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EMBEDDINGS OF SHAPE CLASSES OF COMPACTA IN THE TRIVIAL RANGE

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We show that for compacta X, $Y \subset R^n$, $n \geqq 5$, satisfying the small loops condition and having dimensions in the trivial range with respect to n, $Sh(X) = Sh(Y)$ if and only if $R^n - X \approx R^n - Y$. As a corollary we obtain the following result whose statement is void of shape: If X, $Y \subset R^n$, $n \geqq 5$, are homeomorphic compacta satisfying the small loops condition and having dimensions in the trivial range with respect to n, then $R^n - X \approx R^n - Y$.

1. Main results and introduction. In this paper we are concerned with the general problem of classifying the collection C_Z of compacta in a space Z for which the following property holds: $Sh(X) = Sh(Y)$ is equivalent to $Z - X \approx Z - Y$ ($\approx$ means "is homeomorphic to") for all X, $Y \in C_Z$. Our results apply to compacta in R^n whose dimensions are in the trivial range with respect to n. After defining a fundamental homotopy condition and stating our main results, we will discuss some related work.

Let X be a compactum in a manifold M. We say that X satisfies the *small loops condition* (SLC) if for any neighborhood U of X, there is a neighborhood V of X in U and an $\epsilon > 0$ such that each map of S^1 into $V - X$ of diameter less than ϵ is null homotopic in $U - X$. We say that k is in the *trivial range* with respect to (w.r.t.) n if $2k + 2 \leqq n$ (or equivalently $k \leqq [n/2] - 1$).

THEOREM 1. *Let X, $Y \subset R^n$, $n \geqq 5$, be compacta satisfying* SLC *whose dimensions are in the trivial range* w.r.t. n. *Then, $Sh(X) = Sh(Y)$ if and only if $R^n - X \approx R^n - Y$.*

Theorem 1 generalizes the main result of [6] which we recapture in the following corollary. (The paper [6] improved upon [4] which was the first trivial range work of the nature of Theorem 1.) The 1–ULC hypothesis of Geoghegan and Summerhill is a local condition that traditionally has been used to show that topological embeddings are flat or unknotted, whereas the SLC, the cellularity criterion, and the global 1–alg property are three intimately related (see Proposition 1.5 of [5]) global conditions that have traditionally been used to show that topological embeddings have nice complements (are weakly flat) or homeomorphic complements. We will give an example illustrating this point following Corollary 3.

COROLLARY 1. (Geoghegan and Summerhill) *Let* X, $Y \subset R^n$, $n \geqq 5$, *be compacta whose dimensions are in the trivial range* w.r.t. *n such that* $R^n - X$ *and* $R^n - Y$ *are* 1–ULC. *Then,* $Sh(X) = Sh(Y)$ *if and only if* $R^n - X \approx R^n - Y$.

The next two corollaries are purely topological, i.e., their statements are void of shape. However, it is interesting to note that an attempt to prove these corollaries directly seems to naturally lead one into shape theory!

COROLLARY 2. *Let* X, $Y \subset R^n$, $n \geqq 5$, *be compact* ANR's *whose dimensions are in the trivial range* w.r.t. *n and that satisfy the* SLC. *Then, X and Y have the same homotopy type if and only if their complements are homeomorphic.*

COROLLARY 3. *Let* X, $Y \subset R^n$, $n \geqq 5$, *be homeomorphic compacta whose dimensions are in the trivial range and that satisfy* SLC. *Then,* $R^n - X \approx R^n - Y$.

It is now known that two cells in R^n with 1–ULC complements are equivalently embedded, and that two cells in R^n satisfying the cellularity criterion **[8]** have homeomorphic complements. (In fact, **[8]** establishes Theorem 1 for compacta with trivial shape.) Analogously, Bryant **[2]**, **[3]** and Štan'ko **[13]** have shown that two trivial range compacta in R^n with 1–ULC complements are equivalently embedded, and Corollary 3 shows that two trivial range compacta in R^n satisfying the SLC have homeomorphic complements.

By considering embeddings of shape classes, we are able to prove "weak flattening" theorems for a wide class of compacta. Also, shape theory allows us to obtain converses for "weak flattening" theorems, i.e., under certain conditions, homeomorphic complements implies equal shape. In our opinion, this is one important justification for the theory of shape.

