $|M| \cdot |N| = |(M \otimes N)^{**}|$ where $M^* = \mathrm{Hom}_R(M, R)$. Then Ref $(R)$ is a commutative monoid. Let Pro $(R)$ be the submonoid of those $M$ which are projective. Then Ref $(R)$/Pro $(R)$ is a group (see [6]).

The natural homomorphism from $R[x]$ onto $R$ and its splitting map from $R$ to $R[x]$ induces a split exact sequence

$$0 \longrightarrow \mathrm{Ref}\,(R[x])/(\mathrm{Ref}\,(R) + \mathrm{Pro}\,(R[x])) \longrightarrow \mathrm{Ref}\,(R[x])/\mathrm{Pro}\,(R[x]) \longrightarrow \mathrm{Ref}\,(R)/\mathrm{Pro}\,(R) \rightarrow 0\,.$$

Let $\mathrm{Ref}'\,(R[x]) = \mathrm{Ref}\,(R[x]/(\mathrm{Ref}\,(R) + \mathrm{Pro}\,(R[x]))$.

THEOREM 7. *Let $R$ be a Noetherian integrally closed domain with quotient field $K$. Then the sequence*

$$0 \longrightarrow \mathrm{Ref}'\,(R[x]) \longrightarrow B'(R[x]) \longrightarrow B'(K[x])$$

*is exact. If $K$ is perfect then* $\mathrm{Ref}'\,(R[x]) \cong B'(R[x])$.

*Proof.* Let Cl $(R)$ be the elements of Ref $(R)$ of rank $= 1$ and let Pic $(R)$ be the subgroup of projective modules in Cl $(R)$. Consider the next diagram. By a theorem of $B$. Auslander (Theorem 2 in [8]) the first horizontal row is exact. By 7.19 p. 147 of [5] the first vertical row is exact. The last three vertical rows are split exact. Thus the bottom horizontal row is exact. If $K$ is perfect then $B'(K[x]) = 0$ and $B'(R[x]) \cong \mathrm{Ref}\,(R[x])/(\mathrm{Ref}\,(R) + \mathrm{Pro}\,(R[x]))$. This completes the proof.

$$\begin{array}{ccccccc}
 & 0 & & 0 & & 0 & & 0 \\
 & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
0 \to & \mathrm{Cl}\,(R)/\mathrm{Pic}\,(R) & \to & \mathrm{Ref}\,(R)/\mathrm{Pro}\,(R) & \to & B(R) & \to & B(K) \\
 & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
0 \to & \mathrm{Cl}\,(R[x])/\mathrm{Pic}\,(R[x]) & \to & \mathrm{Ref}\,(R[x])/\mathrm{Pro}\,(R[x]) & \to & B(K[x]) & \to & B(K[x]) \\
 & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 & 0 & \to & \mathrm{Ref}'\,(R[x]) & \to & B'(R[x]) & \to & B'(K[x]) \\
 & & & \downarrow & & \downarrow & & \downarrow \\
 & & & 0 & & 0 & & 0
\end{array}$$

Let $R$ be a commutative von Neumann regular ring and let $\mathfrak{S}$ be the collection of ideals $I$ in $R$ maximal with respect to the property that $I$ is generated by idempotents. Then $\mathfrak{S}$ is the set of maximal ideals of $R$ and $R$ is isomorphic to a subring of $\prod_{I \in \mathfrak{S}} R/I$ (direct product).

THEOREM 8. *Let $R$ be a commutative von Neumann regular ring, then $B(R) \cong B(R[x])$ if and only if $R/I$ is a perfect field for each maximal ideal $I$ of $R$. If $R$ is a Boolean ring then $B(R[x]) = 0$.*

*Proof.* Consider the diagram

$$\begin{array}{ccccc}
0 \longrightarrow & B(R) & \longrightarrow & \prod_{I \in \mathfrak{S}} B(R/I) \\
 & \uparrow & & \uparrow \\
0 \longrightarrow & B(R[x]) & \longrightarrow & \prod_{I \in \mathfrak{S}} B(R/I[x]) \\
 & \uparrow & & \uparrow \\
0 \longrightarrow & B'(R[x]) & \longrightarrow & \prod_{I \in \mathfrak{S}} B'(R/I[x]) \\
 & \uparrow & & \uparrow \\
 & 0 & & 0
\end{array}$$

The idempotents of $R[x]$ are precisely the idempotents of $R$ so $\mathfrak{S}$ is the Boolean spectrum of $R$ and the set $\{I \cdot R[x] \mid I \in \mathfrak{S}\}$ is the Boolean spectrum of $R[x]$. In [15] it is shown that the first two rows of the diagram are exact, thus the third row is exact. If $R/I$ is perfect for all $I \in \mathfrak{S}$ then $\prod_{I \in \mathfrak{S}} B'(R/I[x]) = 0$ and $B'(S[x]) = 0$.

