

ERRATA

Correction to

LOCALE GEOMETRY

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As regards [1] §4 (examples), the categories Lc and Lpc are not correctly defined. The equality on page 334 (line 4) should read $X'(fx, fy) \geq X(x, y)$. The equality on page 334 (line 9) should read $(fx, fy, fz) \geq (x, y, z)$. The equality in axiom C 5 should read:

$$C 5. \quad \sup_w (x, y, w) \wedge (x, z, w) \leq X(y, z) \vee (x, y, z) \vee (x, z, y),$$

with appropriate alterations to the lemmas. With this larger class of geometries the results of the article remain valid. Moreover, the new category inclusion $Lc \subset Lpc$ satisfies the colimit claims of §4 and, consequently, has a right adjoint (whose counit is a set bijection). It can be shown (using the above C 5) that $E_n \cong E_1^n$ in Lc when $L = 2$ and E_n denotes n -dimensional Euclidern space with the usual 2-valued convexity structure.

Correction to

REGULAR FPF RINGS

S. PAGE

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In [2] Proposition 3 states that for a left FPF left nonsingular ring any left ideal is essential in a direct summand of the ring. Unfortunately the proof is lacking as was pointed out by E. P. Armendarie. The proof given only works for two sided ideals. The final results of the paper are in fact valid. The arguments of [2] do characterize the left self-injective left FPF regular rings. It is also easy to see (as is pointed out in [2]) that a strongly regular left FPF is left-injective. In [3] it is shown that if R is nonsingular and left FPF, then $Q(R)$, the maximal left quotient ring is also left FPF. So we know the structure of the maximal quotient ring. We will show that, if R is a left EPF regular ring, Proposition 3 does hold.

In what follows R is a ring with zero singular left ideal and maximal left quotient ring Q . We first show that $Q \otimes_R Q \cong Q$ by establishing the following lemma:

LEMMA A. *Let R be a left nonsingular left FPF ring and let $q \in Q$. Then $R + Rq$ embeds in a finitely generated free module.*

Proof. An idempotent e in Q is called abelian if for R -submodules I and J of Qe such that $I \cap J = 0$, $\text{Hom}_R(I, J) = 0$. Now each idempotent of Q can be written as a finite sum of orthogonal abelian idempotents because Q is a self-injective regular ring of bounded index. The injective hull of Rq is Qe for some idempotent e . Let $e = \sum_{i=1}^n e_i$, where the e_i 's are abelian and orthogonal. Clearly, $R + Rq$ embeds in $R(1 - e) \oplus \sum_{i=1}^n (Re_i + Rqe_i)$. Next look at $Re_i + Rqe_i \subset Qe_i$. We will show that $Re_i + Rqe_i$ embeds in a free module for each i . To this end, for convenience, we will assume e is abelian. Now we can reduce to the case where Re is faithful. To do this note that the left annihilator of $Re + Rqe$, ${}^{\perp}(Re + Rqe)$, is ${}^{\perp}((Re + Rqe)R)$, a two sided ideal. The two sided version of Proposition 3 of [2] implies that $R \cong R_1 \times R_2$ where $(Re + Rqe)R$ is essential in R_1 . We can, therefore, assume without loss of generality that $R = R_1$. This makes Re faithful and so $Re + Rqe$ is a generator. This gives the existence of functions f_1, \dots, f_K , to R so that $R = \sum_{i=1}^K \text{Image } f_i$. Let $W = \bigcap_{i=1}^K \ker f_i$. Let F be the sum of K copies of R , and $Q(F)$ the canonical hull of F . Let f be the map of $Re + Rqe$ to F given by f_i on the i th coordinate. We have $W = \ker f$. Since everything in sight is nonsingular, W is not essential in $Re + Rqe$. Let $W \oplus U$ be essential in $Re + Rqe$. Since $1 \in \sum_{i=1}^K \text{Im } f_i$, there exists r_1, r_2 in R so that for $w \neq 0$ in W , $wf_i(r_1e + r_2qe) \neq 0$ for some i . Also since the image of U is essential in $\text{im } f$, we see that $Wf(U) \neq 0$, in $Q(F)$. It follows, because all modules under consideration are nonsingular, that for some nonzero submodule $W_1 \subset W$, $\text{Hom}_R(W_1, U) \neq 0$, which contradicts the fact that e was abelian, unless $W = 0$. The fact that $W = 0$ implies that f_i 's give rise to an embedding. Finally, treat $R(1 - e)$ in the same way.

THEOREM. *Let R be a left nonsingular left FPF ring. Then Q is flat as a left R module and $Q \otimes Q \cong Q$.*

Proof. Lemma A gives the essential ingredients to apply the proof of Theorem 5.17 [1].

PROPOSITION. *Let R be a regular left FPF ring. Let $e = e^2 \in Q$. Then Re is a projective R module.*

Proof. By Theorem 2.8 of [4] it suffices to show $Q \otimes_R Re$ is a Q projective. Now we have $0 \rightarrow Re \rightarrow Q$ exact and Q is flat over R , so $0 \rightarrow Q \otimes Re \rightarrow Q \otimes Q$ is exact. The isomorphism $Q \otimes Q \cong Q$ gives $Q \otimes Re \cong Qe$, and hence is Q projective.

COROLLARY. *For any idempotent $e \in Q$, $Re \cap R$ is a summand of R .*

Proof. The sequence $0 \rightarrow Re \cap R \rightarrow R \rightarrow R(1 - e) \rightarrow 0$ splits.

We can now prove Proposition 3 of [2] for regular FPF rings. If L is a left ideal of R , then L is essential in a summand Qe of Q . Hence L is essential in Re , hence essential in $Re \cap R$, a summand of R .

REFERENCES

1. K. R. Goodeal, *Ring Theory*, Mono. and Text in Pure and Applied Math., **33**, Marcel Dekker, New York.
2. S. Page, *Regular FPF Rings*, Pacific J. Math., **79** (1978), 169-176.
3. ———, *Semi-prime and non-singular FPF rings*, to appear.
4. F. L. Sandomierski, *Nonsingular rings*, Proc. Amer. Math. Soc., **19** (1968), 225-230.

Correction to

ON EQUISINGULAR FAMILIES OF ISOLATED SINGULARITIES

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Theorem 3.1 is incorrect. There are families of plane curves which are Zariski equisingular but do not satisfy condition \mathcal{E} . The error is in the proof of Lemma 3.5. In fact, as the example below shows, there are parametrized families of space curves, where the special fiber is not obtained by specializing the values of the parameters, but has embedded points. The arguments of the rest of the section are correct, and they give the following weaker result (we use the notations of the paper).

THEOREM. *Let $(X_0, 0)$ be a germ of a reduced plane curve, with the following property: there is a representative $\mathcal{X} = (\zeta, X_\mu, D_\mu, \sigma)$ of the versal μ -constant deformation of X_0 such that for all $u \in D_\mu$, $f^{-1}(u)$ coincides with the H -transform of $\zeta^{-1}(u)$ where $Z^\pi \rightarrow X_\mu$ is the*