The following trivial range theorem can be proved by using Theorem 2.4 of **[5]** and some work of Wall **[14]**: If X, $Y \subset R^n$, $n \geqq 5$, are compacta in the trivial range that satisfy the SLC and that have the shape of a finite complex, then $Sh(X) = Sh(Y)$ implies that $R^n - X \approx R^n - Y$. (Ross Geoghegan pointed out the relevance of **[14]** to this result.) Although this theorem replaces the 1–ULC hypothesis of Geoghegan and Summerhill with the more desirable SLC, it adds the condition that the compacta have the shape of a finite complex. It is the purpose of this paper to show that that condition is unnecessary and to prove the converse.

Notice that one direction of Corollary 2 follows from the theorem mentioned in the last paragraph combined with the recent result of West **[15]** that every compact metric ANR has finite type. Our proof will not use **[15]**.

The authors would like to thank John Bryant for a key observation concerning the proof of Theorem 1 for the case $n = 5$.

2. Proof of Theorem 1. We begin by making a definition. Let $X \subset R^n$ be a compactum whose dimension k is in the trivial range w.r.t. n. Then, X is *homotopically stable* in R^n if given a neighborhood U of X, there is a neighborhood $V \subset U$ of X such that regardless of base point $\pi_i(V, V - X) \approx 0$ for $0 \leqq i \leqq n - [n/2]$. (We will ignore the base point in this paper.)

Before proving our first lemma, let us state a couple of sublemmas. The first sublemma follows from Theorem 1.3 of **[5]**.

SUBLEMMA 1. *If* $X \subset R^n$, $\dim X \leqq n - 3$, *is a compactum which satisfies the* SLC, *then* $\pi_i(U, U - X) \approx 0$, $i = 0, 1, 2$, *for each neighborhood* U *of* X.

The next sublemma follows from Theorem 10 (page 342) of **[10]**. (H_* denotes singular homology with Z coefficients.)

SUBLEMMA 2. *Let* V *be an orientable* n*-manifold and let* $X \subset V$ *be a closed subset such that* $\dim X = k$. *Then,* $H_i(V, V - X) \approx 0$ *for* $i \leqq n - k - 1$. (*Thus, if* $k \leqq [n/2] - 1$, *then* $H_i(V, V - X) \approx 0$ *for* $i \leqq n - [n/2]$.)

LEMMA 1. *Let* $X \subset R^n$ *be a compactum,* $\dim X = k \leqq n - 3$, *that satisfies the* SLC. *Then,* X *has arbitrarily small neighborhoods* V *such that* $\pi_i(V, V - X) \approx 0$ *for* $i \leqq n - k - 1$. (*Thus, if* $k \leqq [n/2] - 1$, *then* X *is homotopically stable.*)

Proof. Choose an arbitrarily small neighborhood V of X satisfying Sublemma 1. Let $\tilde{V}$ denote the universal cover of V with projection $p: \tilde{V} \to V$. Denote $p^{-1}(X)$ by $\tilde{X}$. Since $\pi_2(V, V - X) \approx 0$, it follows from the homotopy lifting property that $\tilde{V} - \tilde{X}$ (denoted by $\widetilde{V - X}$) is the universal cover of $V - X$ with projection $p \mid (\tilde{V} - \tilde{X})$. It follows from Sublemma 1 and the homotopy lifting property that $\pi_i(\tilde{V}, \widetilde{V - X}) \approx 0$ for $i = 0, 1, 2$. Sublemma 2 implies that $H_i(\tilde{V}, \widetilde{V - X}) \approx 0$ for $i \leqq n - k - 1$. Hence, the relative Hurewicz Theorem yields $\pi_i(\tilde{V}, \widetilde{V - X}) \approx 0$ for $i \leqq n - k - 1$. For $i = 3, 4, \cdots, n - k - 1$, consider the diagram

$$\begin{array}{ccccccccc}
\pi_i(\widetilde{V - X}) & \to & \pi_i(\tilde{V}) & \to & 0 & \to & \pi_{i-1}(\widetilde{V - X}) & \to & \pi_{i-1}(\tilde{V}) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\pi_i(V - X) & \to & \pi_i(V) & \to & \pi_i(V, V - X) & \to & \pi_{i-1}(V - X) & \to & \pi_{i-1}(V).
\end{array}$$

An application of the five lemma to this diagram finishes the proof.

For completeness, we will briefly sketch a classical type proof, based on some remarks in [2], of the next lemma. Alternately, one can prove Lemma 2 by techniques in §3 of [6] and we prefer that method to the more classical one outlined below. Let M_n^{n-3} denote the set of points in R^n at most $n-3$ of whose coordinates are rational.

LEMMA 2. *Let $X \subset R^n$ be a k-dimensional compactum, $2k+1 \leqq n$. Then, given $\epsilon > 0$, evey map of X into R^n can be ϵ-approxin ated by an embedding g such that $R^n - g(X)$ is* 1–ULC.