If $B'(R[x]) \neq 0$ then $B'(R/I[x]) \neq 0$ for some $I$. For this $I$ the field $R/I$ is not perfect. If $R$ is a Boolean ring then $R$ is von Neumann regular and $R/I$ is the field with two elements for each $I \in \mathfrak{S}$. Thus

$B(R) \cong B(R[x])$ and $B(R) = (0)$ so $B(R[x]) = 0$.

2. First we consider two examples also contained in [14]. Let $F$ be the algebraic closure of the field with $p$-elements and let $R = F[t]$ be the ring of polynomials in one variable over $F$. Then $R$ is a principal ideal domain with a nonperfect quotient field $K$. By Theorem 7 $B'(R[x])$ is a subgroup of $B'(K[x])$. On p. 390 of [3] a nontrivial element of $B'(R[x])$ is given.

Next let $R = [Z\sqrt{2}(s^2+1), s]$ where $s$ is an indeterminate and $Z$ is the ring of rational integers. The integral closure $\bar{R}$ of $R$ is $Z[\sqrt{2}, s]$ and the conductor $c = (s^2+1)\bar{R}$. It is shown in [13] that $B'(R[x]) \cong Z/(2)[t]$ but that $B(K/R) \cong B(\bar{R}/R) = 0$ where $K$ is the quotient field of $R$. Now $K$ is a field of characteristic $= 0$ so $B'(K[x]) = 0$ and $B(K[x]) \to B(K(x))$ is a monomorphism.

Apply Lemma 5 with $S = K$ and we have

$$B'(R[x]) \cong B(K[x]/R[x]) \cong B(K[x]/\bar{R}[x]) \cong Z/(2)[t] .$$

Thus $R$ is a domain with $B(R) \to B(K)$ a monomorphis yet $B(R[x]) \to B(K[x])$ has an infinite kernal.

Next let $Z$ denote the rational integers, it is well known that $B(Z) = 0$. Thus $B(Z[x]) = 0$ since $Z$ is a principal ideal domain of characteristic $= 0$. However, $B(Z[x]/(2x-1)) = B(Z[1/2]) \neq 0$ since the ordinary quaternion algebra represents a nonzero class in $B(Z[1/2])$.

Now let $L$ be a finite extention of the perfect field $k$. Then $L \cong k[x]/(p(x))$ for some irreducible polynomial $p(x) \in k[x]$. Choose $k, L$ so that the natural map $B(k) \to B(L)$ is not onto. Then one has the commuting diagram

$$\begin{tikzcd} B(k[x]) \arrow[r] & B(L) \\ & B(k) \arrow[ul] \arrow[u] \end{tikzcd}$$

Since $B(k) \cong B(k[x])$ we have $B(k[x]) \to B(L)$ is not onto. The ideal generated by $p(x)$ is maximal in $k[x]$.

Next let $\bar{Z}_p$ be the $p$-adic completion of the ring $Z$ of integers for a finite prime $p$. Let $R$ be the completion of the localization at $(p)$ of $\bar{Z}_p|t|$. We get the following diagram

Also $R$ is a principal ideal domain of characteristic $= 0$ so $B'(R[x]) = 0$. Since $B(R/(p)) \cong B(R)$ by Theorem 2 we have $B(R/(p) \cong B(R) \cong B(R[x])$, and thus $K = 0$. However $R/(p)$ is not a perfect field so $B'(R/(p)[x]) \neq 0$ (p. 390 of [3]). Thus $B(R[x] \to B(R/(p)[x])$ is not onto.

$$\begin{tikzcd}
 & & 0 & 0 & \\
 & 0 \arrow[r] & B(R) \arrow[u] \arrow[r] & B(R/(p)) \arrow[u] \arrow[r] & 0 \\
0 \arrow[r] & K \arrow[u] \arrow[r] & B(R[x]) \arrow[u] \arrow[r] & B(R/(p)[x]) \arrow[u] & \\
0 \arrow[r] & K \arrow[u] \arrow[r] & B'(R[x]) \arrow[u] \arrow[r] & B'(R/(p)[x]) \arrow[u] & \\
 & 0 \arrow[u] & 0 \arrow[u] & 0 \arrow[u] &
\end{tikzcd}$$

If $R$ is a local ring with maximal ideal $m$ is the induced homomorphism $B(R) \rightarrow B(R/m)$ onto? If $R$ is a Noetherian integrally closed domain with characteristic $R = 0$ is $B(R) \cong B(R[x])$? If $R$ is a Noetherian integrally closed domain is $\mathrm{Ref}'(R[x]) = 0$?

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