Proof. Let $M(X, Y)$ $[E(X, Y)]$ denote the set of all mappings [embeddings] of X into Y. Then, $E(X, R^n)$ is a dense G_δ subset of $M(X, R^n)$, c.f., page 57 of [7]. The proof of Theorem V5 of [7] shows that $M(X, M_n^{n-3})$ is a dense G_δ subset of $M(X, R^n)$. Thus, $E(X, M_n^{n-3}) = E(X, R^n) \cap M(X, M_n^{n-3})$ is a dense G_δ subset of $M(X, R^n)$. But if $g \in E(X, M_n^{n-3})$, then $R^n - g(X)$ is 1–ULC by the proof of Theorem 2 of [2].

REMARK. M. A. Štan'ko [12] has shown that every embedding of a k-dimensional compactum X, $k \leqq n-3$, into R^n can be approximated with an embedding g such that $R^n - g(X)$ is 1–ULC. This result could be used to replace the last part of the above proof.

LEMMA 3. *Let X be a homotopically stable compactum in R^n, $n \geqq 5$, and let $f: X \to R^n$ be a map such that* $\operatorname{dist}(x, f(x)) < \epsilon$ *for each $x \in X$. Let 0 be an open set containing $f(X)$. Then, there is a homeomorphism $h: R^n \to R^n$ such that*

(1) *h = identity outside of the ϵ-neighborhood of X,*
(2) *$h(X) \subset 0$, and in fact*
(3) *$h \mid X \simeq f$ in 0 (the homotopy may oscillate badly in 0).*

Proof. Case 1. ($n \geqq 6$): (The first part of this is similar to Lemma 2 of [2], however, our hypothesis is too weak to insure that h can be chosen so that $h \mid X$ approximates f. Consequently, the existence of the homotopy specified by (3) is crucial.)

Extend f to a map $\bar{f}$ of R^n to R^n. Let N be a small polyhedral neighborhood of X equipped with a triangulation T of small mesh. (We require that $\bar{f}(N) \subset 0$.) Let T_* be the $([n/2]-1)$-skeleton of T and T^* be the dual $(n-[n/2])$-skeleton. Approximate $\bar{f} \mid N$ with a PL map $\hat{f}$ that is in general position with respect to T (cf. Theorem 1.6.10 of [13]). Then, $\hat{f} \mid X$ is homotopic to f inside of 0 and $\hat{f} \mid T_*$ is a PL embedding. Now there is an ϵ-push h_1 of (E^n, T_*) by [1] such that $h_1 \mid T_* = \hat{f} \mid T_*$.

Since X is homotopically stable, we may employ Stallings' engulfing Theorem **[11]** to shift X off of T^* with a homeomorphism h_2, where h_2 is ambient isotopic to the identity by an isotopy that is fixed outside of a small neighborhood $W \subset N$ of X. (Of course this isotopy may oscillate fiercely inside of W.) Now we may use the local join structure between T^* and T_* to obtain a small push h_3 of $(R^n, h_2(X))$ such that $h_3 h_2(X)$ is very close to T_*. Let $h = h_1 h_3 h_2$. By construction, h satisfies (1) and (2). It remains to see that $h \mid X \simeq f$ in 0.

We have already observed that $f \simeq \hat{f} \mid X$ inside of 0 and so it will suffice to show $\hat{f} \mid X \simeq h \mid X$ inside of 0. First notice that the identity $1_X: X \to X$ is homotopic (in fact, pseudo-isotopic) to a map $p: X \to T_*$ by a homotopy r_t whose track lies in N. In particular, let r_t, $0 \leqq t \leqq \frac{1}{2}$, be the restriction to X of the ambient isotopy that realizes h_2, and let r_t, $\frac{1}{2} \leqq t \leqq 1$, be the pseudo-isotopy obtained by pushing $h_2(X)$ into T_* straight down the local join structure between T^* and T_*. Now, notice that $\hat{f} \mid X$ is homotopic to $\hat{f}p = h_1 p$ by the homotopy $\hat{f}r_t$ that takes place in $\hat{f}(N) \subset 0$.

We will be through if we also show that $h \mid X$ is homotopic to $\hat{f}p = h_1 p$ inside 0. Well, $h_3 h_2 \mid X$ is certainly homotopic to p via the pseudo-isotopy g_t which slides $h_3 h_2(X)$ straight down the local join structure. But then $h_1 h_3 h_2 \mid X = h \mid X$ is homotopic (inside 0) to $h_1 p$ by the homotopy $h_1 g_t$ as desired.

Case 2. ($n = 5$): (the proof of this case involves a modification of the techniques of proof of Lemmas 1, 2 and 3 of **[3]**. Again we cannot hope to have $h \mid X$ approximate f, consequently the existence of the homotopy specified by (3) is important.)

Extend f to a map $\bar{f}$ of R^n to R^n. Let N be a polyhedral neighborhood of X such that $f(N) \subset 0$. We now make a claim.

CLAIM A. There exists a cover of X of order 1 by simply connected open sets $U_1, \cdots, U_r$ such that $U_i \cap U_j$ is connected for each $i, j = 1, \cdots, r$. Furthermore, $U_i \subset N$, $i = 1, \cdots, r$.

The proof of Claim A follows the lines of proof of Lemma 1 of **[3]**, with one modification necessary. This modification occurs at the point where we want to kill $\pi_1(\partial M_i)$ by the surgery trick of trading 2-handles. In particular, suppose we have reached the stage that we have a PL simple closed curve C in ∂M_1 which represents a generator of $\pi_1(\partial M_1)$ and suppose we have the PL 2-cell D in N such that $\partial D = C$. At this point, Bryant uses the fact that $E^n - X$ is 1–ULC to shift D off of X with a small shift. We may use our homotopically stable hypothesis and Stallings' engulfing to shift D off of X by a shift which takes place in a small neighborhood of X. The proof now proceeds as in Lemma 1 of **[3]**. Although we cannot conclude that the resulting U_i's are small, as in **[3]**, we nevertheless establish Claim A.

Consider the cover $\{U_1, \cdots, U_r\}$ of X given by Claim A. One may piecewise linearly embed the nerve P of $\{U_i\}$ in $U = \bigcup_{i=1}^{r} U_i$ as in **[3]**. Approximate $\bar{f}|N$ with a PL map $\hat{f}$ that is in general position with respect to P. Then, $\hat{f}|X$ is homotopic to f inside of 0 and $\hat{f}|P$ is a PL embedding. Now there is an ϵ-push h_1 of (E^n, P) by **[1]** such that $h_1|P = \hat{f}|P$.

Let W be a regular neighborhood of P in N such that $h_1(W) \subset$ 0. *(Then W deformation retracts to P.)* This brings us to another claim.

CLAIM B. Suppose that $\{U_1, \cdots, U_r\}$, P, and W are as above. Then, there is an ambient isotopy e_t of R^n which is the identity outside U such that $e_1(X) \subset W$.

The proof of Claim B is very much like the proof of Lemma 2 of **[3]**; however, a few comments are in order. Notice that it is not necessary that the V_{ij}'s be simply connected (only connected). We would be in trouble if this were necessary! We can construct the homotopy $g: M^2 \times I \to U$ even though the U_i's are not small; however, g will not necessarily be an ϵ-homotopy. The properties of $\{U_i\}$ allow us to apply Zeeman piping to get an ambient isotopy e_t^1 of E^n such that e_t^1 is the identity outside U and $e_1^1(W) \supset g(M^2 \times 0)$. The homotopically stable hypothesis and Stallings' engulfing suffice to yield and ambient isotopy e_t^2 that moves only in a small neighborhood of X and that shifts X off of $\tilde{M}^{n-3}$. *Let e_t^3 be the isotopy pushing across the join structure between* M^2 and $\tilde{M}^{n-3}$ that carries $e_1^2(X)$ into $e_1^1(W)$. Then, e_t may be taken to be the composition of the three isotopies e_t^2, e_t^3 and e_t^1, respectively.

We now define h to be $h_1\, e_1$. It is clear that h satisfies (1) and (2); hence, it remains only to establish (3). We have already observed that $f \simeq \hat{f}|X$ inside of 0 and so it will suffice to show that $\hat{f}|X \simeq h|X$ inside of 0. First notice that the identity $1_X: X \to X$ is homotopic to a map $p: X \to P$ by a homotopy r_t whose track lies in N. In particular, r_t is the homotopy $e_t|X$ followed by a deformation retraction of W to P. Notice that $\hat{f}|X$ is homotopic to $\hat{f}p = h_1 p$ by the homotopy $\hat{f}r_t$ which takes place in $\hat{f}(N) \subset 0$.

We will be through if we also show that $h|X$ is homotopic to $\hat{f}p = h_1 p$ inside 0. Well, $e_1|X$ is certainly homotopic to p via $g_t e_1$ where g_t is the deformation retraction of W to P. But then $h_1 e_1|X = h|X$ is homotopic (inside 0) to $h_1 p$ by the homotopy $h_1 g_t e_1$ as desired.

LEMMA 4. *Let X be a homotopically stable compactum in R^n, $n \geqq 5$, and let $f: X \to R^n$ be a map which is homotopic to 1_X in an open neighborhood U of $X \cup f(X)$. Then, given an open set V where $f(X) \subset V \subset U$, there is a homeomorphism $h: R^n \to R^n$ such that*

(1) *h = identity outside of U,*

(2) $h(X) \subset V$, *and in fact*
(3) $h \mid X \simeq f$ *in* V.

Proof. Let e_t denote the homotopy connecting 1_X and f. Let ϵ be one-half the distance between the track of e_t and $R^n - U$. Then, by using Lemma 2 to approximate f and enough levels of e_t, we may obtain a sequence of embeddings $f_1, f_2, \cdots, f_k$ of X into R^n satisfying the following properties.

(1) $f_1 = 1_X$
(2) $f_k \simeq f$ in V
(3) $E^n - f_i(X)$ is 1–ULC for $i = 1, 3, \cdots, k$
(4) $d(f_i, f_{i+1}) < \epsilon$ for $i = 1, 2, \cdots, k-1$, and
(5) $d(f_i(X), R^n - U) < \epsilon$ for $i = 1, 2, \cdots, k$.

Now, by applying **[2]** or **[3]** $(k-2)$-times, we obtain a homeomorphism h_1: $R^n \to R^n$ such that

(1) $h_1 f_2 = f_k$
(2) $h_1 \mid R^n - U =$ identity.

But now we are in position to apply Lemma 3 where $(X, f, R^n, 0, \epsilon)$ of that lemma corresponds to $(X, f_2, R^n, h_1^{-1}(V), \epsilon)$. Let h_2 be the resulting homeomorphism of R^n. Then, it is clear that $h = h_1 h_2$ is the desired homeomorphism.

The following lemma corresponds to Lemma 4.1 of **[6]**. (It is perhaps worth mentioning here that the statement of Lemma 4.1 of **[6]** as well as the statements of several other results in **[6]** are redundant in that the requirement "strong Z_{n-k-2}-set $(k \geqq 0, n \geqq 2k+2)$" could be replaced by "strong $Z_{[n/2]-1}$-set.")

LEMMA 5. *Let* $X, Y \subset R^n$, $n \geqq 5$, *be homotopically stable compacta whose dimensions are in the trivial range such that* $Sh(X) = Sh(Y)$. *Let* $\{f_i, X, B\}$ *and* $\{f'_i, Y, A\}$ *be fundamental sequences (in* R^n*) which are homotopy inverses to one another. Let* U_0 *be a neighborhood of* X *and let* h *be a homeomorphism of* R^n *such that* $Y \subset h(U_0)$ *and* $h^{-1} \mid Y$ *is homotopic to* $f'_i \mid Y$ *in* U_0 *for almost all* i. *Then, for every neighborhood* V_0 *such that* $Y \subset V_0 \subset h(U_0)$, *there is a homeomorphism* q *of* R^n *such that* $q = h$ *outside* U_0, $X \subset q^{-1}(V_0)$ *and* $q \mid X$ *is homotopic to* $f_i \mid X$ *in* V_0 *for almost all* i.

Proof. Construct $\hat{f}$: $X \to V$ such that $\hat{f} \simeq h \mid X$ in $h(U_0)$ just as is done in the proof of Lemma 4.1 of **[6]**. Now appeal to Lemma 4 to obtain the desired q.

Proof of Theorem 1. First assume that $Sh(X) = Sh(Y)$. Lemma 1 assures us that X and Y are homotopically stable. One can show that $R^n - X \approx R^n - Y$ by meshing a basic system $\{T_i\}$ of neighborhoods for X

with a basic system $\{W_i\}$ of neighborhoods for Y just as is done in the proof of Lemma 4.2 of [6]. In that process, our Lemma 3 plays the role of Lemma 4.1 of [6].

Now assume that $R^n - X \approx R^n - Y$. By Lemma 2, we can find copies $\tilde{X}$ and $\tilde{Y}$ of X and Y, respectively, in R^n such that $R^n - \tilde{X}$ and $R^n - \tilde{Y}$ are 1–ULC (hence satisfy the SLC). Now the direction of Theorem 1 already proved implies that $R^n - \tilde{X} \approx R^n - X$ and $R^n - \tilde{Y} \approx R^n - Y$. Theorem 1.2 of [6] implies that $Sh(\tilde{X}) = Sh(\tilde{Y})$. Therefore, $Sh(X) = Sh(Y)$ as desired.

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